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The birth of wave mechanics (1923–1926)

La naissance de la mécanique ondulatoire

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ABSTRACT

In 1923, in three articles published in the *Comptes Rendus of the Académie des Sciences*, Louis de Broglie proposed the concept of wave–particle duality. Physicists from many countries seized upon this idea. In particular, Schrödinger developed de Broglie's qualitative idea by writing down the equation that the wave must satisfy in the non-relativistic approximation. A relativistic version of this equation was proposed in 1926 by several scientists, and other ones found a solution to the Schrödinger equation as an expansion in powers of the Planck constant.

RÉSUMÉ

En 1923, dans trois articles publiés dans les *Comptes rendus de l'Académie des sciences*, Louis de Broglie proposa le concept de dualité onde–particule. Les physiciens de multiples pays s'emparèrent de cette idée. En particulier, Schrödinger précisa l'idée qualitative de de Broglie en écrivant l'équation à laquelle l'onde doit satisfaire dans l'approximation non relativiste. Une version relativiste de cette équation fut proposée en 1926 par plusieurs chercheurs et, la même année, d'autres chercheurs obtinrent une solution de l'équation de Schrödinger comme série de puissances de la constante de Planck.

Quantum physics was born in 1900, when Planck found a formula for the spectrum of the black-body radiation by making the hypothesis that the energy exchanged between radiation and material proceeds by quanta $E = h\nu$, and determined the value of the constant h by fitting the experimental results with his formula. But it was Einstein who realized the revolutionary character of the idea of quantization [1], and in 1905 he wrote the famous paper where he introduces the notion of *Lichtquanten*, quantum of light, which was named "photon" twenty years later. To accommodate the many optical phenomena that can be understood only assuming that light is a wave, as shown by Young, Fresnel, and Maxwell, he introduced the idea of wave–particle duality of light as early as 1911, in the famous Salzburg conference [2].

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After the groundbreaking discovery by Bohr, in 1913, that the idea of quantization allows one to render an account of the spectrum of the hydrogen atom, quantum mechanics developed as a heuristic science, which made use of various ad hoc assumptions to explain various phenomena. It was de Broglie who, in 1923, initiated a series of progress that would put the formalism of quantum mechanics on a solid ground, by introducing the concept of matter waves.

In the year 1923, the 31-year-old Louis de Broglie wished to "extend to all particles the coexistence of waves and particles discovered by Einstein about light." In three notes [3] indeed, he justified this duality statement and established the relation between the momentum p of a freely propagating particle and the wavelength λ of the associated wave, now known as the de Broglie relation[1]:

$$\lambda = h/p \tag{1}$$

where h is the Planck constant.

The derivation given by de Broglie, based on special relativity, is quite different from that given in modern textbooks. It is more clearly explained in his thesis [4] and more clearly yet, in English, in his Nobel lecture [5]. It will be summarized now.

In analogy with Einstein's postulate that relates the energy E of a photon with its frequency ν by the relation $E = h\nu$, de Broglie associates with a particle at rest a wave function

$$\Psi = \exp\left(\frac{2\pi \mathrm{i}}{h} m_0 c^2 t\right) \tag{2}$$

which makes use of the relativistic energy at rest m_0c^2. If the particle moves at velocity v, the wave function has the same expression, except that the time t is to be replaced by the time t' in the moving frame. Introducing the coordinates x and t in the fixed frame through a Lorentz transformation, the wave function takes the form

$$\Psi = \exp\left[\frac{2\pi \mathrm{i}}{h} \frac{m_0 v}{\sqrt{1-\beta^2}}\left(\frac{c^2 t}{v} - x\right)\right] \tag{3}$$

where $\beta = v/c$.

Since the relativistic expression of the momentum is $p = m_0/\sqrt{1-\beta^2}$, formula (3) corresponds to a wavelength $\lambda = h/p$, which is relation (1). The phase velocity c^2/v is larger than the speed of light c and physically meaningless, but it can be checked [3] that the group velocity $U = \mathrm{d}\nu/\mathrm{d}(\nu/v)$ is equal to v.

After the serendipitous discovery by Davisson and Germer of electron diffraction on a crystal lattice of nickel [6], an undulatory phenomenon associated with a wavelength in agreement with the de Broglie relation (1), de Broglie obtained the Nobel prize, in 1929 (and Davisson got it in 1937).

De Broglie's first two notes [3a,3b] were presented by Jean Perrin, who was going to be awarded the Nobel Prize in 1926. Despite this distinguished support, one may doubt that de Broglie's notes would have been easily accepted by other scientific journals. Indeed, de Broglie's basic idea was to apply his relation to the photon considered as an "atom of light" (*atome de lumière*), supposed to have a very weak, but non-vanishing mass! When de Broglie wanted to transform his theory into a doctoral thesis, the principal examiner was Paul Langevin. According to Whitaker [7] Langevin was skeptical and asked for Einstein's opinion. Einstein's advice was positive, but perhaps not on all points. Eventually, there were no "atoms of light" in de Broglie's thesis [4].

De Broglie's ambition was not restricted to freely propagating particles. He immediately applied the wave description to the electrons in the atom, and re-derived Bohr's quantization rules [3]. Indeed, for a classical motion of a charge $-e$ on a circular orbit with radius r, around the nucleus with charge $+e$, the Coulomb force $e^2/(\pi\varepsilon_0 r^2)$ equilibrates the centrifugal force $p^2/(mr)$ (in the non-relativistic limit). This yields a relation between the radius and the momentum, which can be expressed as h/λ, according to (1). Now, de Broglie imposes the condition that the length of the orbit be an integer number of wavelengths, an "almost necessary" condition of stability of the orbit, as written in the first paper. This selects a discrete series of radii, which are the same as that found by imposing Bohr's quantization condition. This is a remarkable result, which justifies Einstein's positive appreciation of de Broglie's thesis work.

Having a wave, it was natural to ask the question of the wave equation governing the evolution of that wave. It is Schrödinger who solved the problem, in a series of papers [8] published in 1926, just after Heisenberg [9], on the one hand, Born and Jordan [10], on the other one, published their matrix version of quantum mechanics. Modern quantum mechanics was born when Schrödinger demonstrated the equivalence between the matrix formalism and his wave formulation. A remarkable feature of Schrödinger's formalism is the fact that it also yields a treatment of several particles systems, however only in the non-relativistic approximation.

What about relativity? As early as 1926, a relativistic treatment, valid only for a single particle with spin zero, was obtained independently by various authors from seven different countries. Five articles were in German [8d,11–14] and two

[1] Actually relation (1) was not explicitly written in 1923. In Ref. [2b], it was said that the wave has the phase velocity $V = c^2/v$ and the frequency $\nu = m_0c^2(1 - v^2/c^2)^{-1/2}/h$, where m_0 is the mass of the particle at rest and c is the velocity of light. Since $V = \lambda\nu$ and $p = m_0 v(1 - v^2/c^2)^{-1/2}$, relation (1) results. It is explicitly written in de Broglie's Nobel lecture [3].

in French [15,16]. The result is generally known as the Klein–Gordon or Klein–Gordon–Fock equation,[2] though Shiff [17] calls it Schrödinger's relativistic equation. For a free particle (in the absence of potential), the wave function $\Psi(x, y, z, t)$ satisfies

$$\nabla^2\Psi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi = \frac{4\pi^2 m^2 c^2}{h^2}\Psi \tag{4}$$

where m and c have their usual meaning. This relation can be written $p_\mu p^\mu \Psi = (m^2/c^2)\Psi$ where μ denotes the four space-time coordinates. For a particle of charge e in a quadrivector potential A_μ, the momentum p_μ is just to be replaced by $p_\mu - eA_\mu$. The case of the Coulomb interaction created by a point charge can be treated exactly [17]. It corresponds to a π^- meson bound to a nucleus [18]. The resulting fine structure is quite different from that observed in the hydrogen atom (where the particle is an electron and has a spin!) The correct relativistic treatment of the hydrogen atom was made possible in 1928 by Dirac's equation.

In the same year 1926, the solution to Schrödinger's equation as an expansion in powers of the Planck constant h was proposed nearly at the same time by three authors: Wentzel [19], Brillouin [20], and Kramers [21]. Wentzel's and Kramers illustrated their articles by various examples, while Brillouin's note was restricted to the principle of the series expansion, as was usual among French physicists.

It is remarkable that de Broglie, 50 years after his discovery, despite the fantastic success of Schrödinger's equation in theoretical chemistry and atomic physics, was still reluctant to accept Born's probabilistic views [22]. "Usual quantum mechanics gives us but an exact statistical view without revealing us the true nature" [of the coexistence of waves and particles]. The "true nature" that de Broglie had in mind, was the *pilot wave* theory, outlined in a note in 1927 [23] and then elaborated by de Broglie himself and by David Bohm. The de Broglie–Bohm theory is philosophically gratifying for those who would like a deterministic world, and many scientists are still investigating its consequences. However, the probabilistic formalism of quantum mechanics is sufficient for all known applications. Moreover, experimental violations of Bell's inequalities prove that interpretations in the spirit of de Broglie's or Einstein's views either must be rejected, or have a fundamental non-locality built into them [24]. Far from being useless, this fundamental debate has drawn the attention of physicists onto the extraordinary character of the behavior of individual quantum objects, and lead to the present development of quantum technologies.

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[2] Actually the first published article seems to be that of Klein, while Gordon's and Fock's articles are illustrated by applications that are lacking in the other versions.