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Laszlo Tisza and the two-fluid model of superfluidity

*Laszlo Tisza et le modèle à deux fluides de la superfluidité*

Sébastien Balibar

Laboratoire Pierre-Aigrain, Département de physique de l'École normale supérieure (ENS), PSL Research University, Université Paris-Diderot, Sorbonne Paris Cité, Sorbonne Universités, UPMC Université Paris-6, CNRS, 75005 Paris, France

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ABSTRACT

The “two-fluid model” of superfluidity was first introduced by Laszlo Tisza in 1938. On that year, Tisza published the principles of his model as a brief note in *Nature* and two articles in French in the *Comptes rendus de l'Académie des sciences*, followed in 1940 by two other articles in French in the *Journal de physique et le Radium*. In 1941, the two-fluid model was reformulated by Lev Landau on a more rigorous basis. Successive experiments confirmed the revolutionary idea introduced by Tisza: superfluid helium is indeed a surprising mixture of two fluids with independent velocity fields. His prediction of the existence of heat waves, a consequence of his model, was also confirmed. Then, it took several decades for the superfluidity of liquid helium to be fully understood.

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R É S U M É

Le « modèle à deux fluides » de la superfluidité a été introduit en 1938 par Laszlo Tisza. Cette année-là, Tisza publia les principes de son modèle sous la forme d'une note brève dans la revue *Nature* et de deux articles en français dans les *Comptes rendus de l'Académie des sciences*, qui furent eux-mêmes suivis en 1940 de deux autres articles en français dans le *Journal de physique et le Radium*. En 1941, le modèle à deux fluides fut reformulé par Lev Landau sur une base plus rigoureuse. Différentes expériences ont confirmé l'idée révolutionnaire de Tisza : l'hélium superfluide est, en effet, un mélange surprenant de deux fluides, dont les champs de vitesse sont indépendants. Sa prédiction de l'existence d'ondes de chaleur, qui est une conséquence de ce modèle, fut confirmée, elle aussi. Ensuite, il fallut plusieurs décennies pour que la superfluidité de l'hélium soit complètement comprise.

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1. Historical context: superfluidity in 1938

Let me first summarize the historical context of Tisza's work.

E-mail address: sebastien.balibar@lpa.ens.fr.

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On 10 July 1908, Heike Kamerlingh Onnes liquefied helium gas for the first time in his Leiden laboratory [1,2]. Ten years after the discovery of liquid hydrogen by James Dewar, helium was the last so-called “permanent gas”, and Kamerlingh Onnes discovered that it was actually not permanent, since it liquefied at 4 K under atmospheric pressure. By pumping on liquid helium, Kamerlingh Onnes was able to cool it down to 1 K. He described his discovery in two communications to the Netherlands Academy of Sciences (KNAW) [1] and in a note to the *Comptes rendus de l'Académie des sciences* [3], which was received on 17 August and published one week later. He noticed that liquid helium did not freeze, and he did not observe any particular change in the liquid properties down to 1 K, the world temperature record at that time. For this discovery, he was awarded the Nobel prize in 1913. In the meantime, he had also discovered superconductivity. Low temperature physics had begun.

The first indication of a change in the properties of liquid helium was observed in 1932 by John C. McLennan in his Toronto laboratory. At that time, cryostats and refrigerators were made of transparent glass and McLennan noticed that when pumping to cool it down, liquid helium stopped boiling below 2.2 K [4], as if some mysterious order took place below 2.2 K in this extremely cold liquid. Willem H. Keesom was a former student of Kamerlingh Onnes working in the same Leiden laboratory with his daughter Anna, and they studied together the temperature dependence of the specific heat of liquid helium. They found a singularity at 2.2 K, another indication that a transition to some ordered state happened there [5]. It must have been rather astonishing to find two different states in a very pure liquid made of small round atoms with very weak interactions and no chemical properties. Anyhow, this singularity in the temperature variation of the specific heat having the shape of the Greek letter λ , they called 2.2 K the “lambda temperature T_λ ” or “ λ -point” and the change occurring at T_λ the “ λ -transition”. Above T_λ , the helium liquid was called “helium I” and “helium II” below.

Soon afterwards (1936–1937), the same Willem H. Keesom with his daughter Anna showed that liquid helium was conducting heat surprisingly well below the same 2.2 K [6]. A few months later, J.F. Allen, R. Peierls, and Z. Uddin also observed a surprisingly high thermal conductivity in Cambridge [7]. Later on, one understood that this very high thermal conductivity was the reason why liquid helium did not boil below T_λ . Indeed, if thermal conductivity is very high, there are no significant temperature differences in the liquid, consequently no regions where the probability of bubble nucleation is higher than elsewhere, so that the liquid evaporates from its free surface, but no boiling occurs. It happens that many years later I worked for my PhD on the quantum evaporation of liquid helium and discovered that it is analogous to the photoelectric effect [8,9]. But let us come back to the 1930s.

At that time, some researchers probably thought that the large thermal conductivity might be a consequence of convection being very efficient if the liquid viscosity was very small [10]. Indeed, an undergraduate student – A.D. Misener – and two technicians – J.O. Wilhelm and A.R. Clark – working in Toronto under the authority of E.F. Burton [11] measured the damping of a rotating cylinder to deduce the viscosity of liquid helium. Once more, they observed the absence of boiling below the λ -point, but, most importantly, they found that viscosity apparently decreased sharply below T_λ [10]. Liquid helium was thus attracting more and more attention at the end of 1937, and the history of superfluidity started [10].

It started with two famous notes published side by side by *Nature* in its January 8th (1938) issue. The first one [12] – “Viscosity of liquid helium below the lambda point”, received on 3 December 1937, by P. Kapitza (Institute for Physical Problems, Moscow) – appeared on page 74. The second one [13] – “Flow of liquid helium II”, received on 22 December 1937, by J.F. Allen and A.D. Misener – appeared on page 75. A.D. Misener had moved from Toronto to Cambridge, where he wanted to graduate under the supervision of J.F. Allen. For a detailed comparison between the scientific content of these two notes, and for comments on priority in relation with the dates of receipt, see ref. [10]. Let us now summarize the scientific content of these two notes.

P. Kapitza had observed the flow of liquid helium through a slit between two well-polished cylinders pressed against each other. This slit was very thin (0.5 micrometers) and he observed that “the flow of liquid above the λ -point could be only just detected over several minutes, while below the λ -point the liquid helium flowed quite easily, and the level in the tube settled down in a few seconds. From the measurements we can conclude that the viscosity of helium II is at least 1500 times smaller than that of helium I at normal pressure.” After some considerations on turbulence, he concluded that “by analogy with superconductors, ... the helium below the λ -point enters a special state which might be called superfluid.” This is a famous sentence where Kapitza introduced the word “superfluid” for the first time. His intuition was quite remarkable because superfluids and superconductors are indeed analogous states of matter, but Kapitza wrote this sentence long before the BCS theory of superconductivity was established, *a fortiori* before any demonstration of such an analogy.

The Cambridge note by Allen and Misener presented a detailed study of the flow through two micro-capillaries with different sections at two different temperatures (1.07 and 2.17 K) and ten different pressures. Their main findings were that, contrary to Poiseuille’s law, which describes laminar situations, the velocity was nearly independent of pressure, also independent of the capillary section. They concluded that “the observed type of flow... in which the velocity becomes almost independent of pressure, most certainly cannot be treated as laminar or even as ordinary turbulent flow. Consequently any known formula cannot, from our data, give a value of the viscosity which would have much meaning.” At that time, everyone else kept considering that liquid helium was a liquid with a small viscosity. But Allen and Misener discovered that, below T_λ , the hydrodynamics of helium required an entirely new interpretation. Here is the real experimental breakthrough that triggered all the following developments.

2. Fritz London and Bose–Einstein condensation

Before reading these two notes on viscosity, that is, in 1936, Fritz London had already tried to find the type of order taking place in this liquid below T_λ . F. Simon [14] and F. London [15] had realized that, in liquid helium, the magnitude of quantum fluctuations must be rather large. The kinetic energy associated with these quantum fluctuations, which is usually called “zero-point energy”, is in the order of $E_k = \hbar^2/[2m(a-d)^2]$, that is, it is inversely proportional to a small confinement length $(a-d)$ where a is the interatomic distance and d the atom diameter. It is also inversely proportional to the atom mass m , which is small again. Since the van der Waals interaction E_i is weak because the polarizability of the He atoms is weak, F. Simon and F. London found that E_k is comparable to E_i , which explains why the molar volume of liquid helium is anomalously large, also why liquid helium does not freeze, even when approaching the absolute zero, except at pressures larger than 25 bar.

F. London [15], followed by H. Frölich [16], had considered a strange possibility: liquid helium II could be a body centered cubic crystal with only half of the network sites occupied by atoms [17]. Equivalently, it could be considered as two entangled crystalline structures, respectively one of atoms and the other of vacancies, which are holes, absent atoms.¹ But, in 1938, F. London understood that this tentative model could not be “maintained” and that “an entirely different interpretation was needed” [18]. It happened after one more note was sent to *Nature* by J.F. Allen, this time with H. Jones, a young theorist [19].

Entitled “New phenomena connected with heat flow in helium II”, the new letter by Allen and Jones described the discovery of what is now known as the fountain effect. They discovered that, when heat was applied to liquid helium II on one side of a micro-capillary, the pressure increased proportionally to the heat current so that the level of the free surface went up on the warm side. A liquid jet could even occur if the pressure was high enough. For London, it was no longer possible to doubt that this liquid had totally anomalous properties for which a radically new interpretation was needed. On 5 March 1938, London sent a note to *Nature*, which was published on 9 April. There, he explained that liquid helium II was not crystalline, before proposing that it was undergoing a transition related to the Bose–Einstein condensation at T_λ [18].

By extending a calculation by Satiendra Nath Bose [20] to a set of atoms, Albert Einstein had found that, below a critical temperature T_c , a set of atoms obeying the “Bose–Einstein statistics” should accumulate in a single quantum state [21]. This “Bose–Einstein condensation” (BEC) was the result of order taking place in momentum space, not in real space. One could also say that, below T_c , all atoms started behaving the same way because they formed a macroscopic wave of indistinguishable atoms. For Einstein, this strange “condensation” was the rather surprising result of a calculation, but he did not really believe in it. As is well known, he said: “Die Theorie ist hübsch aber ob auch etwas wahres dran ist?” (“The theory is pretty, but is there anything true in it?”) Furthermore, Uhlenbeck had raised an objection to it. But in November 1937, an international conference took place in Amsterdam to celebrate the 100th anniversary of van der Waals. London participated there in a discussion between Uhlenbeck, Ehrenfest, Kramers, and Einstein, where Uhlenbeck withdrew his objection to the Bose–Einstein theory [22].

After reading the new note by Allen and Jones [19], London thought again to the Bose–Einstein condensation (BEC) [22], and he made a rough estimate of the BEC temperature T_c by considering liquid helium as an ideal gas with the same density. He found $T_c = 3.09$ K, a value close to $T_\lambda = 2.2$ K. He also calculated the temperature variation of the specific heat near the BEC transition point, and found again a behavior close to the one observed in liquid helium. After all, a liquid with a large molar volume could perhaps be not so different from a gas [22]. London hesitated in publishing his results, but he finally sent a note to *Nature* on 5 March 1938, where he wrote: “...the degeneracy of the Bose–Einstein gas has rather got the reputation of having a purely imaginary existence [...] a model which is so far from reality that it simplifies liquid helium to an ideal gas [...] [but] it seems difficult not to imagine a connection with the condensation phenomenon of Bose–Einstein statistics.” This note is famous because it signals the historical jump of quantum mechanics from the microscopic physics of atoms to macroscopic systems like a liter of liquid helium, which is visible to the naked eye. In reality, London had already thought with his brother Heinz about macroscopic wave functions to describe superconductors (see, for example, [23]).

Fritz London had also discussed his revolutionary idea with Laszlo Tisza, who had joined him a few months before (September 1937). Both London and Tisza were refugees flying away from nazism in Eastern Europe, and they had been attracted to Paris by Paul Langevin and his collaborators [10,24]. Laszlo Tisza immediately agreed with London’s tentative theory and added to it new ideas in order to interpret more experimental results, especially the fountain effect.

3. The five articles written in Paris by Laszlo Tisza

After discussing with Fritz London, Laszlo Tisza had one more revolutionary idea. Starting from London’s work, he proposed that, for an ideal Bose gas below its BEC transition temperature T_c , a macroscopic number of atoms n of the helium atoms should be “condensed” on the ground state, while the rest $(n_0 - n)$ should be distributed on all excited states. Supposing that this assumption was correct, the fraction n/n_0 should decrease with temperature according to

¹ Eighty years, later this assumption looks not so far from what is now called “supersolidity”, which was considered for a few years as a possible explanation of the anomalous mechanical properties of solid helium, before being rejected [17].

$$\frac{n}{n_0} = (T/T_0)^s \quad (1)$$

where the exponent $s = 3/2$ for an ideal gas, but it should be larger for a liquid with strong interactions between atoms. It meant that, below the BEC temperature T_c , the ideal gas should be a mixture of two components: a “superfluid component” made of condensed atoms and a “normal component” made of non-condensed atoms. The concentration of the “superfluid component” should vary continuously from 0 at T_c to 1 at $T = 0$. As a consequence, at any temperature between T_c and 0, the liquid should be made of a definite mixture of the superfluid component with the normal component. Nowadays, this view is considered too simple for a liquid because atom–atom interactions must play a role on the nature of both the ground state and the excited states, and the atoms lose their individual character. Tisza was aware of some of these difficulties, but he continued. The condensed atoms being on the ground state, they could not contribute to any momentum exchange, consequently to any dissipation, contrary to the non-condensed atoms, which should be responsible for dissipation, that is for viscous behavior, whatever it is. It allowed him to understand the apparent contradiction which had been noticed between the Toronto experiment using a macroscopic cylinder and the experiments by Kapitza and by Allen, where a microscopic slit or a micro-capillary were used. Indeed, in the Toronto experiment, both the condensed and the non-condensed atoms should be moving, while in the two others only a superflow of condensed atoms should take place. He then predicted that this superflow should produce a gradient in the concentration of condensed atoms, which should be accompanied by a temperature gradient according to Eq. (1).

Finally, Tisza understood the origin of the fountain effect as the inverse process of the above-mentioned superflow: “If one maintains a temperature difference between the ends of a capillary, a gradient of density of the excited atoms and, thus, of pressure is produced.” He noticed a similarity between this pressure gradient and the osmotic pressure difference across a semi-permeable membrane. And he finished his article by writing:

A detailed discussion of the problem will be given in the *Journal de physique*. I am greatly indebted to Dr. F. London for the opportunity of seeing his paper before publication.

L. Tisza, Laboratoire de physique expérimentale, Collège de France, Paris, 16 April.

This historical note [25] to *Nature* is considered as the birth of the “two-fluid model”, but, at that time, London could not agree with Tisza’s new ideas [26]. To imagine that liquid helium II was made of two fluid components that are mixed but that could flow independently of each other was impossible for him. But Tisza developed his model in the two other notes he sent to the *Comptes rendus de l’Académie des sciences* a few months later. In 1940, Tisza published another two articles in the *Journal de physique et le Radium* [27,28], as he had announced. Even later (1941), Landau reformulated Tisza’s two-fluid model on a more rigorous basis [29,30].

In his first note to the *Comptes rendus* [31], which was written in French and presented by Paul Langevin on 14 November 1938, Tisza first considered the nature of the two components in his two-fluid model. He derived the two equations that describe the hydrodynamics of liquid helium below T_λ and arrived at a discovery: instead of diffusing as in any classical liquid, heat propagates as a wave, and he calculated the velocity of these heat waves. This prediction had to become a crucial test of his model. He also described how his model agrees with the measurements of the anomalous thermal conductivity of liquid helium [6,7].

As for Tisza’s second note to the *Comptes rendus* [32], it describes the viscosity of the normal component, which he calculated within his approximations. Tisza explained why his model was in agreement, not only with the Toronto experiment [4] and a later one by W.H. Keesom and G.E. Mac Wood [33] where the two components were moving, but also with the two experiments by Kapitza and by Allen and Misener, where only the superfluid component was moving. He also explained why, in a quantum liquid like helium at low temperature, viscosity was disappearing at low temperature, contrary to classical liquids where viscosity increases as temperature decreases. He then explained the fountain effect as a thermomechanical effect with some resemblance to the osmotic pressure across a semi-permeable membrane. Finally, he explained the “Rollin creeping” [48], which is why, in a beaker, superfluid helium was able to creep up along the sides so that the beaker empties spontaneously. It was a consequence of the existence of a thin superfluid film covering the beaker walls.

In October 1939, despite the growing stress following the invasion of France by the nazi army, which soon forced the Langevin laboratory to move to Toulouse [26], Tisza sent a couple of articles to the *Journal de physique et le Radium* [27,28]. In this journal, he could further develop his two-fluid model without the length limitation that was imposed by the *Comptes rendus* [27]. He thus added a graph showing the agreement between his theory and the viscosity measurements by Keesom and Mac Wood [33]. He also added a paragraph explaining why, contrary to Bose liquids, Fermi liquids could not be superfluid. He also asked questions about turbulence in a superfluid, but he was far from the recent developments of this field that show its complexity. In order to get more information on his thermomechanical effect, he finally proposed to study the propagation of small temperature variations below 0.8 K. This proposal demonstrated his remarkable intuition, but he had probably not anticipated that, a few years later, such experiments would support some of Landau’s criticism on his work.

4. Lev Landau and later developments

This is not the right place to describe the life of Lev Landau who had written a leaflet comparing Stalin to Hitler in may 1938 and who had been consequently thrown in its sadly known Lubyanka prison by the NKVD. Landau survived thanks to

Kapitza and came back to the physics of quantum liquids at a time when World War II had forced scientists in the West to stop their scientific research. But research at the Kapitza Institute continued and, in 1941, Landau wrote two versions of an article, “On the theory of superfluidity”, one in English to *Physical Review* [29], and one in Russian to the *USSR Journal of Physics* [30]. In this famous article, Landau reformulated Tisza’s two-fluid model on a more rigorous basis. The principles and consequences were similar, except for the nature of the normal component of the quantum liquid. Landau had the intuition that it was made of collective excitations, not of single atoms on excited states, as Tisza had assumed. In order to fit the measured specific heat, he had to assume the existence of two different kinds of excitations: phonons and “rotons”. Given these assumptions, Landau proposed a dissipation mechanism that should appear when the flow velocity exceeds a “critical velocity” of order 60 m/s. He also predicted that there should exist two different types of waves in superfluid helium: ordinary sound and heat waves that he called “second sound”. This second sound was similar to the heat waves predicted by Tisza, except that their respective velocities were different in the low temperature limit. They were discovered in 1944 by Peshkov [34], who thus confirmed the validity of the two-fluid model, but further measurements in 1946 [35] proved that Landau was right for their low-temperature dependence.

Now Landau’s article deserves a few more comments. Landau never admitted that Tisza could have any priority in the introduction of the two-fluid model. He considered that “the explanation advanced by Tisza not only has no foundation in his suggestions but is in direct contradiction to them”. In 1949, Landau insisted in *Physical Review* [36] by saying that he had “missed seeing Tisza’s short letters to the *Comptes rendus* [...] However his entire quantitative theory is entirely incorrect.” Landau’s attitude is surprisingly unfair for several reasons.

It is hard to believe that Landau did not know the two notes that Tisza had published in the *Comptes rendus*. Indeed, Landau and Kapitza had a very close collaboration, not only a deep friendship and a mutual recognition to each other. In 1941, both Kapitza and Landau had written a letter to *Physical Review* [37,29]. They probably sent their letters together, because the date of receipt is the same and they were published next to each other in the review. It happens that, in his own letter [37], Kapitza cites the two notes to the *Comptes Rendus* by Tisza. How Landau could have ignored Tisza’s notes? Could it be because Landau did not understand French? I asked Alexei A. Abrikosov, who was awarded the Nobel prize for his work on superconductivity and was a close collaborator of Lev Landau, about that and he answered to me on 15 January 2001:

Dear Dr. Balibar,

Landau was very able to languages. He knew German, English, French and Danish. Therefore he could read Tisza’s papers in French, the more as that Lifshitz, whom he often ordered to read papers, instead of doing that himself, didn’t know French...

Sincerely yours, Alex Abrikosov

Moreover, Landau never cited London and actually never believed that there could be any relation between superfluidity and Bose–Einstein condensation. And Landau’s 1941 paper is not “entirely correct” either. Landau had to modify the spectrum of his “rotons” in 1947 [38]; I. Tamm had suggested that rotons could be microscopic vortices, but Landau realized in 1947 [38] that rotons correspond to a particular region of the phonon spectrum. In 2004, P. Nozières explained that rotons are related to local order in the liquid, “ghosts of a Bragg peak” [39]. As for the demonstration that excitations in superfluid helium are collective modes, it had to wait for the theoretical work of Bogoliubov [40] and for experimental measurements using neutron scattering even later. Most importantly, it is surprising to realize that Landau introduces quantum hydrodynamics without mentioning any difference between Fermi fluids and Bose fluids.

In 1946, London proposed to study liquid helium 3, a Fermi liquid, as a test of his prediction of a relation between superfluidity and BEC. In 1949, D.W. Osborne, B. Weinstock, and B.M. Abraham [41] demonstrated that liquid helium 3 was indeed not superfluid at temperatures comparable to T_λ . In 1972, D.D. Osheroff, R.C. Richardson and D.M. Lee [42] discovered superfluid transitions in helium 3, which require the formation of He3–He3 pairs as in the case of electrons in superconductors. These transitions happen below 2.5 mK, a temperature colder than T_λ by three orders of magnitude. With the help of A.J. Leggett [43] for the theory of helium 3, D.D. Osheroff, R.C. Richardson, and D.M. Lee demonstrated that the two observed states of superfluid helium 3 correspond to two types of pairing with different symmetries, which had been predicted 10 years before by P.W. Anderson, W.F. Brinkmann, and P. Morel [44], and by R. Balian and N.R. Werthamer [45]. D.D. Osheroff, R.C. Richardson and D.M. Lee received a Nobel prize in 1996, A.J. Leggett in 2003. Now, the relation between superfluidity and Bose–Einstein condensation was not universally accepted until 1995, when E.A. Cornell and C. E. Wieman,² in Colorado, and W. Ketterle,³ at MIT, discovered BEC in quantum gases. They shared one more Nobel prize, in 2001. Superfluids have many other interesting properties, which I cannot describe here, the most important one being the quantization of their vortices.

5. Conclusion

In summary, Landau had a major contribution to the understanding of superfluidity, but it followed London’s discovery of its relation to BEC and the introduction of the two-fluid model by Tisza, which deserves more recognition. The study

² For a review, see the Nobel lecture in [46].

³ For a review, see the Nobel lecture in [47].

and full understanding of superfluidity required many decades of work in the whole world. As for the recognition of its discovery, it is probable that if London had not died prematurely in 1954, he would have shared Landau's Nobel prize in 1962. Kapitza's Nobel prize in 1978 could also have been shared with J.F. Allen.

One thing looks clear to me: in the 1930s, articles were published more quickly, and English was not as dominant in scientific publications as it is today. But scientific discoveries were already difficult to attribute to single scientists and the competition worldwide was already intense.

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