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## Comparative study of the alpha decay of Hg isotopes using different forms of nuclear potentials



### *La désintégration alpha des isotopes du mercure, à travers différents modèles de potentiel nucléaire*

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## ABSTRACT

The alpha decay half-lives of Hg isotopes within the range  $A = 171$ – $212$  have been studied using 25 different versions of nuclear potentials to select the suitable form of nuclear potential for alpha decay studies. The computed standard deviations suggested that the apt potential is BW 91 with a deviation 0.133. The next low deviation is shown by Proximity 1966, Proximity 1984, and Proximity 2003-I, II with deviations less than 0.2. Concerning other potentials, the fact we observed is that almost all the potentials possess a standard deviation less than one. The universal curve studied for alpha decay is observed to show straight line behavior irrespective of the nuclear potential used. Since the predicted alpha half-lives match well the experimental values, the half-lives of certain new Hg alpha emitters have been predicted by the present method.

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## R É S U M É

La période radioactive alpha des isotopes du mercure de nombre de masse  $A$  compris entre 171 et 212 a été évaluée à partir de 25 modèles différents de potentiel nucléaire adapté à la radioactivité alpha. Le calcul des écarts types suggère que le potentiel pertinent est BW 91, avec un écart de 0,133. La deuxième place revient à Proximity 1966, suivi de Proximity 1984 et de Proximity 2003-I, II, avec des écarts types inférieurs à 0,2. Les autres potentiels ont presque tous un écart type inférieur à 1. La courbe universelle caractéristique du rayonnement alpha est une droite, quel que soit le potentiel utilisé. Comme les périodes calculées sont en bon accord avec les valeurs expérimentales, on a calculé aussi, par la même méthode, les périodes de quelques émetteurs alpha nouveaux.

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## 1. Introduction

Alpha decay has been a topic of great interest among the theoreticians and experimentalists since its discovery by Rutherford [1]. Basically, alpha decay is considered to be a Coulomb repulsion effect that becomes dominant for heavy and superheavy nuclei (SHN) as the Coulomb force increases with the size of the nucleus. The qualitative features of alpha decay were first explained by George Gamow [2] in 1928 based on quantum tunneling and soon after Gurney and Condon [3] explained alpha decay by means of wave mechanics. Later, several microscopic and macroscopic approaches have been put forward [4–11] on the basis of Gamow's theory. The phenomenological models like generalized liquid drop model (GLDM) [12], generalized density-dependent cluster model (GDDCM) [13], unified model of  $\alpha$  decay and  $\alpha$  cluster (UMADAC) [14], and coupled channel approach [15], have been developed by defining a phenomenological connection between alpha decay half-lives and  $Q$  values [16].

In the calculation of  $\alpha$ -decay half-lives, the pre-formation factor is an indispensable quantity. It is important to mention that the microscopic description of pre-formation factor plays a key role in the understanding of the decay process, even though it requires a precise knowledge of the initial quantum state, not always available. Among the various approaches, the shell model [17–19], the BCS method [20], and the hybrid (shell model +  $\alpha$ -cluster) model [21], which involves the microscopic description of pre-formation factor should be mentioned with great importance. The microscopic description of alpha decay has shown that the continuum part of the nuclear spectrum plays an important role in alpha decay processes. To obtain a proper pre-formation probability, one has to include the continuum [20], suggested by BCS method, which is in contrast with the shell model calculations that include only a few bound states [17–19]. The hybrid model [21] is essentially a shell model that treats a large shell model basis up to the continuum states through the wave function of the spatially localized  $\alpha$  cluster and explains well the experimental decay width.

In developing models, the main hurdle lies in the selection of the proper interaction potential. The theoreticians adopted various forms of interaction potentials to develop models that can interpret the experimental results satisfactorily and can predict new results [22–28]. Proximity potential, which was first used by Shi and Swiatecki [29], subjected to several modifications, has been widely used as the nuclear potential for alpha decay studies [22].

In recent times, the alpha decay studies using different forms of nuclear potential attained central importance as the main objective of the works is to select an accurate interaction potential that can describe all the features of alpha decay. The alpha decay studies carried out by Yao et al. [30] to fulfill this requirement suggested that the apt potential is generalized proximity potential 1977, which has been used by Santhosh et al. [31–37] in the past years to study alpha as well as cluster decay in the heavy and superheavy region. Even though studies to meet this specific requirement, i.e. to select an appropriate potential for alpha decay studies, are rare, calculations have been done by Wang et al. [38] using 20 different mass models and 18 empirical formulas. Through the evaluation of  $\alpha$ -decay half-lives of 344 isotopes of nuclei with  $Z = 52$ –107, Ghodsi et al. [39] studied and predicted proximity 1977 as the best form of nuclear potential. Even though certain works are available in this scenario, there is still uncertainty regarding the proper selection of the interaction potential to perform alpha decay studies. So, our present work is an extension of the previous works [40–42] in which we tried to select a suitable potential form for alpha as well as cluster decay studies. In the present manuscript, we have evaluated the alpha decay half-lives of Hg isotopes within the range  $A = 171$ –212, using 25 different versions of nuclear potentials; this includes the prediction of new Hg alpha emitters that are not verified experimentally yet.

A brief description of the model and the different nuclear potentials used for the study are presented in Section 2. The results and the discussion are given in Section 3, and the conclusive remarks of the study are provided in the last section.

## 2. The model

The interacting potential barrier for the touching configuration and separated cluster and daughter nucleus is taken as,

$$V = \frac{Z_1 Z_2 e^2}{r} + V_N(r) + \frac{\hbar^2 \ell(\ell + 1)}{2\mu r^2} \quad (1)$$

Here  $Z_1$  and  $Z_2$  are the atomic numbers of the daughter nucleus and emitted cluster,  $r$  is the distance between the centers of the daughter nucleus and the emitted cluster and is given as  $r = s + C_1 + C_2$ , where  $C_1$  and  $C_2$  are the Süsmann central radii of the daughter nucleus and the emitted cluster and  $s$  is the distance between the near surfaces of the cluster and daughter nucleus. The term  $\ell$  represents the angular momentum,  $\mu$  is the reduced mass and  $V_N(r)$  is the nuclear potential. The above equation is for spherical parent and daughter nuclei. The inclusion of deformation values of the parent and the daughter nuclei will decrease the width and height of the potential barrier, which in turn tends to increase the barrier penetrability.

For the internal part (overlap region), the potential energy barrier is obtained by using the simple power law interpolation method as done by Shi and Swiatecki [29] and is given as:

$$V = a_0(L - L_0)^n, \quad \text{for } s < 0 \quad (2)$$

Here  $L = s + 2C_1 + 2C_2$  and  $L_0 = 2C$ . The constants  $a_0$  and  $n$  are determined by the smooth matching of the two potentials at the touching point.

Using a one-dimensional WKB approximation, the barrier penetrability  $P$  is given as

$$P = \exp \left\{ -\frac{2}{\hbar} \int_a^b \sqrt{2\mu(V - Q)} dz \right\} \quad (3)$$

Here the reduced mass  $\mu$  is given by  $\mu = mA_1A_2/A$ , where  $m$  is the nucleon mass and  $A_1, A_2$  are the mass numbers of the daughter nucleus and emitted cluster, respectively. The turning points  $a$  and  $b$  are determined from the equation  $V(a) = V(b) = Q$ . The above integral can be evaluated numerically or analytically.

The half-life is given by

$$T_{1/2} = \left( \frac{\ln 2}{\lambda} \right) = \left( \frac{\ln 2}{\nu P} \right) \quad (4)$$

where  $\nu = \left( \frac{\omega}{2\pi} \right) = \left( \frac{2E_\nu}{\hbar} \right)$  represents the number of assaults on the barrier per second and  $\lambda$  is the decay constant.  $E_\nu$ , the empirical vibration energy, is given as [43]

$$E_\nu = Q \left\{ 0.056 + 0.039 \exp \left[ \frac{(4 - A_2)}{2.5} \right] \right\}, \quad \text{for } A_2 \geq 4 \quad (5)$$

In our model, the pre-formation probability  $S$  can be calculated as the penetrability of the internal part (overlap region) of the barrier [44] and is given by

$$S = \exp(-K) \quad (6)$$

where the action integral is given as

$$K = \frac{2}{\hbar} \int_a^0 (2\mu\{V - Q\})^{1/2} dz \quad (7)$$

where  $a$  is the turning point defined as  $V(a) = Q$ , and  $z = 0$  represent the touching configuration.

For the calculation of the nuclear potential,  $V_N(r)$ , 25 different versions of nuclear potentials are used, and the details of these nuclear potentials can be found in Ref. [42].

### 3. Results and discussion

We have calculated the alpha decay half-lives of  $^{171-212}\text{Hg}$  isotopes using different forms of nuclear potentials. The half-lives are evaluated using 25 different forms of nuclear potentials, which include different versions of Proximity potentials; Proximity 1977 [45–47] and its different modifications with different sets of surface tension coefficients,  $\gamma$  values (Proximity 1966 [48], Proximity 1976 [49], Proximity 1979 [50], Proximity 1981-I, II, III [51]; Proximity 1984 [52], Proximity 1988 [53], Proximity 1995 [54], Proximity 2003-I, II, III [55], Modified proximity 1988 (Mod-Prox-88) [56]), Proximity 2000 [57–59], Modified proximity 2000 (Prox-00DP) [60], Proximity 2010 [61], and the Bass potentials (Bass 1973 [62,63], Bass 1977 [64], Bass 1980 [28]), Broglia and Winther 1991 (BW 91) [28], Christensen and Winther 1976 (CW 76) [65], Aage Winther (AW 95) [66], Ngo 1980 [67], and the New Denisov Potential (Denisov) [68] so as to select a suitable form for alpha decay studies. The  $Q$  value of the reaction is calculated using the equation,

$$Q = \Delta M_p - (\Delta M_\alpha + \Delta M_d) + k(Z_p^\varepsilon - Z_d^\varepsilon) \quad (8)$$

where  $\Delta M_p$ ,  $\Delta M_d$ ,  $\Delta M_\alpha$  are the mass excesses of the parent nucleus, the daughter nucleus, and the alpha particle, respectively. The screening effect of the atomic electrons [69] is included in the term  $k(Z_p^\varepsilon - Z_d^\varepsilon)$  where  $k = 8.7$  eV,  $\varepsilon = 2.517$  for  $Z \geq 60$ , and  $k = 13.6$  eV,  $\varepsilon = 2.408$  for  $Z < 60$  [70]. To compute the  $Q$  values, mass excess values are taken from [71]. We found that for the isotopes  $^{203-206}\text{Hg}$ , the  $Q$  value obtained is negative, and hence alpha decay is not permitted for these isotopes. We know that the half-life depends on the barrier penetrability, which is related to the total potential  $V$ , and the  $Q$  value of the reaction. For a particular parent nucleus, the  $Q$  value for alpha emission is fixed. Since the  $Q$  value is fixed, the half-life depends only on the barrier penetration probability, which in turn depends on the total potential  $V$ . In the total potential  $V$ , the Coulomb part will be the same and the variation in half-life corresponds to the difference in the form of nuclear potential used. So, to choose the right potential for alpha decay studies, we have done half-life evaluations by changing the nuclear potentials and checked for what form of potential the half-life matches well with the experimental results.

Here we have performed the half-life evaluations for zero angular momentum transfers as the  $\ell$  values involved in alpha decay are small, of the order of  $5\hbar$  ( $\approx 5\hbar$ ), and its contribution to half-life is shown to be small [72]. The comparison study performed by us for alpha emission from  $^{171-212}\text{Hg}$  is represented in Fig. 1. From the graphs we can see that the inclusion of various nuclear potentials does not produce any deviation in the general trend, though we observed variations in the half-lives. In alpha decay as well as cluster decay, a minimum half-life indicates the magicity of the daughter nucleus, and a

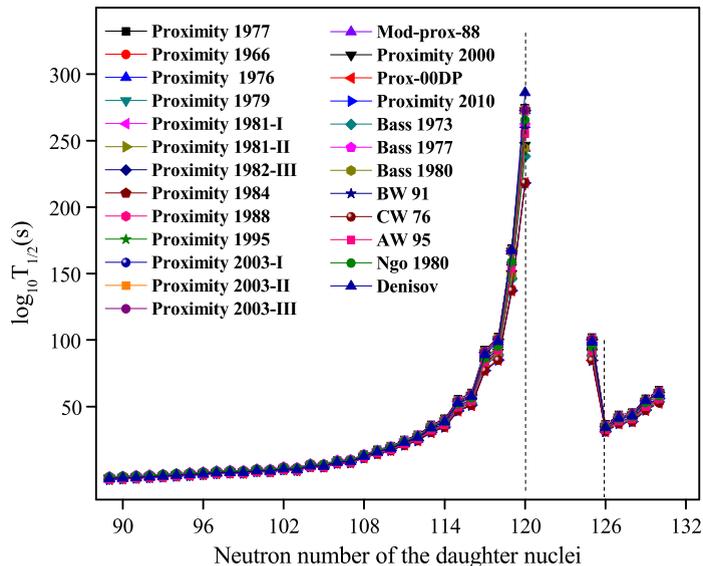


Fig. 1. Plot of the computed  $\log_{10} T_{1/2}$  values versus the neutron number of daughter nuclei for alpha emission from  $^{171-212}\text{Hg}$  isotopes.

maximum half-life indicates the magicity of the parent nucleus. In Fig. 1, we can see the existence of a profound minimum at  $N = 126$ , which indicates the neutron magicity of the daughter nucleus at  $N = 126$ . In addition, we have noticed a peak at  $N = 120$ , which corresponds to the near double magicity of the parent nucleus  $^{202}\text{Hg}$  ( $Z = 80, N = 122$ ). Our predictions on near magicity at  $N = 122$  is well supported by the spherical mean-field calculations of Nakada and Sugiura [73], in which the authors indicated the possibility of neutron magicity at  $N = 124$ . We have listed the alpha decay half-lives for  $^{171-192}\text{Hg}$  calculated using the above-mentioned 25 nuclear potentials in Table 1, with the experimental data [74] provided in the last column. In our model, the spectroscopic factor  $S$  is calculated as the penetrability of the internal part of the barrier [44]. The total penetrability calculated using the equation,

$$P = SP_S \tag{9}$$

where  $P_S$  is the external penetrability. Consider the decay of the  $^{171}\text{Hg}$  emitting alpha particle. The spectroscopic factor  $S$  is obtained as 0.854 and the external penetrability,  $P_S$ , is obtained as  $4.968 \times 10^{-18}$ . Therefore, the total penetrability,  $P$ , is

Table 1  
Comparison of the alpha decay half-lives predicted using different nuclear potentials with the experimental data.

Parent nuclei	$Q_\alpha$ (MeV)	$\log_{10} T_{1/2}$ (s)							Expt.
		Proximity 1977	Proximity 1966	Proximity 1976	Proximity 1979	Proximity 1981-I	Proximity 1981-II	Proximity 1981-III	
$^{171}\text{Hg}$	7.698	-3.106	-3.981	-4.635	-4.272	-4.186	-4.326	-4.292	-4.229
$^{172}\text{Hg}$	7.558	-3.299	-3.578	-4.219	-3.866	-3.781	-3.919	-3.886	-3.636
$^{173}\text{Hg}$	7.408	-2.830	-3.131	-3.759	-3.414	-3.331	-3.468	-3.435	-3.097
$^{174}\text{Hg}$	7.266	-2.373	-2.695	-3.310	-2.974	-2.893	-3.027	-2.995	-2.678
$^{175}\text{Hg}$	7.108	-1.844	-2.191	-2.792	-2.464	-2.386	-2.518	-2.486	-1.975
$^{176}\text{Hg}$	6.930	-1.222	-1.597	-2.183	-1.865	-1.789	-1.919	-1.888	-1.693
$^{177}\text{Hg}$	6.768	-0.635	-1.037	-1.609	-1.300	-1.227	-1.354	-1.323	-0.928
$^{178}\text{Hg}$	6.610	-0.041	-0.469	-1.029	-0.728	-0.657	-0.781	-0.752	
$^{179}\text{Hg}$	6.393	0.821	0.355	-0.189	0.102	0.171	0.049	0.078	
$^{180}\text{Hg}$	6.292	1.225	0.742	0.208	0.494	0.560	0.440	0.468	
$^{181}\text{Hg}$	6.317	1.092	0.618	0.089	0.371	0.436	0.316	0.345	
$^{182}\text{Hg}$	6.029	2.342	1.814	1.303	1.574	1.636	1.519	1.547	
$^{183}\text{Hg}$	6.071	2.127	1.609	1.103	1.371	1.432	1.315	1.343	
$^{184}\text{Hg}$	5.695	3.890	3.300	2.815	3.071	3.128	3.015	3.042	
$^{185}\text{Hg}$	5.806	3.322	2.756	2.273	2.527	2.583	2.470	2.497	
$^{186}\text{Hg}$	5.237	6.273	5.596	5.138	5.377	5.429	5.322	5.347	
$^{187}\text{Hg}$	5.262	6.111	5.439	4.988	5.223	5.273	5.167	5.191	
$^{188}\text{Hg}$	4.740	9.249	8.475	8.045	8.268	8.315	8.213	8.236	
$^{189}\text{Hg}$	4.666	9.718	8.929	8.508	8.726	8.771	8.670	8.693	
$^{190}\text{Hg}$	4.101	13.861	12.961	12.561	12.766	12.808	12.712	12.734	
$^{191}\text{Hg}$	3.701	17.344	16.365	15.980	16.177	16.216	16.122	16.144	
$^{192}\text{Hg}$	3.417	20.173	19.138	18.766	18.956	18.992	18.901	18.922	

Table 1 (continued)

Parent nuclei	$Q_\alpha$ (MeV)	$\log_{10} T_{1/2}$ (s)						Expt.
		Proximity 1984	Proximity 1988	Proximity 1995	Proximity 2003-I	Proximity 2003-II	Proximity 2003-III	
<sup>171</sup> Hg	7.698	-3.897	-3.787	-4.296	-4.071	-3.866	-3.852	-4.229
<sup>172</sup> Hg	7.558	-3.495	-3.379	-3.890	-3.668	-3.465	-3.450	-3.636
<sup>173</sup> Hg	7.408	-3.048	-2.928	-3.439	-3.220	-3.020	-3.003	-3.097
<sup>174</sup> Hg	7.266	-2.612	-2.490	-2.999	-2.783	-2.586	-2.568	-2.678
<sup>175</sup> Hg	7.108	-2.107	-1.986	-2.491	-2.277	-2.083	-2.064	-1.975
<sup>176</sup> Hg	6.930	-1.513	-1.395	-1.892	-1.682	-1.491	-1.471	-1.693
<sup>177</sup> Hg	6.768	-0.953	-0.841	-1.328	-1.121	-0.933	-0.911	-0.928
<sup>178</sup> Hg	6.610	-0.385	-0.283	-0.756	-0.552	-0.368	-0.344	
<sup>179</sup> Hg	6.393	0.439	0.523	0.073	0.273	0.454	0.479	
<sup>180</sup> Hg	6.292	0.827	0.899	0.463	0.661	0.840	0.866	
<sup>181</sup> Hg	6.317	0.703	0.776	0.340	0.537	0.714	0.743	
<sup>182</sup> Hg	6.029	1.899	1.933	1.542	1.735	1.907	1.937	
<sup>183</sup> Hg	6.071	1.695	1.734	1.338	1.530	1.702	1.733	
<sup>184</sup> Hg	5.695	3.386	3.354	3.037	3.223	3.390	3.423	
<sup>185</sup> Hg	5.806	2.843	2.833	2.492	2.678	2.844	2.879	
<sup>186</sup> Hg	5.237	5.682	5.517	5.342	5.521	5.680	5.716	
<sup>187</sup> Hg	5.262	5.526	5.370	5.186	5.365	5.522	5.560	
<sup>188</sup> Hg	4.740	8.561	8.186	8.231	8.403	8.554	8.594	
<sup>189</sup> Hg	4.666	9.016	8.603	8.688	8.858	9.007	9.048	
<sup>190</sup> Hg	4.101	13.046	12.266	12.729	12.892	13.035	13.078	
<sup>191</sup> Hg	3.701	16.450	15.312	16.139	16.298	16.436	16.480	
<sup>192</sup> Hg	3.417	19.223	17.770	18.917	19.073	19.207	19.253	

Parent nuclei	$Q_\alpha$ (MeV)	$\log_{10} T_{1/2}$ (s)					Expt.	
		Mod-Prox-88	Proximity 2000	Prox-00DP	Proximity 2010	Bass 1973		Bass 1977
<sup>171</sup> Hg	7.698	-3.609	-3.667	-4.826	-4.761	-4.210	-2.729	-4.229
<sup>172</sup> Hg	7.558	-3.212	-3.266	-4.407	-4.346	-3.813	-2.317	-3.636
<sup>173</sup> Hg	7.408	-2.772	-2.822	-3.945	-3.888	-3.373	-1.860	-3.097
<sup>174</sup> Hg	7.266	-2.345	-2.391	-3.496	-3.443	-2.945	-1.415	-2.678
<sup>175</sup> Hg	7.108	-1.851	-1.893	-2.981	-2.931	-2.449	-0.900	-1.975
<sup>176</sup> Hg	6.930	-1.272	-1.310	-2.380	-2.332	-1.867	-0.294	-1.693
<sup>177</sup> Hg	6.768	-0.729	-0.762	-1.816	-1.771	-1.319	0.275	-0.928
<sup>178</sup> Hg	6.610	-0.180	-0.209	-1.249	-1.206	-0.765	0.850	
<sup>179</sup> Hg	6.393	0.613	0.588	-0.433	-0.392	0.038	1.685	
<sup>180</sup> Hg	6.292	0.983	0.963	-0.050	-0.010	0.414	2.071	
<sup>181</sup> Hg	6.317	0.862	0.848	-0.169	-0.126	0.294	1.935	
<sup>182</sup> Hg	6.029	2.004	1.993	0.999	1.038	1.452	3.141	
<sup>183</sup> Hg	6.071	1.808	1.802	0.803	0.845	1.256	2.925	
<sup>184</sup> Hg	5.695	3.409	3.407	2.435	2.472	2.883	4.620	
<sup>185</sup> Hg	5.806	2.894	2.898	1.916	1.957	2.363	4.065	
<sup>186</sup> Hg	5.237	5.555	5.561	4.614	4.651	5.073	6.887	
<sup>187</sup> Hg	5.262	5.409	5.420	4.470	4.509	4.927	6.724	
<sup>188</sup> Hg	4.740	8.209	8.223	7.298	7.334	7.788	9.693	
<sup>189</sup> Hg	4.666	8.624	8.644	7.720	7.758	8.215	10.127	
<sup>190</sup> Hg	4.101	12.275	12.299	11.395	11.431	11.946	14.005	
<sup>191</sup> Hg	3.701	15.317	15.345	14.452	14.489	15.050	17.240	
<sup>192</sup> Hg	3.417	17.773	17.806	16.919	16.957	17.550	19.855	

Parent nuclei	$Q_\alpha$ (MeV)	$\log_{10} T_{1/2}$ (s)					Expt.	
		Bass 1980	BW 91	CW 76	AW 95	Ngo 1980		Denisov
<sup>171</sup> Hg	7.698	-3.725	-4.044	-4.104	-4.573	-2.862	-4.190	-4.229
<sup>172</sup> Hg	7.558	-3.328	-3.606	-3.719	-4.194	-2.461	-3.795	-3.636
<sup>173</sup> Hg	7.408	-2.887	-3.144	-3.291	-3.772	-2.014	-3.355	-3.097
<sup>174</sup> Hg	7.266	-2.458	-2.701	-2.875	-3.362	-1.579	-2.925	-2.678
<sup>175</sup> Hg	7.108	-1.962	-2.191	-2.393	-2.887	-1.075	-2.427	-1.975
<sup>176</sup> Hg	6.930	-1.379	-1.594	-1.828	-2.329	-0.482	-1.840	-1.693
<sup>177</sup> Hg	6.768	-0.832	-1.034	-1.296	-1.803	0.076	-1.285	-0.928
<sup>178</sup> Hg	6.610	-0.279	-0.471	-0.759	-1.272	0.640	-0.724	
<sup>179</sup> Hg	6.393	0.522	0.348	0.019	-0.501	1.460	0.093	
<sup>180</sup> Hg	6.292	0.895	0.726	0.382	-0.140	1.840	0.476	
<sup>181</sup> Hg	6.317	0.772	0.592	0.263	-0.256	1.707	0.350	
<sup>182</sup> Hg	6.029	1.925	1.775	1.386	0.969	2.894	1.537	
<sup>183</sup> Hg	6.071	1.726	1.562	1.193	0.689	2.682	1.332	
<sup>184</sup> Hg	5.695	3.342	3.228	2.771	2.456	4.353	3.009	
<sup>185</sup> Hg	5.806	2.822	2.682	2.263	1.886	3.806	2.467	

(continued on next page)

**Table 1** (continued)

Parent nuclei	$Q_\alpha$ (MeV)	$\log_{10} T_{1/2}$ (s)						Expt.
		Bass 1980	BW 91	CW 76	AW 95	Ngo 1980	Denisov	
$^{186}\text{Hg}$	5.237	5.504	5.460	4.892	4.702	6.594	5.280	
$^{187}\text{Hg}$	5.262	5.356	5.299	4.747	4.538	6.433	5.124	
$^{188}\text{Hg}$	4.740	8.172	8.234	7.524	7.477	9.379	8.124	
$^{189}\text{Hg}$	4.666	8.589	8.664	7.937	7.906	9.812	8.571	
$^{190}\text{Hg}$	4.101	12.249	12.518	11.573	11.754	13.679	12.541	
$^{191}\text{Hg}$	3.701	15.294	15.743	14.610	14.971	16.918	15.883	
$^{192}\text{Hg}$	3.417	17.751	18.356	17.066	17.578	19.543	18.600	

**Table 2**

The standard deviation obtained for different nuclear potentials.

Potential	Standard deviation	Potential	Standard deviation
Proximity 1977	0.379	Proximity 2003-III	0.206
Proximity 1966	0.150	Proximity 2000	0.363
Proximity 1976	0.672	Prox-00DP	0.877
Proximity 1979	0.329	Proximity 2010	0.819
Proximity 1981-I	0.256	Bass 1973	0.313
Proximity 1981-II	0.382	Bassv1977	1.395
Proximity 1981-III	0.350	Bass 1980	0.303
Proximity 1984	0.177	BW 91	0.133
Proximity 1988	0.265	CW 76	0.267
Mod prox-88	0.411	AW 95	0.748
Proximity 1995	0.355	Ngo 1980	1.219
Proximity 2003-I	0.174	Denisov	0.291
Proximity 2003-II	0.195		

obtained as  $4.243 \times 10^{-18}$  and hence the decay half-life is  $7.8346 \times 10^{-4}$  s. These values are obtained by taking Proximity 1977 as the nuclear potential. In Table 1, we have excluded the calculations of  $^{173-212}\text{Hg}$  as the half-lives of alpha emission from these isotopes are greater than  $10^{20}$  s ( $T_{1/2} > 10^{20}$  s). On analyzing the table, it can be seen that the half-lives are different on using different nuclear potentials. It seems that the proximity potential versions; Proximity 1976, Proximity 1979, Proximity 1981-I, II, III, Proximity 1995, Proximity 2003-I, Proximity 1973; Bass 1973, BW 91, CW 76, AW 95, and Denisov are appropriate for alpha decay studies. But the right way to make the right choice of potential is by obtaining the standard deviation of all the 25 forms of nuclear potentials with the experimental data [74]. The standard deviations  $\sigma$  of the logarithmic values of the calculated half-lives are obtained using the equation

$$\sigma = \left\{ \frac{1}{n-1} \sum_{i=1}^n (\log_{10} T_i^{\text{cal}} - \log_{10} T_i^{\text{exp}})^2 \right\}^{1/2} \quad (10)$$

The results of the above calculation are provided in Table 2. On analyzing the results, we can see that all the potentials except Bass 1977 have standard deviations less than one. Among the different versions of proximity potentials, it is seen that except Proximity 1976, Prox-00DP, and Proximity 2010, all other proximity potentials show a standard deviation less than 0.5. Prox-00DP is the proximity potential which shows high deviation among them, i.e. 0.877. If we choose the apt proximity potential form, the least standard deviation is given by Proximity 1966,  $\sigma = 0.150$ . The next low value is for Proximity 2003-I, and Proximity 1984. The values are 0.174 and 0.177, respectively. Proximity 1981, and Proximity 2003-II, III also possess a standard deviation less than 0.3. In general, if we analyze the table, we can confidently say that the modifications of the Proximity 1977, having slightly different sets of  $\gamma$  values, should be considered as the appropriate proximity potential forms. But among the 25 nuclear potentials, the nuclear potential that reproduces and interprets the experimental results authentically is BW 91, with  $\sigma = 0.133$ , the least value among these 25 potential versions. Other than the above-mentioned proximity potentials and BW 91, we would like to mention that CW 76 and Denisov also possess a significantly low deviation and can reproduce the experimental data fairly well.

Though we mentioned a number of potentials with low deviation, we choose ten potentials with  $\sigma < 0.3$ . Proximity 1966, Proximity 1981-I, Proximity 1984, Proximity 1988, Proximity 2003-I, II, III, BW 91, CW 76 and Denisov are selected as the suitable potentials for studying alpha decay from parent nuclei in the heavy region. For understanding the predictive power of different nuclear potentials, we have plotted the deviation of these nuclear potentials. Fig. 2 shows the deviation of the above best 10 nuclear potentials having  $\sigma < 0.3$ . For a comparison, we have also plotted the deviation for the rest of the potentials having  $\sigma > 0.3$ , given in Fig. 3. Again from the figures, BW 91 is depicted as the best option of nuclear potential to study alpha decay. Also, the proximity potentials Proximity 1966, Proximity 1981-I, Proximity 1984, Proximity 1988, and Proximity 2003-I, II, III are considered as suitable nuclear potentials. Our previous study on the alpha decay of Po isotopes [42] claimed Proximity 2003-I, Proximity 1966, and Proximity 1977 as appropriate potentials. Compared to this,

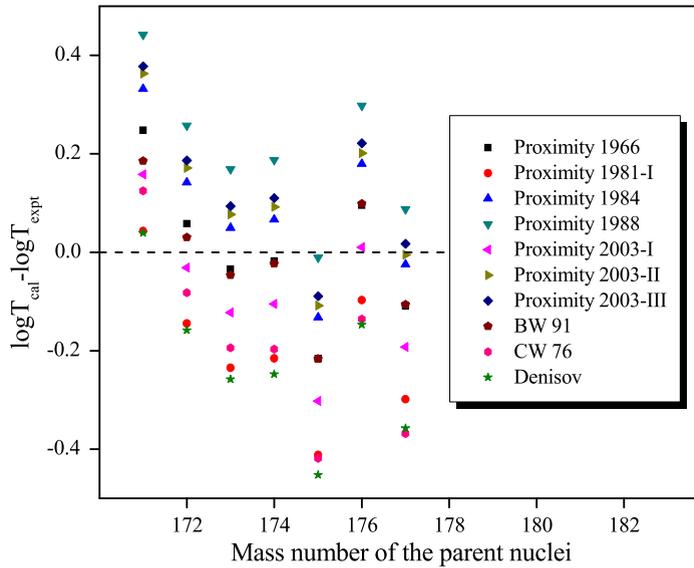


Fig. 2. Deviation of the predicted alpha half-lives with experimental data using different versions of the nuclear potentials for which  $\sigma < 0.30$ .

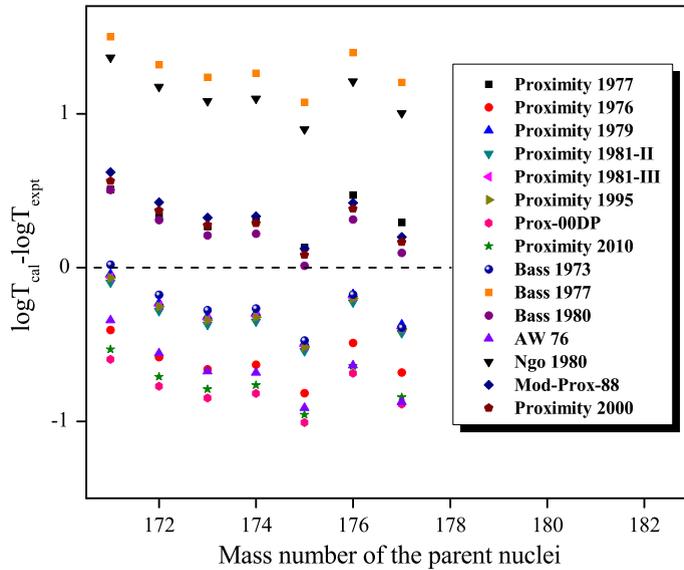


Fig. 3. Deviation of the predicted alpha half-lives with experimental data using different versions of the nuclear potentials for which  $\sigma > 0.30$ .

the results of our present study agree with the previous predictions, but in addition to the previous predictions, we have obtained more potential raised to the level of best potentials.

The agreement we attained with the experimental data established the robustness of the theoretical framework we have used. To check the validity of our approach, we have obtained the universal curve [75] of  $\log_{10} T_{1/2}$  versus  $-\ln P$  for alpha decay using all the nuclear potentials mentioned above, which is shown in Fig. 4. Irrespective of the parent nuclides and the nuclear potentials, a single curve is obtained for all alpha transitions. The graph is observed to show linear behavior with the same slope and intercept as the ones obtained in the case of cluster radioactivity, which again proves the strength of the model.

To study the effect of deformation on the half-lives, we have calculated the alpha decay half-life of  $^{171}\text{Hg}$  using our formalism Coulomb and proximity potential model for deformed nuclei (CPPMDN) [24], in which the effects of quadrupole deformation ( $\beta_2$ ) of the parent and daughter nuclei are included. The deformation values are taken from the mass table of Moller et al. [76]. For  $^{171}\text{Hg}$ , the half-life is found to be  $1.368 \times 10^{-4}$  s. Using the spherical version of CPPMDN, the Coulomb and proximity potential model (CPPM [25]), in which both the parent and daughter nuclei are treated as spherical; the corresponding half-life is obtained as  $4.768 \times 10^{-4}$  s. Comparing both values, it can be seen that on including deformation, the barrier penetrability increases and accordingly the half-life is reduced. This was well supported by the previous

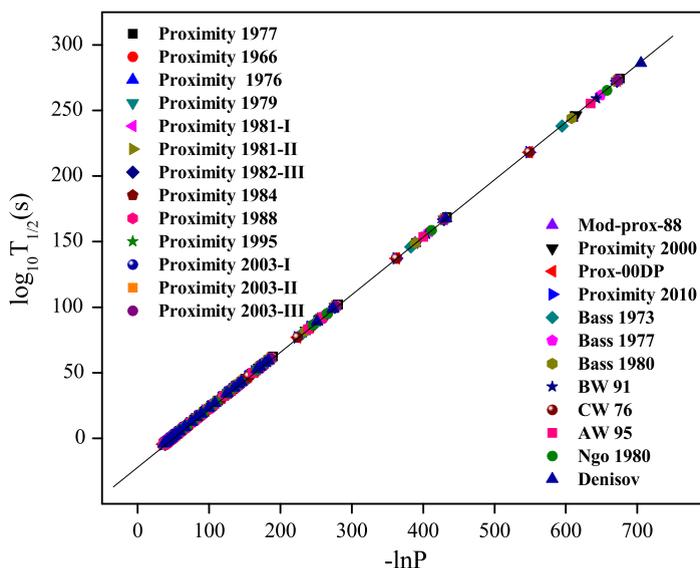


Fig. 4. Universal curve for the alpha decay of  $^{171-212}\text{Hg}$  isotopes using different versions of the nuclear potentials.

studies of Santhosh et al. [24,77]. Also, including the vibrational modes increases the value of penetration probability due to the reduction of the potential barrier, and accordingly the half-life is decreased [78]. Thus, it is worth pointing that the deformation and vibrational states tend to decrease the alpha decay half-lives and hence the study of effect of deformation and vibrational states is quite important in the calculation of the half-life of the decay process.

The agreement attained with the experimental data with significantly low standard deviation motivated us to predict the half-lives of 31 alpha emitters using different forms of nuclear potentials; the values are given in Table 1. We hope that our predictions may be detectable in the future.

#### 4. Conclusion

The half-lives for the emission of an alpha particle from Hg isotopes have been investigated using 25 different versions of nuclear potentials. The extensive calculations using the different nuclear potentials have been performed so as to select an apt potential that can be used for studying alpha decay. The predicted values on comparison with the experimental data suggested BW 91 as the most appropriate potential for alpha decay studies. Apart from BW 91, the study has shown that, among the proximity potential versions, Proximity 1966, Proximity 1984, and Proximity 2003-I, II are able to interpret the experimental results with a good matching. All together we have predicted ten nuclear potentials with  $\sigma < 0.30$  as the suitable potentials for alpha decay studies. The half-lives of certain Hg alpha emitters that are not verified experimentally yet are also predicted using different versions of nuclear potentials. The linear behavior of the universal curve and the experimental matching well established the strength of the theoretical approach used for the study.

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