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Plausible black hole solutions in higher-derivative gravity

Possibles solutions à trous noirs en gravité étendue

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A B S T R A C T

Here we obtain explicit black hole solutions in Extension Gravity models with high-order derivative terms, while the Lichnerowicz-type theorem simplifies our analysis by vanishing Ricci's scalar curvature. We find out two explicit static, spherical solutions that satisfy the presented action: the first one is the same usual Schwarzschild solution and the other one is the new non-Schwarzschild solution. It means that Schwarzschild's solution following the no-hair theorem can describe any black hole object on each gravity theory. Without considering the first law of thermodynamics for it, we show that the non-Schwarzschild solution is depending on its set of constants, and then we consider its entropy and other thermodynamic parameters for specific values of the constants.

R É S U M É

Dans le cadre de modèles de gravité étendue comportant des dérivées d'ordre supérieur, nous obtenons des solutions explicites représentant des trous noirs. Nous trouvons deux solutions sphériques statiques qui vérifient les équations : la première est la solution de Schwarzschild bien connue, alors que l'autre est une solution nouvelle non schwarzschildienne. Ainsi, la solution de Schwarzschild, qui résulte du théorème « sans cheveux », peut décrire un trou noir dans l'une ou l'autre des théories de la gravité. Sans partir de la première loi de la thermodynamique, nous montrons que la solution non schwarzschildienne dépend de certaines constantes, et nous évaluons son entropie et les autres grandeurs thermodynamiques pour des valeurs spécifiques des constantes.

The study of string and of some other effective gravities show that the usual Einstein–Hilbert action only describes a low-energy system, so one can assume Einstein's gravity to be the effective low-energy theory that needs correction with some terms built from the different powers of the curvature tensor and its derivative to illustrate high-energy systems [1]. Adding these terms to the usual Einstein–Hilbert action allows us to dissolve the non-renormalizability of gravity theory, albeit at the price of introducing ghost models [2].

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Studying Einstein's gravity with added curvature terms is worthwhile to shed light on some facts about the gravity field's behavior in the high-energy system. Black holes, among the interesting objects in general relativity, are among the best candidates for this goal. Hereupon in this paper, we study black hole solutions with high-order curvature terms added to the usual Einstein–Hilbert action. We suppose that the general action is given by

$$I = \int \mathrm{d}^4x\sqrt{-g}\left(\alpha R - \beta C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \gamma R^2\right) \tag{1}$$

where α, β, and γ are constants and $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, which is a trace-free part of the Riemann tensor. We shall work in units where $\alpha = c = G = 1$. The equations of motion following (1) are then

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - 4\beta B_{\mu\nu} + 2\gamma R\left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu}\right) + 2\gamma(g_{\mu\nu}\Box R - \nabla_\mu\nabla_\nu R) = 0 \tag{2}$$

where $B_{\mu\nu} = (\nabla^\rho\nabla^\sigma + \frac{1}{2}R^{\rho\sigma})C_{\mu\nu\rho\sigma}$ represents the Bach tensor. The equation of motion (2) can describe massive spin-2 with mass-squared $m_2^2 = 1/(2\beta)$, and massive spin-0 or massless spin-2 with mass-squared $m_0^2 = 1/(6\gamma)$.

We will now attempt to find black hole solutions explicitly for the equations of motion (2) when, without loss of generality, a static, spherically symmetric metric is given by

$$\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right) \tag{3}$$

With tedious calculation, field equation for non-vanishing components, with γ set to zero for the reasons discussed in [3], becomes

$$G_{\mu\nu} + 2\beta\left(\frac{5}{3}g_{\mu\nu}\Box R - R^{\nu\nu}C_{\mu\nu\mu\nu}\right) = 0 \tag{4}$$

where $G_{\mu\nu}$ is usual Einstein tensor. Obviously, for $\beta = 0$, one gets the usual Schwarzschild solution for static, spherically symmetric ansatz (3). Therefore, when $\beta \neq 0$, one has:

$$\text{I. } G_{00} + 2\beta\left(-\frac{5}{3}f(r)\Box R - R^{ii}C_{0i0i}\right) = 0, \quad i = 1, 2, 3$$

$$\text{II. } G_{11} + 2\beta\left(\frac{5}{3}\frac{\Box R}{f(r)} - R^{11}C_{1i1i}\right) = 0, \quad i = 2, 3$$

$$\text{III. } G_{22} + 2\beta\left(\frac{5}{3}r^2\Box R - R^{33}C_{2323}\right) = 0$$

and

$$\text{IV. } G_{33} = G_{22}\sin^2\theta = 0 \tag{5}$$

For $\theta \neq 0$, substituting (IV) into the (III) gives

$$\Box R = \frac{3}{5}\frac{1}{r^2}R^{33}C_{2323} \tag{6}$$

so that the explicit forms of I and II are

$$\begin{aligned} &G_{00} - 2\beta\left[R^{11}C_{0101} + R^{22}C_{0202} + R^{33}\left(\frac{f(r)}{r^2}C_{2323} + C_{0303}\right)\right] = 0 \\ &G_{11} - 2\beta\left[R^{22}C_{1212} + R^{22}C_{0202} + R^{33}\left(\frac{-1}{r^2 f(r)}C_{2323} + C_{1212}\right)\right] = 0 \end{aligned} \tag{7}$$

Solving each one of Eqs. (7) gives two different metric function $f(r)$,

$$\begin{aligned} &f_1(r) = 1 + \frac{c_1}{r} \\ &f_2(r) = c_2 r + c_3 r^2 \frac{1}{2\beta}\left[-3\ln(r)r^2 + 3r^3 + 2\beta\right] \end{aligned} \tag{8}$$

where c_1, c_2 and c_3 are constants. Comparing $f_1(r)$ with the usual Schwarzschild sheds light on the fact that, for $c_1 = -2\,M$, Schwarzschild's solution again could represent the black hole structure in high-order energy, which describes the equations of motion (2). On the other hands, the metric function $f_2(r)$ represents the general non-Schwarzschild solution that is

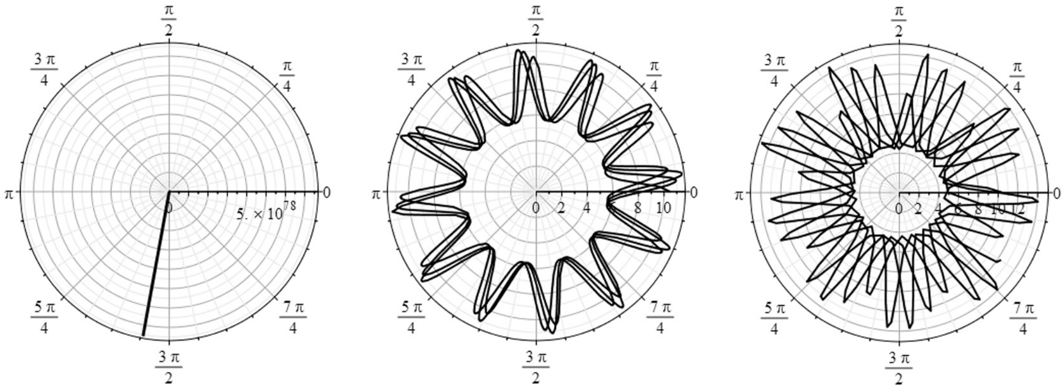

Fig. 1. Comparison of the geodesy of a falling physical particle for three different values of c_2 and c_3 when $\beta = 1$, $r_0 = 10.5$ into the horizon area.

shown in [3]; therefore, we can consider it as a metric function describing the space–time structure around black holes. One obtains some plausible metric functions. For instance, when $c_2 = 0$, $c_3 = -3/(2r_0^2)$ and $\beta \equiv r_0^2$, the metric function $f_2(r)$ becomes

$$f_2(r) = 1 - \frac{3}{2}\left(\frac{r}{r_0}\right)^2 \left[(r-1) - \frac{(r-1)^2}{2} + \frac{(r-1)^3}{3} - \ldots\right] \tag{9}$$

where we use Taylor series for the natural logarithm. An approach similar to (9) but with different coefficients has been investigated in ref. [3]. As shown in Fig. 1, the behavior of a physical particle falling in non-Schwarzschild black holes can be much different. For example, if $c_2 = c_3 = 0$, with respect to other constants of the model, the particle goes to singularity faster than in the usual Schwarzschild solution, while for two other cases when at least one of the constants is opposite zero, the particle follows a strange path. Note that Fig. 1 shows only the particle's behavior in the event horizon, and so the metric function $f_2(r)$ does not comply with the no-hair theorem, since this theory discusses classical properties of black holes out of the event horizon [9].

Entropy and the first law of thermodynamic are most interesting issues to investigate black hole solutions. Hereupon, we consider entropy in the following of our discussion. Since we are working with higher-order derivative theory, generally entropy is not one-quarter of the area of the event horizon but, as mentioned in [4], the entropy of black holes, when the latter are described with a usual Schwarzschild solution, is the same as in Bekenstein–Hawking's entropy formula. For a non-Schwarzschild solution that is illustrated with the metric function $f_2(r)$, we need to use the formula that was derived by Wald [5,6]. This has been evaluated for the ansatz (3) in quadratic curvature gravities in [7] and for $f(R)$ gravities in [8]. In our case, with $\gamma = 0$ and $\beta = r_0^2$, entropy is given by (A_H is area of event horizon)

$$S = \frac{A_\mathrm{H}}{4}\left[1 - \frac{4}{3r_0}c_2 - \frac{16}{3}c_3 + \frac{8}{r_0^2}\left(\frac{1}{3} + \ln(r_0)\right)\right] \tag{10}$$

and the Hawking temperature is

$$T = \frac{1}{4\pi}\left[c_2 + 2r_0c_3 + \frac{3}{2}\left(1 - 2\ln(r_0)\right)\right] \tag{11}$$

So, the Noether charge with respect to Eqs. (10), (11) writes

$$Q = -2A_\mathrm{H}\left[1 - \frac{4}{3r_0}c_2 - \frac{16}{3}c_3 + \frac{8}{r_0^2}\left(\frac{1}{3} + \ln(r_0)\right)\right]\left[c_2 + 2r_0c_3 + \frac{3}{2}\left(1 - 2\ln(r_0)\right)\right] \tag{12}$$

and, finally, the mass of the black hole for the metric function $f_2(r)$ is

$$M = \frac{1}{3}\frac{1}{r_0^2r^2}\left[-36c_2c_3r_0^2r^2\ln(r) - 16c_3^2r_0^2r^3 - 6c_2r_0^2r^2\ln(r) + 48c_3r_0r^2\left(\ln(r)\right)^2 - 12c_3r_0^2r^3 - 36r^3\left(\ln(r)\right)^2 \right.$$
$$\left. - 39c_2r_0r^2\ln(r) + 18r_0r^3\ln(r) + 10c_2^2r_0^2r - 132c_3r_0r^3 + 198r^3\ln(r) - 27r_0r^3 - 4c_2r_0^2 - 234r^3 - 24r_0r\right] \tag{13}$$

Generally, by choosing different values of the constants, we get a different behavior for each thermodynamic parameter, but from the slope of $M(T)$, it can be seen that the specific heat $C = \partial M/\partial T$ is negative for both usual Schwarzschild and non-Schwarzschild black holes.

Following the mentioned discussion, investigating the entropy and other thermodynamic parameters needs considering each case of constants. Here, we only consider the first law of thermodynamics. Analytically, we found that for $c_2 \neq 0$, entropy, the Hawking temperature, and the mass of black holes will be complex. Hereupon, we choose $c_2 = 0$ and $c_3 = -3/2$ as in Eq. (9). We have the freedom to add a constant multiple of the Gauss–Bonnet invariant to the Lagrangian, which shifts the entropy by a parameter independent constant without affecting the equations of motion. Using this approach allows us to find the mass and the temperature of these non-Schwarzschild black holes as a function of entropy:

$$\begin{aligned} M &\approx -2.6784 + 0.147668\, S - 0.0045\, S^2 + 0.00010\, S^3 \\ T &\approx 0.14668 - 0.00890\, S - 0.0004\, S^2 + 1.96\ 10^{-7} S^3 \end{aligned} \tag{14}$$

It can be seen that $\partial M/\partial S \approx 0.147668 - 0.0090S$, which is very close to the expression for the temperature; thus, the non-Schwarzschild black holes are seen to obey the first law of thermodynamics with high precision; the non-Schwarzschild solution satisfies the first law, $\mathrm{d}M = T\,\mathrm{d}S$.

As mentioned above, the slope of $M(T)$ is negative; to prove it for the above fixed set of constants, we have

$$C \approx -453.09 - 2601.8T - 8255.7T^2 \tag{15}$$

In this paper, we considered mathematically Einstein's gravity with some quadratic curvature invariants to find plausible explicit black hole solutions. These black holes may involve condensations of massive scalar, spin-2 and spin-1 modes and massless spin-2 graviton.

We focused our discussion on the static spherically symmetric black hole solutions. Attempting to find the explicit metric function $f(r)$ shows that, for ansatz (3), one has only two explicit black hole solutions, one of which is the usual Schwarzschild solution. Thus, the usual Schwarzschild solution allows one to investigate the black hole structure in high-order energy. In fact, the quantization of usual Schwarzschild solution allows us to explore the behavior of a physical particle in a quantum-gravity medium without loss of generality. The other metric function implies a non-Schwarzschild solution that has not been considered in literature. We applied Wald's formalism and derived an explicit entropy formula. Then the Hawking temperature, the Noether charge, and the mass of a non-Schwarzschild solution are given. We show that, for each set of constants, the non-Schwarzschild solution presents different behaviors. Thus, we use $\{c_2, c_3\} = \{0, -2/3\}$, in order to investigate the first law of thermodynamics to show that the non-Schwarzschild solution obeys the first law. Naturally, the non-Schwarzschild solution has wider classes than those considered here. We wish that our work will contribute to the future research in this direction.

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References

[1] G. 't Hooft, M.J.G. Veltman, One loop divergences in the theory of gravitation, Ann. Inst. Henri Poincaré A, Phys. Théor. 20 (1974) 69.
[2] K.S. Stelle, Renormalization of higher derivative quantum gravity, Phys. Rev. D 16 (1977) 953.
[3] H. Lu, A. Perkins, C.N. Pope, K.S. Stelle, Black holes in higher-derivative gravity, Phys. Rev. Lett. 114 (2015) 171601.
[4] N. Li, X.-L. Li, S.-P. Song, An exploration of the black hole entropy via the Weyl tensor, Eur. Phys. J. C 76 (2016) 111.
[5] R.M. Wald, Black hole entropy is the Noether charge, Phys. Rev. D 48 (1993) 3427, arXiv:gr-qc/9307038.
[6] V. Iyer, R.M. Wald, Some properties of Noether charge and a proposal for dynamical black hole entropy, Phys. Rev. D 50 (846) (1994), arXiv:gr-qc/9403028.
[7] Z.Y. Fan, H. Lü, Thermodynamical first laws of black holes in quadratically-extended gravities, arXiv:1501.00006 [hepth].
[8] J. Li, K. Meng, L. Zhao, Near horizon symmetry and entropy of black holes in gravity and conformal gravity, Gen. Relativ. Gravit. 47 (2015) 116.
[9] N. Gürlebeck, No-hair theorem for black holes in astrophysical environments, Phys. Rev. Lett. 114 (2015) 151102.