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Physics of particulate flows: From sand avalanche to active suspensions in plants

La physique des écoulements de particules : des avalanches aux suspensions actives dans les plantes

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A B S T R A C T

Flows of granular media in air or in a liquid have been a research field for physicists for several decades. Sometimes solid, sometimes liquid, these particulate materials exhibit peculiar behaviors, which have motivated many studies at the frontiers between nonlinear physics, soft matter physics and fluid mechanics. This paper presents a summary of the recent advances in the field, with a focus on the development of continuous approaches, which make it possible to treat granular media as a complex fluid and to develop a granular hydrodynamics. We also discuss how the better understanding of granular flows we have today may help to address more complex materials, such as colloidal suspensions or some biological systems.

R É S U M É

Les écoulements de milieux formés de grains dans l'air ou dans un liquide intéressent les physiciens depuis plusieurs décennies. Tantôt solides, tantôt liquides, ces matériaux divisés ont des comportements singuliers qui sont au cœur de nombreuses études à la frontière entre la physique non linéaire, la physique de la matière molle et la mécanique des fluides. Cet article se propose de faire un point sur les avancées récentes dans le domaine, en se concentrant sur le développement d'approches continues qui permettent de traiter le milieu comme un fluide complexe et de développer une hydrodynamique granulaire. Nous discutons également en quoi la compréhension plus fine des écoulements granulaires que nous avons aujourd'hui permet de mieux appréhender les matériaux plus complexes comme les suspensions colloïdales, voire certains milieux biologiques.

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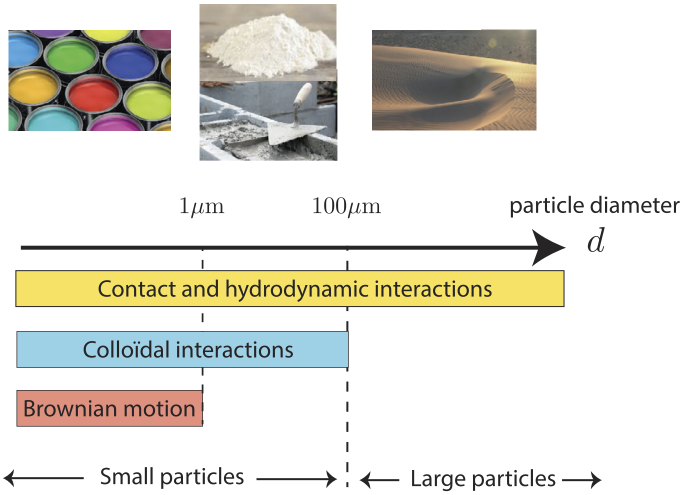

Fig. 1. Examples of granular media and illustration of the different interactions playing a role as a function of the size d of the particles.

1. Introduction: grains at different scales

What is in common between the mortar prepared by a worker in a mixer and a plant, which recovers a vertical posture after being tilted by the wind? Although very different, both situations involve to some extent the flow of a pile of grains. Coarse sand and gravels are avalanching in the rotating mixer while microscopic starch grains are avalanching in the statocytes, the specific cells in plants dedicated to the detection of the gravity direction. Understanding the dynamics of grain flows is thus important in many different situations, from industrial applications (for example in construction industry, pharmaceutical industry, food industry) to natural processes (for example, avalanches, debris flows, soil erosion, plant gravitropism), and at very different scales. The description of dense granular media still represents a real challenge, mainly because this multi-body system involves complex particle interactions [1–3]. The quest for a continuum description for particulate flows is not finished, although advances have been made during the last 20 years, which are discussed in this paper.

The properties of dense particulate flows depend on the particle size as illustrated in Fig. 1, the main reason being that interaction forces between solid grains strongly depend on their size. For particles typically larger than $d > 100$ µm, the dominant forces are due to direct mechanical contacts involving normal compression and tangential frictional forces, and when the grains are immersed in a liquid, to hydrodynamics interactions induced by the motion of the interstitial fluid. For smaller particles, typically $d < 100$ µm, other colloidal forces come into play, such as van der Waals or electrostatic interactions, or entropic forces induced, for example, by polymer brushes at the surface of the grains. At even smaller sizes, when $d < 1$ µm, particles start to be sensitive to the thermal agitation of the solvent, and Brownian motion becomes important. In this review, we start by considering large particles for which only contacts and hydrodynamic interactions play a role, before discussing the case of small particles, where complexity arises from the existence of other interaction forces. In the course of this paper, we will discuss results from experiments using model particles (often spherical beads), and also results from discrete numerical simulations, which consists in solving the motion of each individual grain from Newton laws using model force interactions to describe the contacts, and in solving the flow of the liquid between the grains [4].

2. Large particles: granular media and suspensions

Dry granular media or granular suspensions refer to media made of grains larger than 100 µm in air or fully immersed in a liquid. In this case, the dynamics is controlled by the collisions and the frictional contacts between the grains and, for suspensions, also by the hydrodynamic interactions. The case of partially immersed grains, which corresponds to systems where capillary forces come into play, is not discussed here, but belongs to the class of cohesive materials, which may share resemblance with the powders described in section 3.1. Although they look simple at first sight, granular media still resist a complete description able to predict their behavior in the whole range of observed regimes, from very dilute and agitated media to very dense and jammed systems. In the following, we discuss how a first approach to the rheology of granular flows can be inferred from simple dimensional arguments.

2.1. Rheology of granular media and suspensions

To study the rheology of complex fluids (i.e. the flow behavior in response to an applied force), the traditional approach consists in considering the plane shear configuration. The fluid of interest is confined between two plates separated by a distance h, the top plate moving at a velocity u so that the fluid is sheared at a constant shear rate $\dot{\gamma} = u/h$. Classical rheology consists in measuring how the shear stress τ exerted by the fluid on the top plate varies with the shear rate $\dot{\gamma}$. In the case of granular materials and suspensions, another crucial parameter has to be considered, namely the granular pressure $P^{\rm p}$. When sheared, the packing of grains pushes on the wall and exerts a normal stress $P^{\rm p}$ on the wall. As a consequence, there exist two ways to study the rheology of granular systems. In the classical way, the material is sheared at a constant volume, keeping the distance between the two plates constant. In this case, one has to measure how both the shear stress τ and the normal granular stress $P^{\rm p}$ vary with the shear rate for a given volume fraction of grains ϕ, where ϕ is the ratio of the volume occupied by the grains to the total volume of the sample. However, a second method has proved to be relevant for granular media and consists in shearing the material by imposing the granular pressure (more precisely, by imposing on the top plate of the shear cell the vertical normal stress on the grains). In the latter case, the volume occupied by the grains is free to adjust. In the following, we start by describing the pressure-imposed rheology before presenting the volume-imposed rheology and discussing the connection between the two.

2.1.1. Pressure-imposed rheology

The concept of pressure-imposed rheology has been introduced initially for dry granular media [5] and generalized later to suspensions [6]. The motivation was that, in many situations of interest in applications, the volume fraction of the grains is not controlled. A typical example is the case of granular avalanches when grains flow down a plane (in air or in a liquid) [7]. In this case, gravity imposes the stress distribution, but the volume occupied by the grains adapts, and the flowing granular layer may dilate or contract depending on the conditions (the grains being more or less agitated). Pressure-imposed rheology has been first developed to describe the properties of free surface flows and consists in prescribing a shear rate $\dot{\gamma}$ keeping the granular pressure $P^{\rm p}$ constant. This is achieved in a shear cell, where the top plate is free to move vertically and submitted to a constant vertical force (Fig. 2a). The top plate imposed the granular pressure $P^{\rm p}$ on the grains [5]. The two measured quantities characterizing the rheological behavior of the material in this condition are the shear stress τ and the volume fraction ϕ, which both depend on the shear rate $\dot{\gamma}$ and on the imposed pressure $P^{\rm p}$. Interestingly, in the limit of infinitely rigid particles of density $\rho_{\rm p}$ and diameter d interacting only through frictional contacts, dimensional analysis imposes that the system is controlled by a single dimensionless parameter called the inertial number: $I = \dot{\gamma}\, d/\sqrt{P^{\rm p}/\rho_{\rm p}}$. The inertial number is equal to the shear rate made dimensionless using an inertial time scale based on granular pressure. Consequently, from dimensional analysis again, the shear stress τ has to be proportional to the pressure $P^{\rm p}$ (the only stress scale in the problem), with a coefficient of proportionality (a macroscopic coefficient of friction) being a function of I. For the same reason, the volume fraction ϕ has also to be a function of the inertial number I only, such that:

$$\tau = P^{\rm p}\mu(I) \quad \text{and} \quad \phi = \phi(I), \qquad \text{where } I = \frac{\dot{\gamma}\, d}{\sqrt{P^{\rm p}/\rho_{\rm p}}} \tag{1}$$

The evolution of the two dimensionless functions $\mu(I)$ and $\phi(I)$ as a function of the inertial number I are sketched in Fig. 2a [5]. The macroscopic friction coefficient $\mu(I)$ starts at a constant value $\mu_{\rm c}$ when $I \to 0$, and experiments suggest that it tends to a second constant μ_2 when $I \to \infty$. The volume fraction ϕ is a decreasing function of the inertial number, with a maximum value $\phi_{\rm c}$ when $I \to 0$ [8].

The case of a suspension when the particles are immersed in a liquid of the same density and of viscosity $\eta_{\rm f}$ can be addressed following the same formalism (Fig. 2b). The suspension is sheared by a porous top plate, free to move vertically. The mesh of the plate is smaller than the particle size and the liquid can flow through it. In the limit of the viscous regime, when inertia plays no role, the rheology is controlled by a single dimensionless number called the viscous number $J = \eta_{\rm f}\dot{\gamma}/P^{\rm p}$, being equal to the shear rate made dimensionless using a viscous time scale based on the granular pressure. The friction coefficient and the volume fraction are then given by the following relations:

$$\tau = P^{\rm p}\mu(J) \quad \text{and} \quad \phi = \phi(J), \qquad \text{where } J = \frac{\eta_{\rm f}\dot{\gamma}}{P^{\rm p}} \tag{2}$$

The friction coefficient $\mu(J)$ and the volume fraction $\phi(J)$ are sketched in Fig. 2b. The friction coefficient is an increasing function of the viscous number starting at the same value $\mu_{\rm c}$ as in the dry case and increases linearly when $J \to \infty$. The volume fraction is equal to the same maximum value $\phi_{\rm c}$ as in the dry case in the quasi-static limit $J \to 0$, and decreases when J increases. Direct experimental measurements of $\mu(J)$ and $\phi(J)$ have been possible in a rheometer specifically designed to impose a constant normal stress [6,9].

The rheology of dry granular media and suspensions under pressure imposed conditions thus share similarities and are both described in term of a frictional law. In the quasi-static limit, when $I \to 0$ or $J \to 0$, no distinction can be made between a dry granular medium and a suspension: the maximum flowing volume fraction $\phi_{\rm c}$ and the quasi-static friction coefficient $\mu_{\rm c}$ are the same, suggesting that this limit is controlled by contact properties only. Experiments and numerical simulations shows that $\phi_{\rm c} \approx 0.59$ for spheres interacting with frictional contacts (less than $\phi_{\rm rcp} = 0.64$, the volume fraction of random close packing) and that $\mu_{\rm c} \approx 0.4$. It is important to note that $\phi_{\rm c}$ increases when the interparticle friction coefficient $\mu_{\rm p}$ decreases to zero, and for the case of frictionless particles ($\mu_{\rm p} = 0$), numerical simulations suggest that $\phi_{\rm c}$ coincides

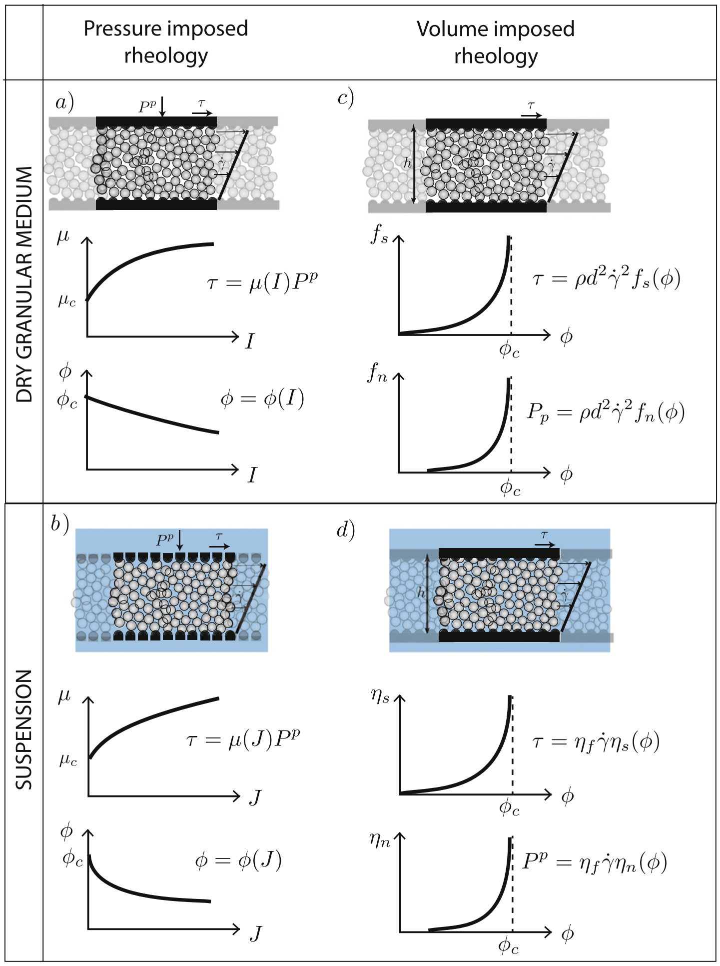

Fig. 2. a) Pressure-imposed rheology for dry granular media, and sketches of the friction coefficient μ and the volume fraction ϕ as a function of the inertial number I; b) pressure-imposed rheology for suspensions, and sketches of the friction coefficient μ and volume fraction ϕ as a function of the viscous number J; c) volume-imposed rheology for granular media, and sketches of the two Bagnold functions f_s and f_n as a function of the volume fraction ϕ; d) volume-imposed rheology for suspensions, and sketches of the shear and normal viscosities η_s and η_n as a function of the volume fraction ϕ.

with ϕ_{rcp} [10]. The quasi-static friction coefficient also decreases when decreasing the interparticle friction coefficient, but remains finite and equal to $\mu_c \approx 0.1$ for frictionless particles [10,11].

2.1.2. Volume-imposed rheology

The more conventional rheological approach consists in shearing the material keeping the distance between the two plates constant, i.e. keeping the volume fraction ϕ of the particles constant. In this configuration, the measured quantities are the shear stress τ and the granular pressure P^p, which are functions of the shear rate $\dot{\gamma}$ and of the volume fraction ϕ. The rigidity of the particles implies that there exists no intrinsic stress scale nor intrinsic time scale. For dry granular material made of particles of diameter d and density ρ_p, dimensional analysis thus implies that the shear and the normal stress τ and P^p are given by the following expressions:

$$\tau = \rho_p d^2 \dot{\gamma}^2 f_s(\phi) \text{ and } P^p = \rho_p d^2 \dot{\gamma}^2 f_n(\phi) \quad (3)$$

where $f_s(\phi)$ and $f_n(\phi)$ are two dimensionless increasing functions of ϕ, which appear to diverge close to the maximum volume fraction ϕ_c (Fig. 2c). The variation of the stresses with the square of the shear rate is often called the Bagnold law, following the pioneering work of Bagnold [12], who first experimentally evidenced the square variation of the stress with the shear rate.

Viscous suspensions, made of the same rigid particles but now immersed in a liquid of viscosity η_f can also be analyzed within the same framework. The suspension prepared at a volume fraction ϕ is sheared at a shear rate $\dot{\gamma}$ (Fig. 2b). In the viscous regime, when the inertia of the particles and of the fluid is negligible, dimensional analysis implies that the shear and normal stresses vary linearly with the shear rate as:

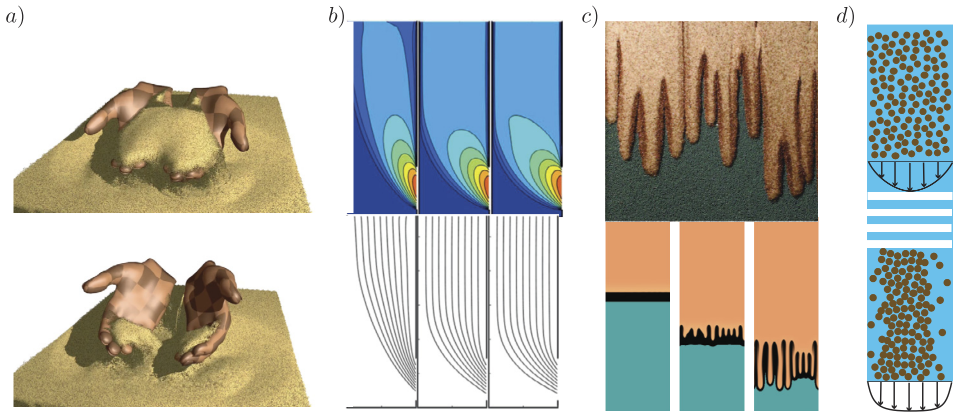

Fig. 3. Examples of simulations of granular flows using a continuum description; a) realistic simulations of flow of sand manipulated by hands from [16,17]; b) prediction of the flow in a silo from [18]; c) fingering instability observed when a granular front made of two sizes of particles flows down an inclined plane from [19]; d) flow of a suspension in a pipe, leading to a migration of the particles toward the center (from [20]).

$$\tau = \eta_f \dot{\gamma}\, \eta_s(\phi) \quad \text{and} \quad P^p = \eta_f \dot{\gamma}\, \eta_n(\phi) \tag{4}$$

where $\eta_s(\phi)$ and $\eta_n(\phi)$ are called the shear and normal relative viscosity respectively, and are two dimensionless increasing functions of ϕ, which also diverge close the maximum volume fraction ϕ_c [13,6,14].

The existence of two different descriptions (volume-imposed and pressure-imposed) can be perturbing for the neophyte interested in granular rheology. The material is described as a frictional material in the pressure-imposed approach, with a yield stress and a shear stress proportional to the pressure, whereas it is described as a viscous or Bagnold liquid in the volume-imposed approach, with no yield stress. This illustrates the fact that the behavior of granular materials strongly depends on the way they are manipulated. The two approaches are of course fully equivalent, and one can easily switch from one description to another, the two functions $\mu(I)$ and $\phi(I)$ (respectively $\mu(J)$ and $\phi(J)$ for suspensions) being related to the two functions $f_s(\phi)$ and $f_n(\phi)$ (respectively $\eta_s(\phi)$ and $\eta_n(\phi)$). Assuming that the function $\phi(I)$ (resp. $\phi(J)$) is monotonic, it can be inverted to obtain $I(\phi)$ (resp. $J(\phi)$). When injected in the definition of the inertial number I (resp. viscous number J), it comes that $f_n(\phi) = 1/I(\phi)^2$ (resp. $\eta_n(\phi) = 1/J(\phi)$) and $f_s(\phi) = \mu(\phi)/I(\phi)^2$ (reps. $\eta_s(\phi) = \mu(\phi)/J(\phi)$). The equivalence between pressure- and volume-imposed rheologies gives also information about the divergence of the functions $f_s(\phi)$, $f_n(\phi)$, $\eta_s(\phi)$, and $\eta_n(\phi)$ close to ϕ_c. The observation that the friction coefficient tends to a finite value in the quasi-static limit $I \to 0$ or $J \to 0$ implies that the two functions $f_s(\phi)$ and $f_n(\phi)$ diverge in the same manner when $\phi \to \phi_c$ (the same with the two functions $\eta_s(\phi)$ and $\eta_n(\phi)$), to insure that the ratio remains finite. Finally, the divergence close to ϕ_c can be written as a power law, i.e. $f_s(\phi) \propto f_n(\phi) \propto (\phi_c - \phi)^{-\alpha_{gran}}$, $\eta_s(\phi) \propto \eta_n(\phi) \propto (\phi_c - \phi)^{-\alpha_{sus}}$. Our experiments show that α_{sus} is close to 2 for frictional particles [13,6,9], but may be larger, about 2.8, for frictionless particles [15]. For the case of dry granular material, the divergence observed in numerical simulations is also close to $\alpha_{gran} \propto 2$.

2.2. Towards hydrodynamics of granular media

The knowledge of the response of a granular medium submitted to a plane shear has been used as a starting point to develop a full tensorial rheological model, able to describe complex flow configurations with shear in different directions. For dry granular media, the generalization of the frictional approach in terms of a friction coefficient depending on the inertial number seems a promising route. For suspensions, the presence of the liquid phase has to be taken into account, and two-phases flow approaches have been developed.

2.2.1. Continuum modeling of dry granular media

For dry granular media, the pressure-imposed approach appears to be the most relevant, as in most configurations, the volume fraction of grains is not prescribed and the system is free to dilate or contract. This has motivated the development of a simple description, based on a three-dimensional generalization of the $\mu(I)$ friction law, assuming that the material is incompressible and that the shear stress tensor is collinear with the rate-of-deformation tensor [21]. This approach is equivalent to a visco-plastic description, in which both the yield stress and the viscosity are pressure-dependent quantities. Such a description has been implemented in fluid mechanics codes, and quantitative predictions have been made for flows on inclined plane, flows in silo [22,18], granular collapses of columns [23,24]. Stability properties for flow on inclined planes have been predicted within this framework [25]. A modified version of the $\mu(I)$ rheology preventing negative pressure has been recently developed, and gives realistic results in situations as complex as hands manipulating dry sand [16,17] (Fig. 3). In geophysics, many applications involve granular layers on slopes, with a typical thickness much smaller than the flow length. In these configurations, a depth-averaged approach is relevant, which consists in writing mass and momentum equations integrated over the flow thickness. A whole corpus of works has been devoted to this approach, implementing the frictional rheology of granular media in depth-averaged equations [26–28]. Recent studies are able to describe segregation phenomena and the appearance of complex fingering pattern at the front of an avalanche flowing down a slope [19] (Fig. 3c). Although several limits exist to this approach that will be discussed later in this paper, an increasing number of studies show that this simple frictional rheology $\mu(I)$ captures important features of granular flows in complex situations. A last important remark concerns the existence of nonphysical instabilities in certain range of parameters of the generalized $\mu(I)$ rheology [29]. It has been shown that a short-wave instability may appear close to the quasi-static regime, or at high inertial number, which is reminiscent of an ill-posedness of the system of equations.

2.2.2. Continuum modeling of suspensions

The case of suspensions is more complex than the case of dry granular media due to the presence of the interstitial fluid. In many situations, a relative motion takes place between the liquid and the grains, and the dynamics of both phases needs to be properly described. Two-phase flow models have been developed, that consist in considering the fluid and the grains as two intricate continuum phases, and in writing the mass and momentum equations for each of them [30–32]. The difficulty lies in the choice of the stresses for each phase and in the choice of the interphase force [33]. The granular rheology discussed in the previous section provides expressions for the granular stresses, which can be injected in two-phase flow models to give predictions in different configurations.

For the expression of the granular stresses, we have the choice between the description in terms of the volume-imposed rheology (the viscous description) or in terms of the pressure-imposed rheology (the frictional description). Although the two descriptions are equivalent, depending on the configuration, it may be more convenient to use one or the other. For example, the flow of a suspension in a pipe when the particles and the fluid have the same density is typically tackled using the volume-imposed rheology and the description in terms of an effective viscous fluid, because the volume fraction is well controlled, whereas the particle stress adjusts. By contrast, the flow of dense grains immersed in a lighter fluid and avalanching down an inclined plane is a configuration where the volume fraction adjusts, and stresses are prescribed by gravity, meaning that the frictional description of the pressure-imposed rheology is more convenient.

An illustration of the success of the two-phase flow approach is the so-called migration phenomenon sketched in Fig. 3d [20]. An initially homogeneous suspension is injected in a pipe. The medium being equivalent to an effective viscous liquid, the velocity profile is a parabola, reminiscent of a Poiseuille flow. However, after a certain distance, the particles migrate to the center, and the concentration is higher in the middle of the pipe than on the wall, and the velocity profile presents a pseudo plug region in the center part. Qualitatively, the migration can be explained by considering the effect of particle pressure gradients. Initially, when the suspension is homogeneous, the shear rate is maximum close to the wall and vanishes in the center. From the expression of the granular pressure – Eq. (4) – this implies that P^{p} is maximum on the wall and vanishes at the center. This gradient of granular pressure creates a net body force on the particle phase from the region of high pressure to the region of low pressure, i.e. from the wall to the center, explaining the migration. This inward flux of particles is compensated by a outward flux of liquid. The migration stops when gradients of volume fraction are such that the granular pressure is homogeneous across the section. This phenomenon can be captured using the two-phase flow approach [34,33,35].

2.3. Microscopic origin of the rheology

We have discussed in the preceding sections the progress made in the continuum description of granular media and suspensions. However, the constitutive laws used in these approaches remain empirical. Relating the macroscopic behavior of the medium to the dynamics and to the properties of the individual grains remains a challenge. For example, predicting the value of the macroscopic friction coefficient from the properties of the grains (shape, interparticle friction coefficient), is still out of reach. However, numerical simulations using discrete element methods has been a powerful tool to investigate the dynamics of the grains and their collective motion, giving keys for developing theoretical analysis.

The most advanced theory has been developed for the dilute and agitated flow regime of granular media, called the granular gases [38]. In this regime the particles interact through collisions. The main difference with a molecular gas lies in the dissipative nature of the collisions. In the 1980s, a kinetic theory for granular gases has been developed [39], which has been much improved since, leading to quantitative predictions for shear flow in gaseous states [40]. However, the standard kinetic theory fails to properly describe dense granular flows when approaching the maximum volume fraction. When the volume fraction increases, the system leaves the pure collisional regime and particles experience enduring contacts, leading to cooperative motions. A first approach to capture the dense regime consists in modifying the kinetic theory. The Extended Kinetic Theory developed in [41] introduces a correlation length L, and considers that the dissipation is controlled by collisions between clusters of size L. This theory correctly predicts flows down inclined planes.

However, to describe and understand precisely the physics of granular flow or suspensions close to the maximum volume fraction, more detailed analysis of the fluctuating motion and the associated correlations have been necessary. Fig. 4 shows both the contact force network between the particles (Fig. 4a) and the fluctuating velocity around the mean applied linear shear profile (Fig. 4b) for two different volume fractions. Force chains are observed, which become more pronounced and extend on a longer domain when the volume fraction approaches the maximum volume fraction. The velocity fluctuations are also much more important and form vortex-like pattern when approaching ϕ_{c}. Theoretical approaches inspired from statistical physics of jammed systems have been developed to characterize this highly correlated fluctuating motion and to

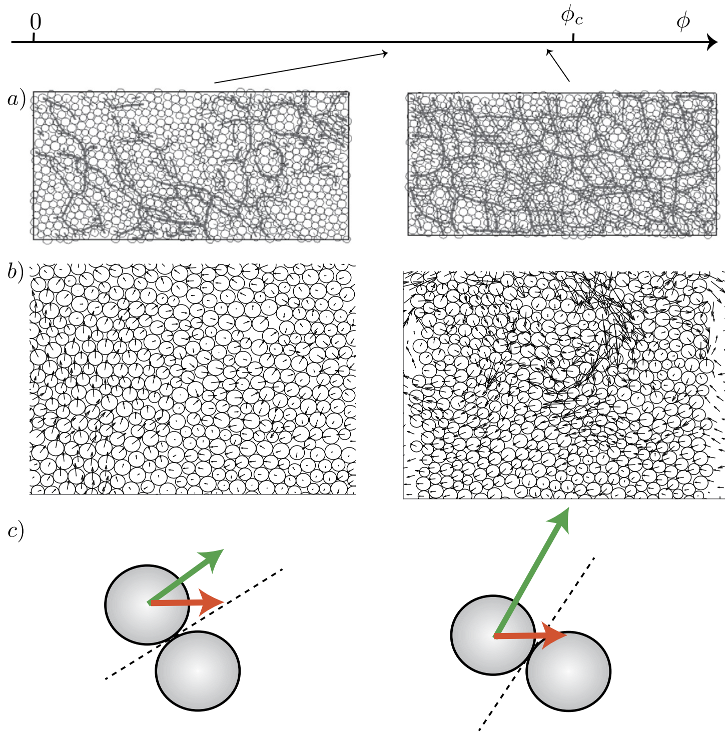

Fig. 4. Evolution of the force network; (a) (data from [36]) and of the fluctuating velocity field; (b) (data from [37]) for two different volume fractions approaching the maximum volume fraction; (c) cartoon illustrating the amplification of the fluctuations when approaching the maximum volume fraction (the so-called lever effect).

predict scaling laws close to ϕ_c [42,43]. The key ingredient is that the dissipation is enhanced because fluctuating motions are amplified close to jamming. This is called the lever effect [44,15]: because the system is close to a rigid transition, there exist only few degrees of freedom to deform, meaning that, to satisfy the macroscopic imposed deformation, the system exhibits more and more tortuous motions when approaching ϕ_c. The cartoon of Fig. 4c illustrates the lever effect with the simple system composed of two beads, one being fixed and the other being pushed horizontally. The rigidity transition corresponds to the two particles being aligned horizontally. In this cartoon, a system far from ϕ_c is represented by a particle far from horizontal (left in Fig. 4c). Imposing a horizontal displacement (the red arrow in the figure) leads to a larger real displacement displayed in green, due to the non-penetrability condition. The case of a system close to ϕ_c corresponds, in this cartoon, to a sphere almost aligned with the horizontal (on the right in Fig. 4c), where from geometrical arguments it is straightforward to show that the real displacement imposed by the same horizontal displacement is amplified. The amplification of the fluctuations then induces more collisions in the dry case, and more viscous dissipation in the suspension case. The lever effect has been used as the starting point in theoretical approaches predicting the divergence of the constitutive laws when approaching ϕ_c [42,43]. No doubt that the progress made in the understanding of the microscopic dynamics at the grain scale will help developing more accurate constitutive laws in the future.

2.4. Beyond simple rheology

The constitutive laws discussed in the previous sections are based on the generalization of the properties of steady uniform plane shear flows. Although real successes have been obtained in predicting complex configurations, this simple approach fails to properly capture the details of unsteady and non-uniform flows.

A first missing ingredient concerns transient flows. It is well known that a granular material can be prepared at rest at different volume fractions, higher or lower than the maximum volume fraction measured under a steady shear ϕ_c. This means that when sheared, a medium prepared at a volume fraction denser than ϕ_c has to dilate (the well-known Reynolds dilatancy), and that a packing looser than ϕ_c has to compact. As a result, a pile initially prepared in a dense state does not behave the same as a pile prepared in a loose state [45]. The influence of the preparation is even more pronounced for immersed granular media, as illustrated in Fig. 5a. A column of grains immersed in water initially prepared in a loose state spreads far and fast when released [46]. When prepared in a dense state, the collapse of the column is much slower and stops early. In this example, the coupling with the liquid phase plays a major role: during the dilatation phase of the dense case, liquid is sucked in between the grains, creating an additional compressive stress, which enhances the friction and decreases the mobility. For the initially loose case, the phenomenon is reversed. The initial compaction phase is associated

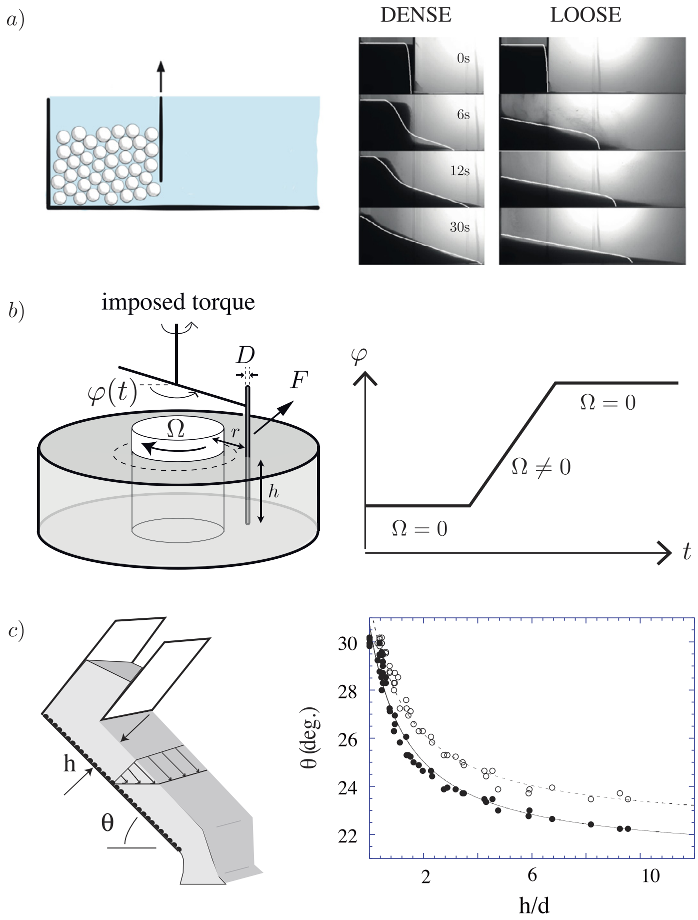

Fig. 5. a) Submarine collapse of granular column showing that the initial preparation of the packing in a dense or in a loose state strongly influences the dynamics [46]. b) Evidence of non-local effects in a Couette geometry: a rod submitted to a force F and immersed in the medium far from the shear band starts moving as soon as the inner cylinder rotates, although no macroscopic motion is observed around it [52]. c) Starting (open circles) and stopping (black dots) angles of a layer of grains on an inclined plane as a function of the thickness h/d of the layer [53].

with liquid being expelled from the medium, which diminishes the compressive stress, enhancing mobility. To capture those effects, an additional law describing the dilatation/compaction process has been proposed and coupled with the two-phase flow equations, leading to predictions for the initiation of submarine granular avalanches [47], or for the dynamics of an object impacting a dense or a loose sediment [48].

Taking into account the variations of the volume fraction represents only a first step towards a detailed description of transient flows. The microstructure of the medium, i.e. the orientation of the contacts, also evolves during transient flows before reaching the steady state. This is, for example, evidenced in experiments where the direction of the shear is reversed [49–51]. When a suspension initially sheared in a direction is suddenly sheared in the reverse direction, the viscosity suddenly drops before increasing again toward its steady value. The drop corresponds to the opening of the contacts, which were oriented in the direction of the initial shear and require a finite deformation to reorient along the new direction when the shear is reversed. Few attempts exists to develop rheological models that take into account the evolution of the microstructure [51].

A second limit of the rheology based on the generalization of steady uniform shear concerned non-local effects. A striking illustration is given by the experiment sketched in Fig. 5b [52]. A granular material confined in between two concentric cylinders is sheared when the inner cylinder is put in rotation at an angular velocity Ω. In the quasi-static regime, a shear band is observed close to the rotating cylinder, which extends typically over 10 particle diameters. A vertical rod is immersed in the apparent static region far from the shear band and submitted to a force F. When $\Omega = 0$, everything is static, and a minimal force F_c is necessary to move the rod, which otherwise remains static. However, as soon as the inner cylinder is put into motion and that a shear band develops, the rod slowly moves into the medium, although the force F is below the critical force $F < F_c$. This observation means that the flow close to the inner cylinder induces a flow further away close to the rod. This is a clear evidence of non-local effects, in the sense that the shear at one position influences the rheology further away in regions where no macroscopic flow is apparent. The appearance of non-local effects in the dense-flow regime is not a surprise, when considering the existence of highly correlated motion and long force chains described in section 2.3. However, taking them into account in rheological models is still a challenge. The development of non-local models is a very active domain of research [54,55].

A last observation not described by simple constitutive laws is the existence of a hysteresis in the transition from solid to liquid regime. A static layer of grains resting on a rough plane starts to flow when the inclination reaches a critical angle θ_{start}, whereas a flowing layer will stop when the slope goes below a critical angle $\theta_{stop} < \theta_{start}$ (Fig. 5c) [53]. On an inclined plane, the critical angles depend on the thickness of the layer, an effect reminiscent of non-local effects. The same hysteresis is observed on a pile or in a rotating drum. To trigger an avalanche, the pile has to reach a critical angle, which is higher than the angle observed after the avalanche. Although the hysteresis of the avalanche angles is a very well-known effect studied for more than 40 years, no consensus exists on its physical origin.

3. Small particles: powders and colloidal suspensions

In the first part of this review, we have seen that for particles interacting only through hard contacts or hydrodynamic interactions, dimensional analysis strongly constrains the possible steady rheological responses of granular matter. In particular, when sheared at fixed volume, the shear stress must be proportional to $\dot{\gamma}^2$ in the inertial regime (Bagnold behavior) and to $\dot{\gamma}$ in the viscous regime (Newtonian behavior), with a pre-factor that only depends on the volume fraction ϕ but not on the shear rate. However, real granular media and suspensions often deviate from this ideal rheological response. They can display a minimum stress to flow (yield stress) at constant volume, a shear-rate dependent viscosity (shear thinning or shear thickening behavior) or a time-dependent rheology (thixotropy). This is especially true for materials composed of 'small' particles, typically of diameters below 100 μm, like powders or colloidal suspensions (Fig. 1). The reason for this departure lies in the existence of other force scales at the particle level, such as cohesive forces, short-range repulsive forces, or thermal fluctuations. Unifying all these phenomena in a unique rheological description is still a challenge [56]. In the following, we give a brief overview of the main rheological behavior of these more complex granular media, with some recent progress in the field.

3.1. Attractive interaction

Cohesion in granular media and suspensions can have different physical origins. For instance, it is well known that adding a small amount of liquid to a dry sand is sufficient for building sand castles, a signature of cohesion arising from the capillary bridges between grains. Another source of cohesion comes from the van der Waals interactions between molecules that affect all particles at small scales. These attractive forces are responsible for the aggregation of small colloids in suspensions when no stabilizing repulsive forces exist. The first main consequence of cohesion in powders or colloidal suspensions is the appearance of a yield-stress in the absence of any external confining stress. In addition, the critical flowing packing fraction ϕ_c of cohesive media is usually smaller at low stress than at large stress, meaning that the response at fixed volume is now shear-thinning. Finally, in colloidal suspensions, the aggregation process itself is often slow and limited by thermal diffusion such that, in the absence of external forcing, the structure of the medium evolves in time. The rheological response therefore depends on the waiting time before the suspension is sheared. At large concentration, the network of aggregative particles can percolate throughout the whole system, providing the system a yield-stress (gel-like response). Under shear, the network of particles can be broken. This competition between the aging of the structure induced by aggregation and its rejuvenation induced by the shear rate gives rise to a rich rheological response, combining thixotropy and non-linear shear rate dependence.

3.2. Repulsive interaction

To prevent aggregation, colloidal suspensions are often stabilized by a short-range repulsive force between particles. This repulsive force can either stem from electrostatic surface charges on the particle surfaces or from a specific polymer coating. This is the case of modern concrete, where the addition of polymers (superplastizers) improves the workability of the fresh concrete and its final strength. It was recently proposed that such a short-range repulsive force F_{rep} induces a frictional transition in the suspension, which has dramatic consequences on the rheology [57,58]. At small shear rate (small stress), the repulsive force prevents the particle from making contact. The suspension thus flows as if particles were frictionless, with a viscosity that diverges at a critical packing fraction $\phi_c^{\mu_p=0} = \phi_{rcp} \approx 0.64$ corresponding to frictionless particles (see §2.1.1) (Fig. 6a). Conversely, at large shear rate (large stress), the hydrodynamic forces overcome the repulsive force and particles are pressed into frictional contact. The rheology of the suspension thus switches to that of a frictional suspension whose viscosity diverges at a critical packing fraction $\phi_c^{\mu_p\neq0} \approx 0.59 < \phi_c^{\mu_p=0}$. A suspension of frictional particles with a short-range repulsive force F_{rep} therefore has two possible rheological branches: a low-viscosity frictionless branch at low stress and a high-viscosity frictional branch at large stress (Fig. 6b). The transition between the two branches gives rise to a shear-thickening behavior, which has been studied both numerically [57,59] and theoretically [58]. Depending on the initial packing fraction ϕ, the shear-thickening transition can be continuous, discontinuous or even leads to the jamming of the suspension, as illustrated in Fig. 6b. The discontinuous transition occurs at a critical shear stress τ_c and a critical shear rate $\dot{\gamma}_c$ given by

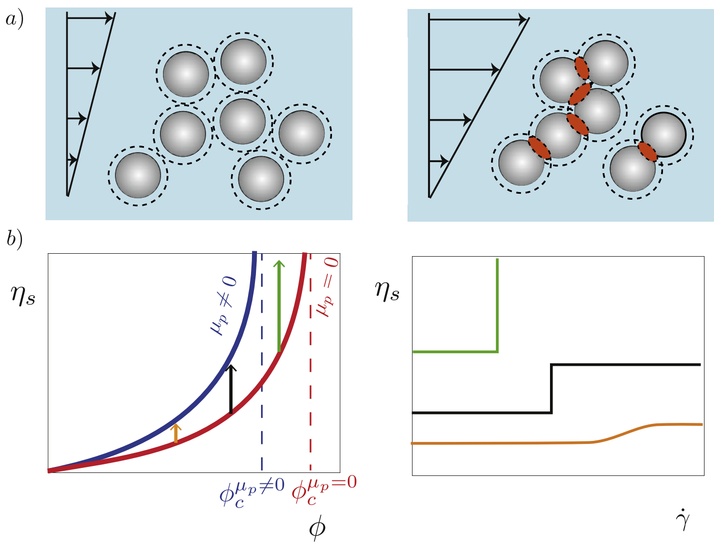

Fig. 6. Frictional transition and shear-thickening in dense suspensions. a) Frictional transition model for frictional particles with a repulsive force. b) (left) Viscosity branches corresponding to the frictionless and frictional states; (right) sketch of the shear stress as a function of the shear rate for the three different concentrations indicated by arrows on the left graph.

$$\tau_c = \beta \frac{F_{\rm rep}}{d^2} \quad \text{and} \quad \dot{\gamma}_c = \beta \frac{F_{\rm rep}}{\eta_s^{\mu_p=0}(\phi) d^2} \tag{5}$$

with $\beta \approx 0.04$ [59].

The above mechanism provides a coherent framework for explaining the dramatic shear thickening behavior observed in some non-Brownian suspensions of particles, such as cornstarch in water. Experimental support for this mechanism was recently obtained from direct measurements of the frictional properties of shear thickening suspensions. At the macroscopic level, Clavaud et al. [11] used rotating drum experiments and a model suspension where the repulsive force can be tuned to deduce the friction coefficient µ of the suspension from the avalanche angle. They showed that the presence of a repulsive force between particles leads to a frictionless state at low stress and a shear-thickening rheology, which are both suppressed when the repulsive force is missing [11]. At the microscopic level, Comtet et al. [60] measured the friction coefficient μ_p between pairs of particles of a shear thickening suspension using a tuning-fork-based Atomic Force Microscope. They evidenced a frictional transition above a critical normal load $F_{\rm rep}$, in good agreement with the shear-thickening transition measured for the macroscopic rheology [60].

3.3. Brownian suspensions

For suspensions with particle diameter typically below 1 µm, the thermal agitation of the particles (Brownian motion) is no longer negligible. This introduces a new dimensionless parameter in the rheology, the Péclet number, which describes the competition between the advection time scale, $\dot{\gamma}^{-1}$ and the diffusion time scale, $d^2/\mathcal{D}$, where $\mathcal{D} = k_B T/(3\pi \eta_f d)$ is the Stokes–Einstein diffusion coefficient. The Péclet number is given by:

$$Pe = \frac{\eta_f \dot{\gamma} d^3}{k_B T} \tag{6}$$

For $Pe \ll 1$, the diffusion time is much smaller than the advection time. The rheology then corresponds to the linear response close to the thermal equilibrium, with a shear-rate-independent viscosity and particle pressure, which are both of Brownian origin. A distinctive property of Brownian (frictionless) hard particles is that the thermal viscosity dramatically increases, by tens of order of magnitude, as the packing fraction approaches a value $\phi_g \approx 0.58 - 0.60$ [61,62]. Remarkably, this value is smaller than the maximal flowing packing fraction $\phi_c^{\mu_p=0} \approx 0.64$ discussed previously for *non-Brownian* frictionless spheres. This large increase of the viscosity at ϕ_g is analogous to the case of the glass transition observed in some molecular liquids when they are cooled below a critical temperature. It comes from the dramatic slowing down of the particle diffusion dynamics close to ϕ_g (the diffusion time for the particle to escape the 'cage' of its neighbors). Above ϕ_g, the viscosity is so large that the suspension develops a yield stress on experimental time scales.

Interestingly, thermal fluctuations in Brownian hard particles set a force scale, $F_{\rm ther} = k_B T/\delta$ where δ is the interparticle gap, which acts as an analogue of a repulsive force between particles [63,64]. As the Péclet number increases, the hydrodynamic forces enables to explore deeper the thermal repulsive shell around the particles, yielding an intermediate shear-thinning behavior. However, at large Péclet numbers, when the hydrodynamic forces overcome the thermal repulsive forces, the particles make contact. The suspension thus switches from frictionless to frictional and the same shear-thickening mechanism as described above can apply [65,66]. When both Brownian motion and repulsive force are taken into account, the critical shear stress for the shear-thickening transition is given by

$$\tau_c = \beta \frac{F_{\rm rep}}{d^2} + \alpha \frac{k_{\rm B} T}{d^3} \qquad \text{with} \qquad \alpha \sim \frac{d}{\delta^*} \tag{7}$$

In this expression, δ^* is the typical interparticle gap at which contact occurs, which may depend on particle roughness or elasticity. The respective importance of thermal motion or repulsive force in dense Brownian suspensions therefore depends on the detailed physics at the contact scale.

4. Active granular media: an example in the vegetal world

4.1. Introduction

So far, we have discussed the flowing behavior of passive granular matter, that is a medium made of inert particles that are driven through interactions with other particles and hydrodynamic stresses. However, the living world also offer fascinating examples of 'granular' assemblies that are made of non-passive or active 'particles', such as schools of fish, bacteria colonies or organelles in the cytoskeleton of the cells [67]. In these systems, the medium is composed of self-driven units that consume or extract energy from the surrounding fluid, such that each particle is animated by its own motion, which can be directional or random. Similar examples in the non-living world include motile colloids, collection of robots or shaken granular media made of anisotropic particles. Strikingly, this input of energy at the local scale can have dramatic consequences on the collective behavior and flow response of this active granular matter, including giant fluctuation of density, out-of-equilibrium phase transition, and emergent patterns. The topic of active matter initially emerged from theoretical and numerical studies on the collective motion of flock of birds, by analogy with phase transitions in condensed-matter physics [68]. More recently, the field gains strong momentum in the soft matter and biophysics community, with the study of motile colloids and active gels. In the following, we describe a peculiar example of active granular matter that we recently uncovered in the vegetal world: the statoliths that give plants the sense of gravity.

4.2. Statoliths: an agitated granular medium at the origin of plants' sensitivity to gravity

From tiny shoots to large trees, all plants are able to sense gravity and reorient their growth toward the vertical direction of the gravitational field [69,70]. This ability is not only important at early stages of development for roots to anchor in the soil and shoots to find light. It is also key all along the plant's life, for the plant to maintain its upright position and not fall against its own weight. The detection of gravity in all plants originates in specialized cells, called statocytes, in which starch-rich particles called statoliths are present (Fig. 7a). These micro-size grains are denser than the surrounding cytoplasm and sediment at the bottom of the cells, thus giving the direction of gravity. When the plant is tilted, the statoliths trigger a series of biochemical reactions that induce an asymmetric growth between the two faces of the organ. This differential growth eventually leads to the bending of the plant toward the vertical direction (Fig. 7a).

How cells detect the statoliths and how this sensing is converted into a bending growth response at the organ's level are still the object of many studies in plant biology. We recently showed that plants are actually not sensitive to the intensity of the gravity field, but only to the inclination against the direction of gravity [72,73]. The gravisensor in plants is thus a position sensor, not a force sensor. This finding is surprising, because it suggests that the pile of statoliths at the bottom of the cell move and respond to even the tiniest tilts. At first sight, such a behavior contradicts our knowledge of the physics of granular media, which stipulates that an assembly of grains cannot move below a critical avalanche angle set by friction and steric constrains between particles (see §2.1).

To address this issue, we directly visualized the motion of statoliths in gravisensing cells (wheat coleoptile) in response to various cell inclinations (Fig. 7b) [71]. When a cell is tilted, statoliths first behave like a classic (immersed) granular avalanche: the statoliths flow in the bulk and the pile angle rapidly relaxes toward a critical angle $\theta_{\rm c}$ in a few minutes (Fig. 7c). However, the long-time behavior of statoliths strongly contrasts with that of a classical granular medium. Instead of being stuck at the critical angle, the statolith pile keeps evolving and slowly creeps. Eventually, the free surface of the pile recovers the horizontal after few tens of minutes, as a liquid would do. Investigation of statolith motion at the particle level suggests that this long-time liquid-like behavior comes from the agitation of the statoliths. Unlike a passive granular material, statoliths exhibit random and large fluctuating motion, which likely helps the grains to unjam and flow even for very small inclinations.

To understand the origin of this agitation, we have compared the avalanche dynamics of the statoliths with the behavior of inert particles of similar size in biomimetic cells. In this case, the only source of fluctuation is thermal fluctuation (Brownian motion), whose intensity is characterized by the inverse gravitational Péclet number

$$Pe_{\rm g}^{-1} = \frac{k_{\rm B} T}{m\, g\, d} \tag{8}$$

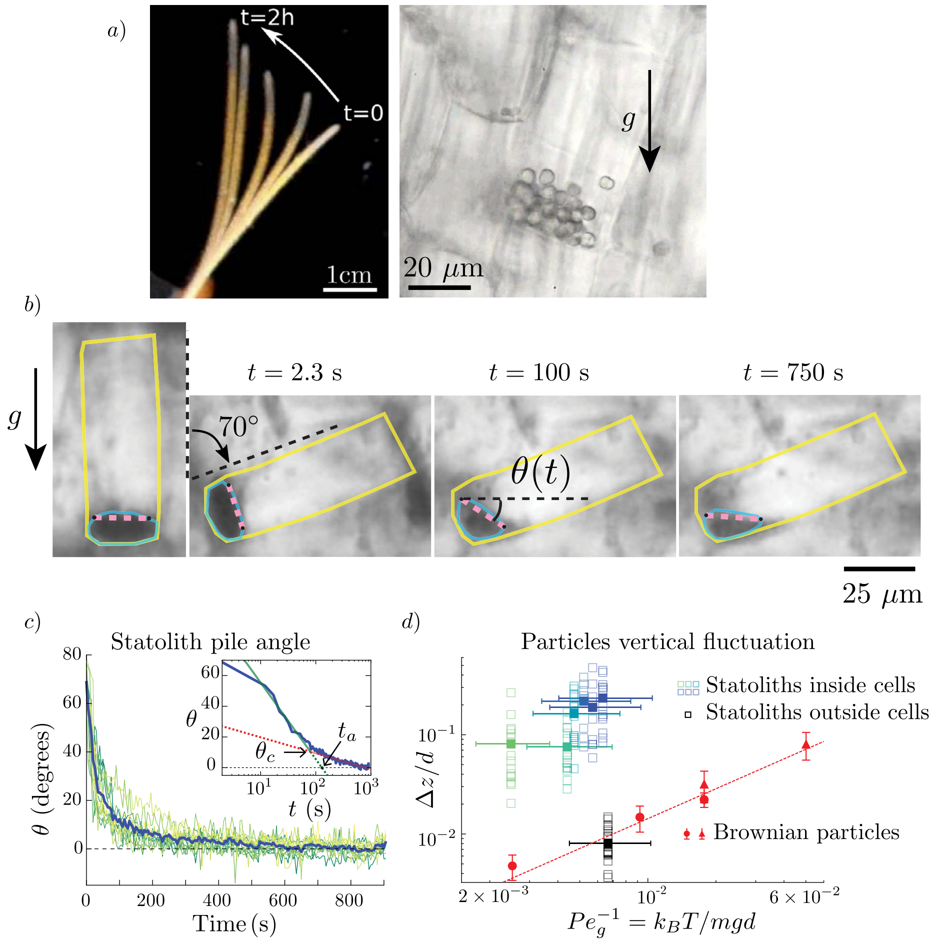

Fig. 7. An example of active granular matter: the gravisensors of plants. a) Gravitropism response of an inclined wheat coleoptile and picture of the gravisensing cells with the statolith pile at the bottom. b) Time-lapse images of a statolith pile avalanche in response to cell tilting. c) Statolith pile angle as function of time showing a first rapid granular avalanche regime followed by a slow creeping regime toward horizontal. d) Statolith vertical fluctuation inside and outside of the cell, compared with Brownian fluctuations of silica microparticles in water for different gravitational Péclet numbers. Adapted from [71].

where $k_B T$ is the thermal energy, m is the particle's mass corrected by the buoyancy, g is the intensity of gravity, and d the particle's diameter. In this biomimetic system, liquid-like avalanches similar to the biological ones can be observed if the thermal agitation is high enough compared to the gravitational energy (large Pe_g^{-1}). However, quantitative comparison between both systems shows that statoliths flow much more rapidly than purely Brownian particles having the same value of the inverse gravitational Péclet number. Everything happens as if statoliths in the plant cells were agitated by an effective temperature ten times higher than the physical thermal temperature (Fig. 7d).

The active nature of statolith agitation is therefore key to explain the remarkable sensitivity of plants to gravity [71]. The statoliths' agitation is large enough to erase the traditional flow threshold of granular media, while small enough to maintain the grains together at the bottom of the cell and give the direction of gravity. Our results show that this agitation is not thermal, but comes from the biological activity of the cytoplasm that surrounds the grains. This example shows how activity can strongly modify the flow and the transport of dense particulate media at small scales. Understanding the physics and the rheology of such active granular matter is an exciting topic for future research [74,75].

5. More open problems!

In this review we have discussed the physics of granular flows, illustrating through a few examples the recent advances made in the field. However, our presentation is far from being exhaustive, and some important problems have not been discussed, which represent novel avenues for future researches. Among them we can mention:

- the transition between the viscous and the inertial regime in suspensions,
- the flow of polydispersed particles, which could lead to segregation phenomena,

- the flow of deformable particles,
- the influence of the particles at the interfaces of liquid, with the dynamics of wetting, of drops, of films,
- the elongational rheology of granular media.

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