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The new International System of Units / Le nouveau Systéme international d'unités

Astronomical distance scales

Échelles de distance astronomiques

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ABSTRACT

This article is an overview of the determination of astronomical distances from a metrological standpoint. Distances are considered from the Solar System (planetary distances) to extragalactic distances, with a special emphasis on the fundamental step of the trigonometric stellar distances and the giant leap recently experienced in this field thanks to the ESA space astrometry missions *Hipparcos* and *Gaia*.

RÉSUMÉ

Cet article donne un aperçu de la détermination des distances astronomiques d'un point de vue métrologique. Les distances sont prises en compte depuis le système solaire (distances planétaires) jusqu'aux distances extragalactiques, avec un accent particulier porté à l'étape fondamentale des distances trigonométriques stellaires et le pas de géant récemment accompli dans ce domaine grâce aux missions d'astrométrie spatiale *Hipparcos* et *Gaia* de l'ESA.

1. Introduction

For centuries astronomers had to content themselves with a 2-dimensional world with virtually no access to the depth of the Universe. The world unfolded before their eyes as though everything was taking place on the surface of a spherical envelope with few exceptions for the nearest sources, such as the Moon whose nearness was made obvious from its repeated passages before the Sun (solar eclipses), the planets or the stars (occultations). The size of this sphere was arbitrary and could not be gauged, let alone the idea that the stars could lie at different distances. Until the 17th century a reliable estimate of the true distance to the Sun and of the size of the Solar System remained out of reach, although a good scale model could be accurately devised and actually crafted in the form of delicately adorned orreries (but not all were on scale).

Regarding the sidereal world and the immense vacuum lying beyond Saturn before reaching the first stars, some realistic ideas started emerging a good century later with the assumption that stars are Suns and share more or less the same

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luminosity. Gregory, Huygens among others came to numbers that at least hinted at the immensity of the world lying beyond the Solar System. However, the first indisputable stellar distances free of any physical assumption about the nature of the stars came out in 1840 through three independent labours, among which that of F.W. Bessel stands out. Once this first direct step has been mastered, astronomers developed gradually a whole set of methods to ascertain the distances of celestial objects, each new step going farther in the cosmos and depending on the reliability of the previous rungs.

This short review aims at an audience of scientists with no particular astronomical background beyond the general knowledge shared by every physicist. Simple and basic formulas that would not appear in an astronomical research paper are given and explained. Only the principles of the methods are provided, illustrated on simple cases, leaving out the real difficulties that are the daily bread of practitioners. The book [1] by M. Rowan-Robinson provides a more technical and comprehensive review of the subject from stars to cosmological distances. Published before *Hipparcos* and HST, the content is a bit outdated, but the description of the issues and the astronomical principles are still valuable and could be complemented with the more recent review of S. Webb [2]. At the Solar System level, the monograph [3] by A. van Helden is the best reference for the historical coverage from Aristarchus to Halley, but includes nothing relevant for the modern period.

The text is organised in two major sections. The first deals with distances within the Solar System with the length of the astronomical unit in kilometres to its recent conceptual mutation to a defining constant with a fixed relation to the SI unit of length. The second part covers the scale of the Universe from the stars to the cosmological distances, with a particular emphasis on the first fundamental rung of the ladder completely rejuvenated over the last twenty years with the two ESA astrometry satellites *Hipparcos* and *Gaia*.

2. Distances in the Solar System: the astronomical unit

2.1. Relative vs. absolute sizes in the Solar system

Astronomical distances have practically never been measured or numerically expressed with standard metric units, like m or km. First this would not be convenient units given the size of the Solar System, let alone the distances to the stars or that to the galaxies. One could claim with good reasons that this can be resolved by a proper choice of multiples, and this will not put astronomy aside from the SI system. This is true and there is a more fundamental ground for the use of an independent and consistent system of units in astronomy.

Except in very limited and relatively recent instances with radar and laser ranging in the Solar System, the measured space quantities in astronomy are always angles and not lengths or distances as it is on the Earth. Therefore, distances are derived quantities and byproducts of astrometric measurements attempting to detect small angular shifts in the direction of a celestial body resulting from its observation from at least two different points, as distant as possible from each other. The baselines, the Earth's radius or the size of the orbit of the Earth around the Sun, were not necessarily known in metric units with an accuracy matching that permitted with the angular measurements. This issue is more important in the Solar System than it is for the stars and the galaxies, for which no extreme fractional error is achievable, even today with *Gaia*, the on-going ESA Astrometry mission, or the HST (Hubble Space Telescope), the only providers of direct and accurate measurements of stellar distances in the visible, although radio astronomy can do even better on a small number of galactic H_2O or OH masers [4].

In the Solar System, the relative size of the planetary orbits was known to a good accuracy even before the discovery of Kepler's third law, relating the orbital period to the distance to the Sun. From pure angular observations, it was possible at the time of Copernicus to build a model of the Solar System showing the orbit of Mars or Venus with their correct scale compared to the Earth with a precision of about 5%. However the absolute scale expressed either in Earth radii, feet, or toises was not possible without loss of accuracy. This situation worsened, in some sense, when orbits could be computed with the laws of gravitation as the relative accuracy greatly improved and the gap between the relative and absolute size widened. The need to use a reference of length disconnected from the standards used for trade or scientific usage became mandatory to benefit fully from the accurate astrometry.

2.2. The astronomical system of units

Starting in the 19th century and made official by IAU in 1938, the astronomical unit was defined as a fundamental constant of the astronomical system of units as a length such that the gravitational constant is the square of the defining Gauss constant,

$$k = 0.017{,}202{,}098{,}95 \tag{1}$$

yielding

$$G = k^2 = 0.000{,}295{,}912{,}208{,}285{,}591{,}102{,}5 \tag{2}$$

with the unit of mass being the solar mass and the unit of time the solar day of 86,400 seconds. Combined with Kepler's third law,

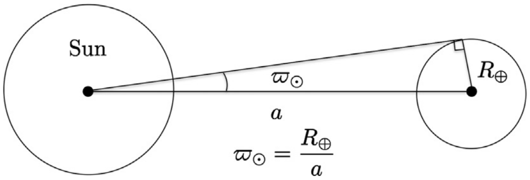

Fig. 1. Definition of the solar parallax.

$$\frac{a^3}{P^2} = \frac{GM_\odot}{4\pi^2} \tag{3}$$

Eq. (1) implies that the mean motion of a massless planet orbiting the Sun at one astronomical unit is k rad·day^{-1}, corresponding to a period of

$$P = \frac{2\pi}{k} = 365.25689832 \text{ days} \tag{4}$$

very close to the sidereal year. Therefore, the *au* defined by Eqs. (1)–(3) agrees with the simple initial idea of the astronomical unit being essentially the mean distance between the Earth of the Sun, or the semi-major axis of its orbit, although this is not formally its definition. With the above definition and units, the law of attraction reads

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{k^2\mathbf{r}}{r^3} \tag{5}$$

This allowed astronomers to produce very accurate numerical or analytical theories of the motion of Solar System bodies and predict their positions without having their absolute distances. The whole system is consistent and angular observations constrain the free constants of the model, primarily the position and velocity vectors of the bodies at an arbitrary epoch.

2.3. The solar parallax

The absolute length of the *au* was derived from dedicated observations as an angular quantity called the solar parallax, whose meaning is shown in Fig. 1, the measurement of the angle $\varpi_\odot$ being equivalent to the distance to the Sun expressed in Earth's radii. The latter being known in common unit, the procedure would yield the size of the Solar System in the same unit. Assessing this length remained a central issue in astronomy until very recently and was even referred to as *the noblest problem in astronomy* by G.B. Airy, the Astronomer Royal from 1835 to 1881. With the law of motion, a single measurement of a distance to one Solar System body suffices in establishing the absolute scale of the Solar System. The very first significant step in this direction was achieved in 1672 during a most favourable (small distance) opposition of Mars, making its parallax as large as possible. On 4 September 1672, Mars approached the Earth at 0.381 au, which is very close to the smallest possible distance of 0.371 au. Observing from two distant points J.D. Cassini in Paris and J. Richer in Cayenne found an equatorial parallax of 10″, or

$$\frac{R_\oplus}{1 \text{ au}} = 10'' \tag{6}$$

giving, $1\,\text{au} \approx 1.32 \times 10^8$ km. This is too small by ≈ 10%, but for the first time astronomers had a sensible estimate of the real size of the Solar System from a method whose principle was sound and could not be disputed.

The rare transits of Venus across the solar disk offered another way of ascertaining the au, as noted first by J. Gregory in 1663 and widely heralded by E. Halley in 1716. The advantage of Halley's proposal is still extant, since he proposed to replace pure angular measurements by timings of the moment when the dark disk of the planet is seen encroaching on the bright solar disk. Given the angular speed of Venus relative to the Sun, it is easy to show that a better accuracy can in principle been reached with the timing than with classical position sights. Halley claimed that the transit duration could be assessed to few seconds of time and consequently the distance to the Sun to one part to few thousandths. International cooperation was put in place for every following occurrence of the Venus transit in 1761, 1769, 1874, 1882 to observe and time the passages from the most remote places on the Earth. This led to adventurous expeditions that have been reported in many books and most is available online or in popular accounts [5,6]. Regarding the astronomical aim, the results were not on a par with the expectations and never reached the accuracy claimed by the illustrious astronomer. The extensive discussion of the four transits by S. Newcomb in 1892 ended up with a solar parallax of $\varpi_\odot = 8\overset{''}{.}79 \pm 0\overset{''}{.}018$ (current determination $8\overset{''}{.}794143\cdots$) or a value for the Sun–Earth distance of $(149.7 \pm 0.3) \times 10^6$ km. This was in some sense a very unsatisfactory situation in regards of the achievements of the planetary theories at the same time and after the triumph of the Solar System dynamics with the discovery of Neptune in 1846.

A fortunate circumstance cast some lights in an otherwise gloomy landscape with the discovery in 1898 of the minor planet Eros (433 Eros) simultaneously at Berlin and Nice, the first of the near-Earth objects to be identified. Eros comes

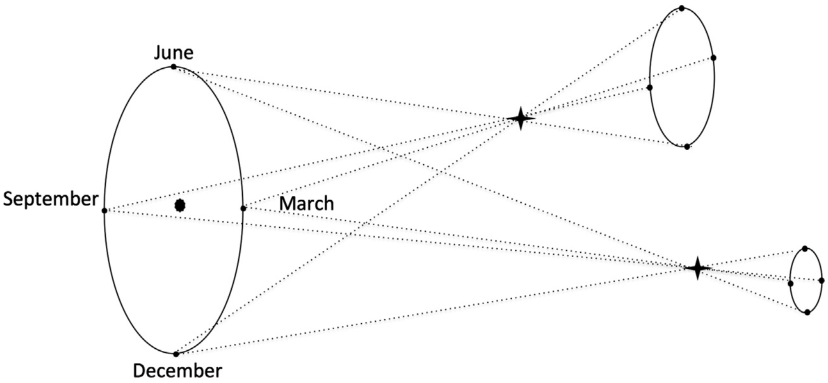

Fig. 2. The parallactic motion for a nearby and a distant star.

within the orbit of Mars and favourable oppositions that repeat every 30 years may bring the planet to 0.2 au[1] from the Earth, closer than any other Solar System object known at that time. The first such passage took place in 1901 and the next good one was in 1931. Again a broad international cooperation was set up to observe and reduce the observations and led to a solar parallax of $\varpi_\odot = 8.''790 \pm 0.''001$. It was the most accurately known value for the solar distance at that time, and this value has remained the standard until the mid-1960s when radar measurements gave a more accurate value for the distance to the Sun.

2.4. The astronomical unit today

Again a direct range measurement based on timing took precedence over classical angular measurements, with a measured quantity that was almost a distance, and no longer an angle. In particular, there were no more reasons to express it as a *parallax*, a formulation inherited from the measurement technique, but as a distance expressed directly in SI units, given the accuracy of the velocity of light. The distance became the primary quantity and the parallax a derived parameter. Later on, the use of spacecraft tracking combined to highly accurate global numerical integrations of the Solar System motions resulted into the best values of the astronomical unit [7–9], which eventually led the International Astronomical Union to recommend in 2009 (Resolution B2, IAU 2009 System of Units) a value of $149{,}597{,}870{,}700 \pm 3$ m for the au.

Eventually, this was turned into a defining astronomical constant in the IAU 2012 Resolution B2 with the astronomical unit being a conventional unit of length strictly equal to 149,597,870,700 m, in agreement with the value adopted in IAU's 2009 Resolution B2 [10]. Accordingly, the BIPM changed this unit from the table of non-SI units whose values in SI units must be obtained experimentally to the table of non-SI units accepted for use with the International System of Units. It is now tied to the meter with a fixed factor. In short, this is now a multiple of the meter and what should be experimentally determined is the scale factor of the Solar System, say the Sun–Earth mean distance expressed in au. A consequence is that to the equation of motion (5) one must substitute

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = -\frac{GM_\odot \mathbf{r}}{r^3} \tag{7}$$

with $GM_\odot$ in $\mathrm{m}^3{\cdot}\mathrm{s}^{-2}$ and the SI units or their multiples for length and time. Modern numerical integrations of the Solar System comply now with this requirement. As far as metrology is concerned, the situation is clarified and it is left to the astronomers now to refine their measurements to give the size of the orbits in meters with the best accuracy.

3. Distance to the stars

3.1. The trigonometric parallaxes

For centuries the problem of stellar distances has puzzled astronomers, although the underlying geometric principles needed to ascertain them were extremely simple and well understood. The basic idea is sketched out in Fig. 2, showing the apparent shift in the star position resulting from the annual motion of the Earth around the Sun.

Provided the stars are not infinitely remote compared to the size of the Earth's orbit, our annual displacement translates into a reflex apparent displacement of the stars on the sky, since during the year the different lines joining the observer to the star are not parallel. The farther the star, the smaller the parallactic ellipse, and more precisely its size is proportional to

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[1] The astronomical unit should be abbreviated as *au* since 2012 as stated in the recommendation of the International Astronomical Union in its Resolution B2. This is also the notation given by the BIPM in its official list of secondary units. It is usual to find instead AU or ua.

the reciprocal of the star distance. The parallax of a star is defined by the angle subtended at the star by one astronomical unit or half the apparent diameter of the Earth orbit when seen from the star. Mathematically, one has, for the parallax ϖ of a star at distance d from the Solar System,

$$\varpi = \frac{a}{d} \tag{8}$$

where $a = 1$ au. The unit of distance is the *parsec*, noted pc, and its common multiples are kpc, Mpc, Gpc. By definition of the parsec, a star at 1 pc has a parallax of $1''0$, meaning that its distance is 206,264 au, corresponding also to 3.26 light-years or 3.1×10^{16} km. With distances in pc and parallaxes in arcsec, one has $\varpi = 1/d$. No star has a parallax reaching one arcsec and the nearest star, Proxima Centauri, with a parallax of $0''77$, is at 1.30 pc from us. In astronomy, one does not measure distances directly, but their angular signature with the parallax. *Hipparcos* and *Gaia* deliver parallaxes from which distances are inferred. But the parallax is the best way to ascertain distances within the Galaxy without any assumption regarding the physics of the source, although this is not the only mean to do it. However, this is the only way to do it on large numbers of stars in a survey mode, while the alternative method needs eclipsing binary stars with good spectroscopy, a relatively rare instance.

Given the definition of this unit of distance, it is clear that both the measured parallaxes and the distances expressed in parsec are independent of the precise knowledge of the au in metric units and would not change with an improvement of the au. This is a direct expression of the angular shift on the sky with a distance given in Sun–Earth distance. Using light-year would have been an option, but nothing in the measurement principle would allow one to link naturally the parallax to the light travel time and simply to the finite speed of the propagation of light. The choice of the pc has been discussed around 1910 and made official by the IAU in 1925. Within the Galaxy distances are given in pc or kpc, while extragalactic distances are in Mpc. Beyond a few Gpc, the redshift, which is the measured quantity, is more usual. Also the distance modulus given in Eq. (15) is convenient when photometric data are involved. The diameter of the Milky Way is about ≈ 30 kpc and the Sun is located at ≈ 8 kpc from the galactic centre. The Andromeda galaxy, our nearest external major galaxy, lies at 0.78 Mpc and has a distance modulus of 24.5 mag.

It is also important to stress that in the heliocentric theory detecting the parallactic motion is a proof of the Copernican doctrine, and conversely its opponents exploited the lack of detection to support alternative theories and to challenge the doctrine. Therefore, the signature of the Earth's motion was primarily searched for fundamental reasons rather than to learn about the size of the Universe. The Tycho planetary system was a partial answer to this absence of evidence and could not be opposed as long the parallax of the fixed stars, or any other proof of the Earth motion, was not seen. This came before the first stellar parallax was measured through the discovery of the stellar aberration in 1727 by J. Bradley, while he was himself engaged into the search of the stellar parallax.

The tiny parallactic motion shows up as a periodic change of spherical coordinates as,

$$\Delta\alpha\cos\delta = -\varpi r(\sin\alpha\cos\lambda_\odot - \cos\alpha\sin\lambda_\odot\cos\epsilon) \tag{9}$$

$$\Delta\delta = -\varpi r[\sin\lambda_\odot(\cos\epsilon\sin\delta\sin\alpha - \sin\epsilon\cos\delta) + \cos\lambda_\odot\sin\delta\cos\alpha] \tag{10}$$

respectively for the star right-ascension and declination. Here, ϖ is the parallax (usually given in second or millisecond of degree), α, δ are the right-ascension and declination, $\lambda_\odot$ the ecliptic longitude of the Sun, ϵ the obliquity of the ecliptic and r the distance of the observer to the Sun in astronomical units (always very close to unity, even with *Gaia*). The path described by the star on the sky is an ellipse of semi-axes ϖ, $\varpi\sin\beta$, where β is the ecliptic latitude. This is a circle at the ecliptic pole that degenerates into a straight segment of length 2ϖ in the ecliptic plane.

In an ideal world, parallaxes could be found by sampling the equatorial coordinates of a star over a year and then extracting the amplitude of the yearly sinusoidal change in one or both coordinates. But the amplitude is at most $0''7$ in the most favourable case and two to three orders of magnitude smaller for a typical galactic star. In addition, there are other sources of change in the star coordinates that must be accounted for, and the parallax is usually a small fraction of the whole motion. Getting accurate absolute parallaxes is nearly hopeless with ground-based observations given the adverse effect of the refraction, the telescope flexure, and the difficulty to refer observations to an invariable frame of reference during the year.

As noted by Galileo, resorting to a small field offered a route to success. Instead of measuring the absolute displacement in a well-defined reference frame, one could detect the tiny parallactic motion with respect to one or a few neighbouring stars with the additional assumption that these reference stars are far more distant than the star whose parallax is searched. In short, the measurement is no longer ϖ, but the difference between the parallax of the nearby star and that of the reference star(s). One gets at the end a relative parallax instead of an absolute parallax, as long as one cannot tell how far the reference star is to correct the result for this bias. After many failed attempts, the German astronomer and mathematician F.W. Bessel made the first successful parallax measurement ever, for the star 61 Cygni, which he found equal to $0''314$ ($0''285 \pm 0''0005$ with *Hipparcos* and $0''28615 \pm 0''00006$ with *Gaia* for 61 Cygni B), or a distance of 3.2 pc (10.3 lightyears).

This marked the start of a systematic and difficult search, which is still on-going today with better instruments placed outside the Earth's atmosphere. This allowed astronomers to get about 100 measured parallaxes by the year 1900 with a relative accuracy better than 50%. The number grew steadily during the 20th century, as shown in Table 1, but this remained a painstaking task with low yielding, although the use of photographic plates from ≈ 1920 onwards relieved observers from

Table 1
Number of the published stellar parallaxes (number of stars with at least one parallax) before *Hipparcos*.

Year	number	notes
1840	3	61 Cygni, Vega, α Cent
1850	20	
1890	40	
1910	300	with 52 photographic parallaxes
1925	2000	photographic plates
1965	6000	Yale catalogue
1980	8000	just before *Hipparcos*

long hours at the eyepiece in the near open air, traded for equivalent long hours at the measuring machine in the comfort of a laboratory. See [11] for a discussion of the state of the art around 1910.

The large scatter of the measurements carried out on the same stars by different observers and different methods gives an idea of the systematic errors. The fourth, and last, version of the Yale Catalogue of Trigonometric Parallaxes [12], the reference in the field before *Hipparcos*, gives the trigonometric parallaxes for 8112 stars with a mode in the quoted accuracy of $0\rlap{.}''004$, an improvement by a factor of three compared to the previous release of 1963. To appreciate the difficulty to obtain parallaxes with ground-based astrometric techniques, in the interval of 32 years between the two publications, only 1,722 stars were added.

In any case, the number of reliable trigonometric parallaxes, say better than 10% in relative uncertainty, stayed below ≈ 2000 before the launch of *Hipparcos* and remained limited to bright stars. This was really very small in comparison with the contemporary sky surveys listing several hundred million stars with positions and some basic photometric information. Of course, the trigonometric parallaxes were not the only distance estimator available, but this was the only way to get a geometric measurement of the parallax free of any assumption on the physics of the stars, and any other method had to be calibrated on a collection of reliable distances and ultimately rested upon this small set of trigonometric parallaxes.

3.2. *The Hipparcos parallax survey*

Hipparcos (see for reviews [13,14]) opened a new era for astrometry thanks to the access to space to do accurate astrometric observations without the limitations caused by the bending and twinkling of light rays by the Earth's atmosphere. The key ideas to carry out absolute astrometric measurements from space can be traced back to the French Astronomer Pierre Lacroute in the mid-1960s. He realised that a census could be carried out with almost uninterrupted observations allowing one to triangulate the heavens with long arcs between pairs of stars. The most important and novel idea was to observe simultaneously two fields of view in very widely separated directions. Combining these two lines of sight onto a single focal plane would lead to a rigid network of stellar positions covering the whole celestial sphere, provided one could guarantee the stability of the angle between the two directions.

This was still basically relative astrometry, but not the way astronomers were used to, thanks to the wide angular separation between the two directions. The phase of the parallactic ellipses in each direction would be different and stars linked by the wide angle would change from time to time. At the end, one could reconstruct the 2-D position of each star at a reference epoch and solve for their motion and absolute parallax, at the expense of a global adjustment incorporating all the observations. After a long process of maturation, feasibility assessment, technical studies and lobbying, the *Hipparcos* project was selected in the European Space Agency's science program in July 1980 and eventually the satellite was launched in August 1989. The scientific goal was to reach a 2 milliarcsec (herein abbreviated as *mas* for $0\rlap{.}''001$) accuracy for about 100,000 stars, selected on the basis of their astrophysical interest, survey coverage at around 8.5 mag and ability of *Hipparcos* to observe them repeatedly during its planned mission of two years. The launch was dramatic and near fatal to the mission due to the failure to fire the apogee motor needed to reach the geostationary orbit. This ended up with the satellite on a wrong orbit, from which virtually nothing valuable was believed achievable and a life-expectancy much reduced due to the daily crossing of radiation belts. Eventually, the mission scenario was adapted in the course of emergency meetings between ESA, the prime contractor and the scientists and successful operations lasted until March 1993, when too much accumulation of high-energy particles fatally impaired the electronic. The final results published by ESA in June 1997 [15] surpassed the expectations placed on the satellite at its acceptance, and this has even been improved ten years later by a new data reduction almost single-handed by the UK astronomer F. Van Leeuwen [16].

The publication gave the astronomical community a brand new astrometric catalogue of 120,000 stars, all accurate in position and parallaxes to about one thousand of a second of arc (two times better than the initial objective) and supplemented with photometric [17] and double star data [18]. On top of that came also a less accurate but much larger catalogue of 2.5 million stars called Tycho-2 [19] resulting from measurements made with the *Hipparcos* star detector and the combination with the almost one-century-old photographic plates of the *Carte du Ciel* sky mapping.

Concentrating on the main topic of this review, *Hipparcos* astrometry was a truly new start for parallax survey. The total number of trigonometric parallaxes rose at once to more than 100,000, with nearly 50,000 better than 20% in fractional errors ($\sigma_\varpi/\varpi < 20\%$) and 20,000 at the 10% level. The reference frame was made inertial by linking the whole system to extragalactic sources, using radio stars common to *Hipparcos* and radio observations, or of observations of quasars relative to nearby *Hipparcos* stars [20]. This was an epoch-making advance in astrometry and in the measurement of stellar distances. Application to luminosity calibrations for a large variety of stellar types followed closely the publication and set the pace to improvements of the second rung of the distance ladder. More generally, the *Hipparcos* data have influenced many areas of astronomy such as the structure and evolution of stars and the kinematics of stars and stellar groups, the distance of the Hyades cluster, the galactic rotation from Cepheid variable stars, albeit the limited sample size of sources and observed volume. The outstanding and in-depth review by Perryman [21] based on most of the papers published in 1997–2007 using the *Hipparcos* catalogue provided an amazing detailed survey of the application of *Hipparcos* to stellar and galactic physics. The stellar distances and accurate proper motions together with the high-precision multi-epoch photometry are the crucial data exploited in these papers.

3.3. *The Gaia parallax survey*

Hipparcos was a resounding and acclaimed international success, allowing the Europeans to quickly submit several more ambitious proposals for space astrometry, at the same time as others were also submitted to NASA or to the Japanese space agency [22]. Only one of these proposals survived the various examinations by selection committees and *Gaia* was eventually selected as a cornerstone mission in April 2000 for a launch around 2011.

The basic concept is directly drawn from *Hipparcos*, but with a much larger telescope (actually two telescopes), a mosaic of 106 CCD detectors replacing the outdated photoelectric detector of *Hipparcos*. Two other instruments were added to carry out spectrophotometry and spectroscopic measurements, the latter to measure the velocity along the line of sight. While the *Hipparcos* catalogue was limited to 100,000 pre-defined stars brighter than 13.2 mag, *Gaia* was designed to realise a survey limited to a sensitivity of 20 mag. *Hipparcos* could only take a star at a time, while *Gaia* is able to record simultaneously several hundred images mapped on its focal plane. About one billion stars, amounting to ≈ 1 percent of the Milky Way stellar content, are expected to be repeatedly observed during the nominal 5-year mission, with a final astrometric accuracy of 25 µas at $G = 15$ mag ($1\ \mu\text{as} = 0.001\ \text{mas} = 10^{-6}$ arcsec).

Gaia's main scientific goal is to clarify the origin and history of our Galaxy, from a quantitative census of the stellar populations and extremely accurate astrometric measurements to derive proper motions and parallaxes. See [23] for the proposal and [24] for a presentation of the actual mission, the spacecraft, the operations, and the data acquisition strategy. The principle of the scanning satellite relies on a slowly spinning spacecraft to measure the crossing times of stellar images transiting on the focal plane. As for *Hipparcos*, there are two fields of view combined onto a single focal plane where astrometric measurements are done. The time relates the one-dimensional star position to the instrumental axes. The relation to the celestial frame is obtained with the satellite attitude, which is solved simultaneously with the star positions in a global solution, as described technically by Lindegren et al. [25].

The *Gaia* satellite was launched on 19 December 2013 and the science data collection started after the in-flight qualification on 25 July 2014. A first batch of results was released on 15 September 2016 with only 14 months of data processed. This release comprised primarily a position catalogue (only two position parameters per source) for 1.14 billion stars, the largest ever such collection. A smaller catalogue combining *Gaia* and *Hipparcos* included parallaxes and proper motions for $\approx 2{,}000{,}000$ stars with a sub-mas accuracy [26]. The release contained also variable stars and a set of 2200 quasars common to *Gaia* and the radio ICRF [27] used to align the *Gaia* and radio frames. Therefore the *Gaia* reference frame and ICRF are nominally identical.

In April 2018, the second release came out with parallaxes for nearly 1.4 billion stars [28], with a median uncertainty of 0.1 mas at $G = 17$ and 0.7 mas at $G = 20$. If we set the threshold for the usefulness of a distance to a relative precision better than 20%, at $G = 17$, *Gaia* DR2 reaches distances up to 2 kpc and 0.3 kpc at $G = 20$. The distribution of the parallax fractional uncertainty is shown in Fig. 3 in the form of ϖ/σ_ϖ. One reads directly on the histogram that about 50 millions stars (out of 1.3 billion in the survey) have a relative precision in distance better than 10%, to be compared to 20, 000 with *Hipparcos*. This again will increase around to 100 million in the more advanced releases.

This makes up the state of the art today regarding our knowledge of the stellar distances within our Galaxy from the geometric measurement of their parallaxes. *Gaia* found that there are 620,000 stars (more precisely unresolved stellar systems) within 100 pc and nearly 5200 at $d < 20$ pc. This gives a typical density of 0.15 star·pc^{-3} in the solar neighbourhood. For a uniform and random distribution with ρ stars per cubic parsec, one computes easily the mean distance between two closest neighbouring stars comes out as

$$< d >= \frac{4\pi\Gamma\left(\frac{4}{3}\right)}{3\left(\frac{4\pi}{3}\right)^{4/3}\rho^{1/3}} \simeq \frac{0.554}{\rho^{1/3}} \tag{11}$$

yielding with the *Gaia* census to $< d > \approx 1.04$ pc for the typical distance between a star and its nearest neighbour, not much different from the distance between the Sun and Proxima Centauri.

Although this data release uses only 22 months of data, while the nominal mission will have 60 and a possible extension of almost five years is likely, this parallax survey is by far the most comprehensive ever done and has no match in terms of size and accuracy, with the exception, regarding the accuracy, of a handful of radio masers observed with the VLBI

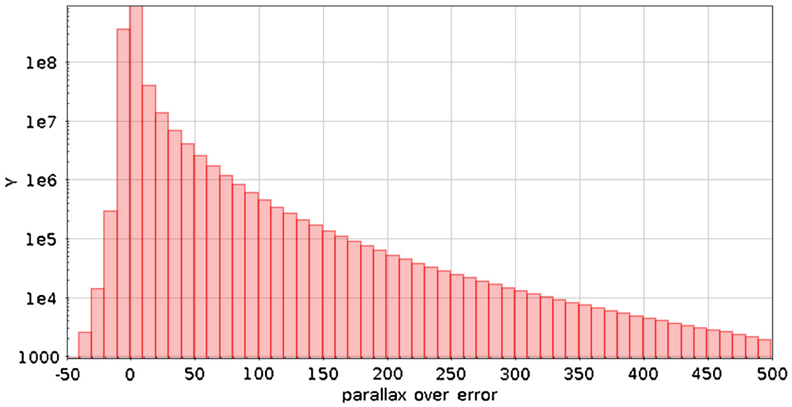

Fig. 3. Distribution of the relative accuracy of the parallaxes in the *Gaia* survey, given as ϖ/σ_ϖ. There are about 50 million stars with a parallax with a relative accuracy better than 10%. Credit: ESA/*Gaia*/DPAC.

technique. For the galactic stars, this is the crowning of nearly two centuries of parallax quest starting with F.W. Bessel in 1838. Until many years in the future there will be not such undertaking to get trigonometric parallaxes directly from astrometric observations and the *Gaia* survey is now actively being (see Sect. 4.3) used to reconstruct the whole distance scale beyond the Galaxy, based on secondary indicators.

I summarise in the next section the principles and ranges of applications of the numerous methods used today by astronomers to estimate distances from photometric indicators, but a major warning must be issued at this stage to any user deriving distances or other astrophysical parameters from the *Gaia* parallaxes. Better to read first the paper by Luri et al. ([29]) to guard against the numerous pitfalls to get a distance or a luminosity from the parallax. Only for the very accurate parallaxes ($\sigma_\varpi/\varpi < 0.1$), the straightforward transformation $d = 1/\varpi$ can be used safely. Otherwise, a better statistical inference must be used to keep the bias under control. As stated earlier, astronomers measure parallaxes and infer distances from them, and the inference is often a risky endeavour.

4. Secondary indicators

4.1. Overall principles

I refer here to distance estimators that are not directly derived from the stellar parallaxes as described in the previous sections, but from other physical of dynamical properties of the stars or of the systems of stars. Most of the methods are very simple in essence, since one tries as a rule to compare the apparent luminosity of a source, how much radiant power is received on the Earth (the easy step), to the true luminosity of the same source (the hard step). Most of the problems with these techniques come from the selection of the standard candles, their reliable identification, the calibration of the method, the systematic effect affecting them, the extinction of light during its journey to the Earth and the assumptions regarding the true luminosity of the sources. The determination of distances farther than the range accessible to trigonometric parallaxes follows more a less a single model with the following steps:

- identify a class of astronomical objects, bright enough to be seen at large distances,
- prove that they have a well-defined luminosity to qualify as standard candles,
- measure the flux on the ground or from space around the Earth,
- find their distances to calibrate their luminosity,
- identify and select similar objects to find the distances of far-away galaxies,
- calibrate a new rung of the ladder with these new distances

Let L be the absolute luminosity of an astronomical source, that is to say the total rate of luminous energy production, and l the flux received on Earth per unit of surface. If we assume a propagation without loss of energy, one has

$$l = \frac{L}{4\pi d^2} \tag{12}$$

where d is the distance between the star and the Earth. If l is measured and L is known or estimated from the star's physical properties, then one can estimate the distance. In practice, luminosities are expressed in a magnitude scale, and the distances in pc are related to the difference between the apparent (m) and absolute (M) magnitude as

$$m = -2.5 \log l + C_1 \tag{13}$$

$$M = -2.5 \log L + C_2 \tag{14}$$

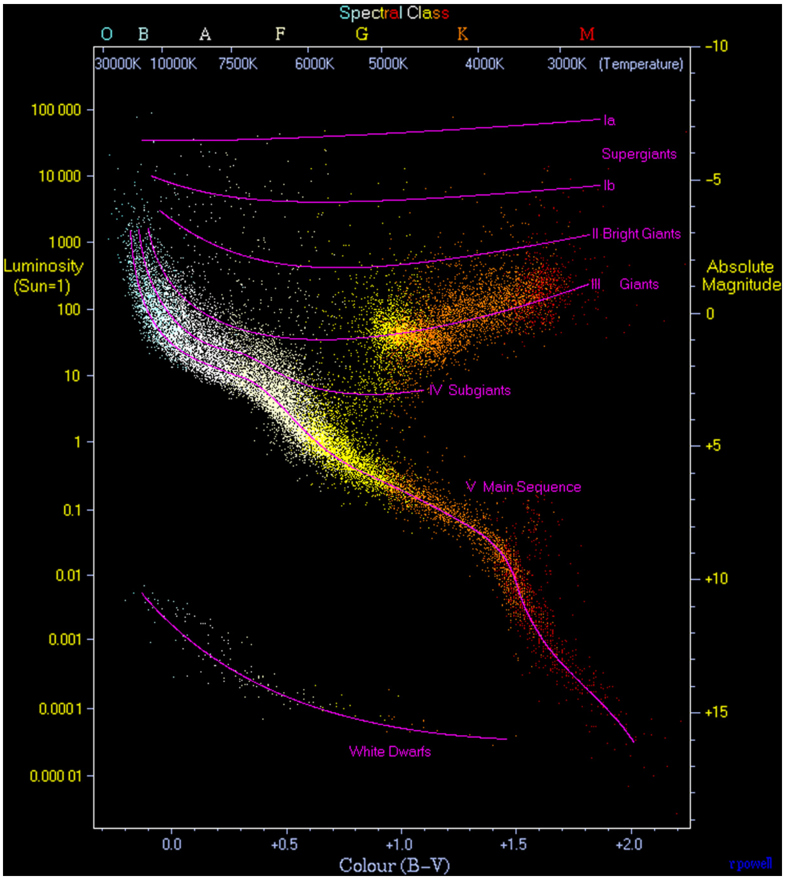

Fig. 4. An observational Hertzsprung–Russell diagram with 22,000 stars plotted from the *Hipparcos* Catalogue and 1,000 stars from the Gliese Catalogue of nearby stars.

for the apparent and absolute magnitudes and with Eqs. (8) and (12)

$$m - M = 5\log d - 5 = -5\log\varpi - 5 \tag{15}$$

with the convention that the absolute magnitude is the apparent magnitude the star would have if it were placed at 10 pc. Here ϖ is in arcsec, or equivalently the distance $d = 1/\varpi$ is in pc. Classically, the magnitude difference $m - M$ is called the *distance modulus* of the source. While rarely used for individual stars in our Galaxy, it makes sense for clusters of stars at several kpc or Mpc such as a globular cluster, a dwarf galaxy surrounding the Milky way, or a distant galaxy like Andromeda, as long as the source is resolved into stars. To illustrate this point, the LMC (Large Magellanic Cloud), well visible in the southern sky, is located at about 50 kpc from the Milky Way. Therefore, its distance modulus is 18.5 mag and a star similar to the Sun ($M = 4.8$) would have an apparent magnitude of 23.3, very faint for many of the telescopes in use today and not visible with *Gaia*. On the other hand, a classical Cepheid pulsating with a period of 4 days has $M \approx -3$ and would be seen as a star of $m = 15.5$ in the LMC, rather easy to detect with a medium size telescope and an accurate target for *Gaia*.

The extinction along the path is probably the most serious issue near the galactic plane, which essentially amounts to saying that the radiant flux decreases faster than the inverse square law. If one has an absorption coefficient of $\Gamma(l, b)$ in mag·pc^{-1} in the direction defined in galactic longitude l and galactic latitude b, Eq. (15) becomes, for a source at distance d,

$$m - M = 5\log d - 5 + \Gamma d \tag{16}$$

For stars, the extinction comes with a reddening, since dust scatters more efficiently the shorter wavelengths and the spectrum appears redder than what is expected for a star with known spectral type and luminosity class. There is a rather well-defined relationship between the reddening (called colour excess), allowing us to make the corrections from stars observed at the same place and in the same direction.

4.2. Distances of clusters

This is the first level of photometric/spectroscopic-derived distance indicator relying on the absolute luminosity of stars and the technique providing the very first estimate of a distance for most of the stars in or out of the Galaxy. It is based on the location of stars on the luminosity–colour diagram, usually referred to as the HR diagram, named after E. Hertzsprung and H.N. Russel, who discovered the feature independently around 1910. A diagram based on *Hipparcos* distances is shown in Fig. 4. Stars tend to fall only into certain regions of the diagram. The most prominent feature is the long concentration along a diagonal crossing the diagram from the top left to the bottom right. This is the location of the main sequence where most of the stars lie during their hydrogen burning phase. In the bottom left, there is a narrow line with white dwarfs, and, above the main sequence, several nearly horizontal lines with the subgiants, giants, and supergiants, a state in the star life when core hydrogen is exhausted. The Sun is found on the main sequence at $B - V = 0.66$ and luminosity of $1L_{\odot}$ by definition. The spectrum of a star and its location in the diagram are highly correlated, to the extent that a solar analogue can be recognised from the absorption lines of its spectra from the depth or absence of characteristic lines such as hydrogen, calcium, oxygen, etc.

If a distant Sun is found from its spectral characteristics, one may say that its luminosity is similar to the Sun's and its absolute magnitude is close to 4.7 in the V passband. Then confronted to its apparent magnitude, a distance may be inferred with Eq. (15), if extinction can be neglected. Using the reddening, the extinction can be included with Eq. (16) to get the distance as well. Due to intrinsic scatter between stars of similar properties, or because of different initial chemical compositions, the presence of an unseen companion, this method is not very accurate when applied to individual stars, although it is useful to get a first estimate of the distance for remote stars.

However, the same principle becomes much more efficient when applied to a cluster of stars [30,31]. Observing a cluster like the Hyades or the Pleiades, one may identify stars that belong to the cluster from their distances, kinematical parameters (they should have the same space velocity), chemical composition, and then exclude field stars not related to the cluster. These stars are thought to have been formed from the same initial cloud at the same epoch. They have the same age and share a similar content of heavy elements. Being similar and at the same distance, their distribution in the HR diagram displays narrow sequences, with little scatter, at least for all the single stars. The difference between the absolute HR diagram and that of a cluster using apparent magnitude is just a vertical translation equal to the cluster distance modulus, since $m - M$ is constant for all the members. By searching for the best fit between the main sequence of the cluster to a calibrated main sequence of the diagram, one gets immediately the distance of the cluster. If extinction is significant, there is also a horizontal translation for the reddening. The calibration must be done beforehand on the closest clusters, like the Hyades from the trigonometric parallaxes or the kinematics of the whole cluster combining the tangent and radial motions. There are many complications in the details (ages, metallicity, He abundance, that differ from the reference cluster and displace the sequence), but the principles are as described and allow one to get distance estimates for most of the not-too-old open clusters found in the galactic plane where main sequence stars are visible. A solar-like star with (absolute magnitude) $V \approx 4.7$ is brighter than (apparent magnitude) $m_v = 20$ up to $d = 10$ kpc, if extinction is neglected. So the method extends the distances achievable without trigonometric parallaxes to a few kpc in the Milky Way.

4.3. Distances from the Cepheids

Using Cepheids as standard candle is the single most important distance indicator for extragalactic distances up to some tens of Mpc. Cepheids are supergiants stars of type F to K with regular brightness variation over periods ranging from 1 to 50 days. The source of variability is well understood as an hydrodynamic instability causing the pulsation of the star, which changes in size and surface temperature during the cycle. As supergiants, Cepheids are intrinsically very bright and can be seen from a very large distance, therefore detectable in external nearby galaxies. With a brightness of $M \approx -5$, a star seen with apparent magnitude $m = 22$ is at a distance of 2.50 Mpc (Eq. (15)), three times as far as the Andromeda galaxy. During the first decade of the 1900s, H. Leavitt studied variable stars in the LMC and SMC (Large and Small Magellanic clouds, two companion galaxies at 50 kpc from the Milky Way), and found about 50 Cepheids in the LMC. She rightly noticed that the period of variability was all the more longer as the star was brighter. Moreover, she showed that the mathematical law relating the apparent magnitude and the logarithm of the period was linear [32]. This was at once a major breakthrough in this field with far-reaching consequences for the understanding of the structure of the Universe. It is fair to say that the early death of H. Leavitt deprived her of a likely Nobel Prize.

Given that these stars were all at the same distance, one could assert that the same relation held for the absolute magnitude, and provided the link between the period and the luminosity (the period–luminosity or PL relation) could be calibrated, one would know the distance to the host galaxy. Since then, many calibrations have been published from census of galactic Cepheids whose distances could be estimated by independent means. They are relatively rare sources and their number is limited to a few thousands, although many new have been discovered by *Gaia*. The population is rather uniform and the basic assumption is that Cepheids in external galaxies behave like those found in the Milky Way. The *Gaia* DR2 variability set comprises 9675 stars classified as Cepheids, against only 599, mainly in the region of the LMC, in the DR1 [33]. This represents the first full-sky census of Cepheids and provides a flavour of *Gaia*'s potential to recover most of the Milky Way Cepheids [34], not hidden by dust clouds. A typical P–L law has the form, with the period P in days,

$$M = a - b \log P \tag{17}$$

or, with a colour correction,

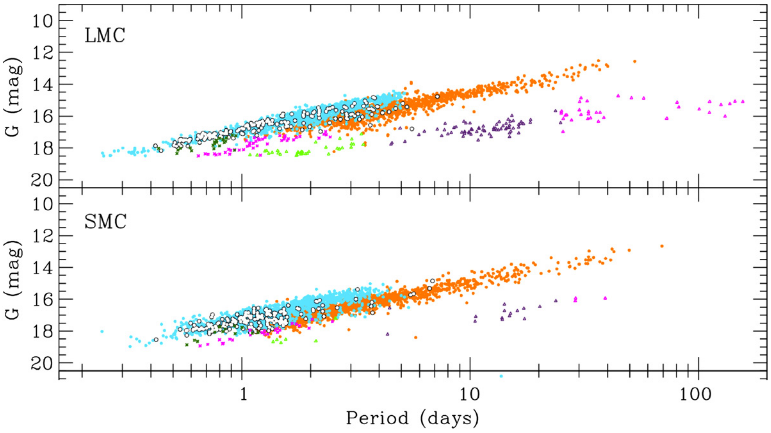

Fig. 5. Period–luminosity relation in apparent G magnitude uncorrected from reddening in the LMC and SMC from the *Gaia* DR2 sample (adapted from [34]).

$$M = a - b \log P + c(B - V) \tag{18}$$

where the most important parameter is the zero-point coefficient a. The coefficients b and c are independent of the distance and result from the analysis of the light curves. Other colour indices than $B - V$ are also used.

Until the advent of *Gaia*, the most recent realisations of the cosmic distance scale rested primarily on the Cepheid calibration using the *Hipparcos* parallaxes with

$$M_V = -2.76 \log P - 1.45 \tag{19}$$

with an estimated error in the range of 5–20%. Very significant is also the HST derived calibration [35]

$$M_V = -3.34 \log P + 2.45(V - I) - 2.52 \tag{20}$$

Using *Gaia* DR1 and distances from the TGAS solution (*Gaia* combined with *Hipparcos* and Tycho), Clementini and collaborators [33] gave a new calibration for classical Cepheids in the V-band as,

$$M_V = -2.678 \log P - (1.54 \pm 0.10) \tag{21}$$

See the paper for the details of the selection and the bias that may result. Taking the numbers as given, one sees that a Cepheid with a period of 50 days has a M_V magnitude of -6.1 and will be visible with the HST and without extinction at 2 Mpc. Similar calibrations have been also obtained for Population-II Cepheids and the fainter, but much more frequent, RR Lyrae pulsating stars. The latter are much more common stars and are useful distance indicators for globular clusters in the Milky Way halo up to 50 kpc. The calibration of the galactic Cepheids from the *Gaia* DR2 is not yet completed, but a partial result for the Cepheids in the Magellanic clouds is shown in Fig. 5 from [34]. The colours code different types of Cepheids with slightly different period–luminosity relations. The plots are impressive by themselves for the number of sources, the resolution between the different populations, and the small scatter around a visual linear fit. As the LMC/SMC distance modulus can be obtained by independent techniques, the curve can be transformed into absolute calibrations (with correction for the reddening) and compared to the galactic Cepheids.

4.4. Towards cosmological distances

As detected so far, the spiral galaxy NGC 3370 contains the farthest Cepheids yet found, at a distance of 29 Mpc. To reach distances where the Hubble flow becomes predominant, other rungs are required for galaxies beyond 500 Mpc. So far, the SNe Ia are the most relevant sources to be used as standard candles for the very large distances. SNe Ia result from the catastrophic instability of a binary white dwarf accreting material from its companion star and exploding when it reaches the Chandrasekahr limit. This well-defined particular condition accounts for the relative uniformity of the observable properties, such as the light curve of the SN Ia and their maximum brightness. They are recognised from other SNs by the shape of their light curve after the maximum, their spectra, and they constitute good standard candles with the peak luminosity $M_V \approx -19.5$, corresponding in energy output to about $10^9 L_\odot$ [36]. They are seen in all types of galaxies with typically one event per galaxy every five centuries. Using Eq. (15), one sees that with the HST they can be seen at few Gpc distances, that is to say at the start of the cosmological distances. But this peak, standardised for different light curves, needs to be calibrated, and again Cepheids are used for this purpose within galaxies at rather small distances of few Mpc, as explained by Sandage and Tammann [37] in a classical paper.

A very important application of the *Gaia* DR2 Cepheids dealing with this topic has been reported in [38] with the combination of the HST photometry of 50 Cepheids located in galaxies at $d < 50$ Mpc where Supernovae Ia have been found and used to extend the distance scale to the Gpc and constrain the Hubble constant. Basically, this fills the necessary step to assess the absolute luminosity of SNe Ia within relatively nearby galaxies from a distance estimate of these galaxies based on another standard candle. *Gaia* Cepheids in the MW are the most coveted sources to achieve this goal given their brightness ($G < 12$) and then their expected high parallax accuracy, about five times better than the HST astrometry. From the HST data and a previous Cepheid calibration using $H_0 = 73.24\ \text{km}\cdot\text{s}^{-1}\cdot\ \text{Mpc}^{-1}$, the authors have calculated the absolute magnitude of the Cepheids in the HST photometric system with the P–L relation,

$$M_H = -5.93 - 3.26(\log P - 1) \tag{22}$$

The analysis done in [38] confirms the existence of a bias in *Gaia* DR2 parallaxes, but larger than the *Gaia* quoted value of -29 µas based on fainter quasars [28]. In the magnitude range of bright Cepheids, they found -46 ± 6 µas instead. This has an implication for the Hubble constant, since the HST value is not in agreement with that determined from Planck's cosmic microwave background (CMB) data, which yields $H_0 = 66.93 \pm 0.62\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$. However, no such tension appears in [39] if we extend the DR2 bias found for the quasars to the bright Cepheids and the Planck value of H_0. The issue is not solved yet, but is just mentioned to show that even with the best tools in the hands of astronomers, as *Gaia* and HST are, the metrology is never simple and extreme care must be exercised everywhere. With *Gaia* parallaxes and their sheer number, a new page just opens up and new papers are expected in the coming years discussing and questioning the cosmic distance scale established with different techniques.

5. Conclusion

Large distances are the realm of astronomers with the characteristic that one cannot experiment but only deal with the information we can collect from the Heavens, primarily of electromagnetic nature, even though new promising messengers such as the neutrino astronomy and the emerging one with gravitational waves are just ahead of us with new challenges. In this brief overview, given the broadness of the field, I have attempted to show that an astronomer dealing with precision measurements must display the same rigour as a metrologist in his/her laboratory, by ceaselessly controlling his apparatus and above all calibrate and calibrate again. The metrological spirit pervades every area of experimental and observational science, whatever the scale of space or time under consideration.

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