INSTITUT DE FRANCE
Académie des sciences

Comptes Rendus
Physique

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Volume 23, Special Issue S1 (2022), p. 365-389

Published online: 6 January 2023
Issue date: 27 October 2023

https://doi.org/10.5802/crphys.102

Part of Special Issue: Astronomie, atmosphères et réfraction
Guest editors: Pierre Léna (Professeur émérite, Observatoire de Paris et Université Paris Cité, membre de l'Académie des sciences) and Luc Dettwiller (Université Jean Monnet Saint-Etienne, CNRS, Institut d Optique Graduate School, Laboratoire Hubert Curien UMR 5516, F-42023, SAINT-ETIENNE, France)

Les Comptes Rendus. Physique *sont membres du Centre Mersenne pour l'édition scientifique ouverte*
www.centre-mersenne.org
e-ISSN : 1878-1535

Comptes Rendus
Physique
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Astronomie, atmosphères et réfraction / *Astronomy, Atmospheres and Refraction*

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L'effet Nouvelle-Zemble et les Fata Morgana. Tracé de rayons dans une atmosphère sans symétrie sphérique

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Abstract. Raytracing in a non-symmetric atmosphere is discussed and illustrated by new simulations of both historical and mythical observations, such as the Novaya Zemlya phenomenon seen by the Dutch in 1597 and Nansen's sighting of it in 1894. An alternative interpretation is given of the mythical hafstramb monster in the Greenland Sea. Recent photographic material that shows mirages of the water surface is analyzed and a simulation presented. Raytracing techniques are reviewed and it is shown that numerical integration by distance is much preferred over other choices of the integration parameter.

Résumé. Le tracé de rayons dans une atmosphère non-symétrique est discuté et illustré par de nouvelles simulations numériques d'observations, historiques et mythiques, tel le phénomène de la Nouvelle-Zemble observé par le Hollandais en 1597 et rapporté par Nansen en 1894. Une interprétation nouvelle est donnée du monstre mythique *halfstramb* dans la mer du Groenland. Des documents photographiques récents, montrant des mirages de la surface de l'eau, sont analysés et leur simulation présentée. Les techniques de tracé de rayons sont revues et il est montré qu'une intégration numérique sur la distance est bien préférable à d'autres choix du paramètre d'intégration.

Published online: 6 January 2023, Issue date: 27 October 2023

1. Introduction

The Novaya Zemlya effect and Fata Morgana are understood as phenomena where light rays are trapped in a temperature inversion that is strong enough to duct them. This may occur when at some height above ground or sea the temperature gradient becomes such that a locally horizontal ray has the same curvature as the surface of the earth. This critical gradient is about 0.1 °C/m for otherwise normal conditions of pressure and humidity. If, moreover, rays are curved downward stronger above this height and less below it, they may travel on oscillating trajectories and follow the duct for as far as it stretches.

The condition for the Novaya Zemlya effect to occur, is that the light of the sun is captured below an inversion and is then ducted toward the observer. Or, as we like to think of it in our simulations wherein rays are traced backward from the observer: the rays will follow the duct for as long as it is strong enough to hold them. Away from the observer, beyond the point where the inversion has weakened enough, rays will either escape or hit the surface. Those that escape may produce the Novaya Zemlya effect if the sun happens to be just in the right spot.

Within the duct there may be obstacles for the light rays such as islands, ships, icebergs and many more things. These will then be seen by the observer and appear to him as mirages or Fata Morganas, their images often distorted beyond recognition.

I will first discuss the basic ingredients of ray tracing such as are required for calculating refractions in an atmosphere of spherical symmetry. Extension to non-symmetric atmospheres is necessary when a ducting inversion is introduced, which must naturally be of finite length. There are more refinements, that are suggested by available examples. These extensions will be made by discussing the best known historical observations and by making simulations of them. The examples I choose are in the first place the observation by the Dutch on Novaya Zemlya in 1597, the event from which the phenomenon takes its name [1]. I have made a simulation of it before, but I will present a new one here. Later in that same year the Dutch also report seeing mirages or Fata Morganas of the distant coast across the Kara Sea and I give a simulation of it.

The first illustration of the Novaya Zemlya effect is due to Fridtjof Nansen [2]. On 16 February 1894 he witnessed it as an image consisting of several horizontal stripes. To my knowledge a simulation of it has not been attempted before.

A mythical appearance in old Norse literature is the *hafstramb* or *merman* [3–6]. It may show itself as a humanlike figure on the horizon. In this case a simulation has been made earlier. Waldemar Lehn has proposed that it might be a mirage of an orca, sticking its snout above the water [7]. I will present an alternative simulation. These historical documents are great literature and their stories have played a role in our understanding of the phenomena that they describe. Making simulations of them turns out to be challenging. Besides that, I include translations of the original narratives.

More recent, of course, is photographic material of mirages, also from outside the Arctic. A special case is the "wall of water" at the horizon. It is found that not just one, but different types of mirage may produce this illusion. There exists a widespread misunderstanding about the word illusion. A Fata Morgana that you see is real. But what you think to recognize in it becomes an illusion if in reality it is not what you imagine to see. Examples in this article are: the Fata Morgana that shows you a distorted image of an iceberg is real enough and can be photographed. But when you think it to be a monster, then that is an illusion. Similarly for the Fata Brumosa and hafgerdingar that are Fata Morganas of the water surface in a distance. The effect is real. The illusion occurs only when you imagine to see a wall of water rising up vertically at the horizon.

2. Elements of raytracing

Before extending to spherically non-symmetric atmospheres, I will shortly review the methods that lend themselves to this purpose. Some of the following material has been borrowed from earlier publications on refraction [8–15], and is reused or adapted here with the kind permission of the publishers.

Figure 1 illustrates the (increments of) variables of the light ray: ϕ = polar angle measured from the Earth's center; $x = R_0\phi$ is the distance along the Earth's surface; β = tilt angle or slope, measured from the local horizontal. Its complement is the local zenith angle: $z = 90° - \beta$; ds = length of the trajectory element. These describe the geometry of the path. The aim of ray tracing is usually to determine path integrals, such as the refraction integral $\xi = \int(\mathrm{d}\beta - \mathrm{d}\phi)$

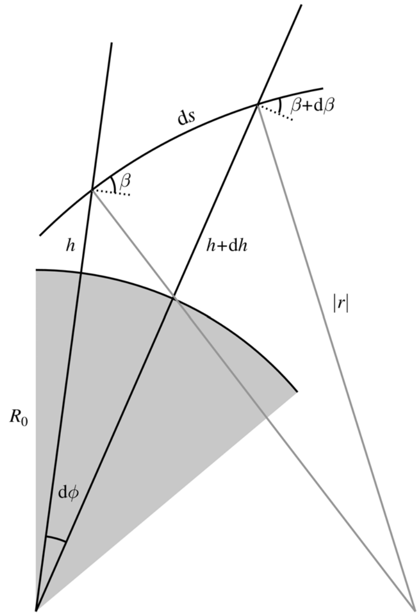


Figure 1. Ray segment and explanation of parameters.

(actually, when counted as positive, refraction $= -\xi$) and the air mass integral $M = \int \rho\,\mathrm{d}s$, where ρ is the density of any constituent of the atmosphere that we might be interested to know, dry air, water vapor, CO_2 etc...

For any path the following differential equations govern its geometry:

$$\mathrm{d}h = (\sin\beta)\,\mathrm{d}s \tag{1a}$$

$$\mathrm{d}\beta = \left(\frac{\cos\beta}{R_0+h} + \frac{1}{r}\right)\mathrm{d}s \tag{1b}$$

$$\mathrm{d}\phi = \frac{\cos\beta}{R_0+h}\,\mathrm{d}s. \tag{1c}$$

To these, the expressions for refraction and air mass may be added.

$$\mathrm{d}\xi = \mathrm{d}\beta - \mathrm{d}\phi = \frac{1}{r}\,\mathrm{d}s \tag{1d}$$

$$\mathrm{d}M = \rho\,\mathrm{d}s. \tag{1e}$$

I have chosen path length, s, as the integration variable in the above equations, in part because this choice provides the most concise forms. In particular, refraction and air mass are just path integrals. Other choices are possible and I will discuss their relative merits later-on.

The most widely used method for solving a set of coupled differential equations such as the above, is Runge–Kutta integration, a family of procedures that dates back to 1895 [16] and to which numerous refinements have since been amended, that are to be found in text books on numerical methods. In these equations, $1/r$ is the local curvature of the ray, and it is here that the physics enters. For a spherically symmetric atmosphere the refractive index, n, does not depend on ϕ. Then, the quantity

$$(R_0+h)\,n\cos\beta = Cst \text{ along the ray.} \tag{2a}$$

This is usually called Bouguer's law.

Table 1. Raytracing integration schemes

Int. variable = h	Int. variable = β	Int. variable = ϕ	Int. variable = s
$\mathrm{d}\beta = \dfrac{\left[1+\frac{R_0+h}{r\cos\beta}\right]}{(R_0+h)\tan\beta}\,\mathrm{d}h$	$\mathrm{d}h = \dfrac{(R_0+h)\tan\beta}{\left[1+\frac{R_0+h}{r\cos\beta}\right]}\,\mathrm{d}\beta$	$\mathrm{d}h = (R_0+h)(\tan\beta)\,\mathrm{d}\phi$	$\mathrm{d}h = (\sin\beta)\,\mathrm{d}s$
$\mathrm{d}\phi = \dfrac{1}{(R_0+h)\tan\beta}\,\mathrm{d}h$	$\mathrm{d}\phi = \dfrac{1}{\left[1+\frac{R_0+h}{r\cos\beta}\right]}\,\mathrm{d}\beta$	$\mathrm{d}\beta = \left[1+\dfrac{R_0+h}{r\cos\beta}\right]\mathrm{d}\phi$	$\mathrm{d}\beta = \left[\dfrac{\cos\beta}{R_0+h}+\dfrac{1}{r}\right]\mathrm{d}s$
$\mathrm{d}s = \dfrac{1}{\sin\beta}\,\mathrm{d}h$	$\mathrm{d}s = \dfrac{\frac{R_0+h}{\cos\beta}}{\left[1+\frac{R_0+h}{r\cos\beta}\right]}\,\mathrm{d}\beta$	$\mathrm{d}s = \dfrac{R_0+h}{\cos\beta}\,\mathrm{d}\phi$	$\mathrm{d}\phi = \dfrac{\cos\beta}{R_0+h}\,\mathrm{d}s$
$\mathrm{d}\xi = \dfrac{1}{r\sin\beta}\,\mathrm{d}h$	$\mathrm{d}\xi = \dfrac{\frac{R_0+h}{r\cos\beta}}{\left[1+\frac{R_0+h}{r\cos\beta}\right]}\,\mathrm{d}\beta$	$\mathrm{d}\xi = \dfrac{R_0+h}{r\cos\beta}\,\mathrm{d}\phi$	$\mathrm{d}\xi = \mathrm{d}\beta-\mathrm{d}\phi = \dfrac{1}{r}\,\mathrm{d}s$
$\mathrm{d}M = \dfrac{\rho}{\sin\beta}\,\mathrm{d}h$	$\mathrm{d}M = \dfrac{\rho\frac{R_0+h}{\cos\beta}}{\left[1+\frac{R_0+h}{r\cos\beta}\right]}\,\mathrm{d}\beta$	$\mathrm{d}M = \rho\dfrac{R_0+h}{\cos\beta}\,\mathrm{d}\phi$	$\mathrm{d}M = \rho\,\mathrm{d}s$

By taking its differential form, dividing out $n(R_0+h)\,\mathrm{d}h$ and using (1a), one finds

$$\frac{\mathrm{d}\beta}{\mathrm{d}s} = \left(\frac{1}{R_0+h}+\frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}h}\right)\cos\beta. \tag{2b}$$

Comparison with (1b) identifies the curvature as

$$\frac{1}{r} = \frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}h}\cos\beta. \tag{2c}$$

In each step, even in each sub-step of the integration, this expression for the curvature is taken along. It ensures that the Bouguer invariant is conserved as long as the atmosphere is spherically symmetric and that for a non-spherical atmosphere the curvature is adjusted to its local value.

It must be noted here that (2c) is often found with an additional minus sign. The philosophy behind it is that we are used to omit the sign when speaking about refraction. In this and previous works, I respect the formal mathematical definition of the curvature, wherein its sign is that of the second derivative of the path. Hence, the radius of curvature, r, of a downward bent path is taken as negative, as in Figure 1, where the radius of the local circle segment is given as its absolute value.

Evaluating the index of refraction requires knowing temperature, pressure and humidity for each height. This may be done by working with a polytropic atmosphere, with piece-wise constant lapse rates. It has the advantage that it allows for an analytic evaluation of the refractive index, such as proposed by Auer and Standish [17] and later in Sinclair and Hohenkerk's implementation of the method, which has become known as the RGO (Royal Greenwich Observatory) method [18,19]. In essence the method was described a century and a half earlier by Jean-Baptiste Biot [20]. This development is discussed by Dettwiller in the articles [21,22] of this special issue.

At the same time, this restriction to a polytropic atmosphere has become a disadvantage. For the understanding of out-of-the-ordinary sunsets and mirages, we want to consider more flexible and realistic temperature profiles. Nowadays, computing memory and speed are largely sufficient for numerically calculating the refractive index and its derivative in any atmosphere.

2.1. *Choice of the integration parameter*

Above, the integration scheme is given for path length, s, as the integration parameter. By (1a)–(1c), one may make a transformation and instead choose integration by height, h, by tilt angle, β, or by distance along the Earth's surface, $\mathrm{d}x = R_0\,\mathrm{d}\phi$. The different schemes are given in Table 1. And since they are connected by simple transformations, of course all methods give identical results, when applied to a spherically symmetric atmosphere and they preserve Bouguer's invariant with the same accuracy, which depends on the (double) precision of your computer program.

There are also limitations, and these are different for the four options. Integrating by height introduces singularities at the horizon, because in all differentials the denominator contains either $\sin(\beta)$ or $\tan(\beta)$, as shown in the first column of Table 1. Even with this problem cured, rays leaving the observer's eye at a negative angle pose another one. Their trajectories must be split into two contributions, one from the observer to the lowest point of the ray and then from the lowest point upward again. For raytracing purposes, integration by height is not to be recommended.

Integration by β, the slope of the ray, is regular for horizontal rays as long as their second derivative is positive. Inferior mirages and even the mock mirage, where rays take a dip into an inversion layer below the observer and escape again from it, are examples of monotonously increasing slope and their raytracing poses no problems. For rays of oscillating trajectories, the method becomes cumbersome and calculations will have to be split into parts of monotonically increasing or decreasing slope, making the extension to non-symmetrical atmospheres prohibitively difficult. For rays that go through a local maximum and are bent back toward the Earth, the method becomes numerically unstable. The denominators of the terms in column 2, Table 1, vanish. Also $\mathrm{d}\beta$ vanishes, see columns 3 where the same term is now the nominator. But when choosing β as the integration parameter, one needs a non-zero step size, however small, which will make the terms in column 2 explode.

Path length, s, seems the most problem-free choice. The equations that govern it, Table 1, column 4, are the simplest in form and neither for horizontal rays or even for vertical ones does the method show discontinuities. Also integration by horizontal distance, $x = R_0\phi$, is suited for all trajectories, with the only exception of a strictly vertical ray.

2.2. *Parametrizing inversion layers*

In describing superior mirages, ideally a measured temperature profile will be available. In the absence of it, a parametrized form may be used for an inversion that in general will reach from ground or sea level up to some height and that higher up will join smoothly onto a standard atmosphere. Two functional forms I find particularly useful. The first is:

$$\mathrm{WS}(h; h_0, a) = \frac{1}{1 + \exp\left(-\frac{h-h_0}{a}\right)}. \tag{3}$$

It is borrowed from nuclear physics, where it is known as the Woods–Saxon function, a common two-parameter parametrization of the radial density of nuclear matter inside an atomic nucleus or of its charge distribution. The function makes a jump from 0 to 1 across $h = h_0$ and the width of the transition region is determined by the parameter a, the *diffuseness*. The function will have to be scaled by multiplying it with ΔT, the adopted temperature jump across the inversion. I have used this function in Refs [11, 12] for simulations of the Novaya Zemlya effect and other sunsets.

Another suitable choice is the hyperbolic three-parameter function

$$\mathrm{FH3}(h; h_1, h_2, a) = \frac{1}{2} + \frac{a}{2(h_2 - h_1)} \ln\left[\frac{\cosh((h-h_1)/a)}{\cosh((h-h_2)/a)}\right], \tag{4a}$$

wherein the second term is just the integral of the hyperbolic tangent from $(h-h_2)/a$ to $(h-h_1)/a$. Normalized as it stands, Equation (4a) represents a unit jump from 0 to 1 between $h = h_1$ and $h = h_2$. In between, the function is linear. The corners are rounded. Just like for the WS function the diffuseness determines the width of the region over which the jump takes place, the parameter a in (4a) determines the "rounding width" of the corners at h_1 and h_2. The smaller a, the sharper the corners. With a slight modification of nomenclature, we will denote it as the *diffusiveness*.

By exponentiating FH3 with some power p, the region between h_1 and h_2 is made to follow a power law:

$$\mathrm{FH4}(h; h_1, h_2, a, p) \equiv [\mathrm{FH3}(h; h_1, h_2, a)]^p. \tag{4b}$$

This FH4 function still represents a jump from 0 to 1 and multiplied with ΔT it is a suitable and flexible way to parametrize an inversion. I have used this hyperbolic form in Ref. [15], which deals with the hafgerdingar mirage. In this reference, the inverse of the diffusiveness was introduced as the *steepness*.

I have named these functions FH3 and FH4 (fonctions hyperboliques) simply by the number of their parameters.

2.3. *Ducts and extension to non-spherical atmospheres*

Inversions come and go and their characteristics may change from hour to hour or even from minute to minute. They also do not extend indefinitely. If they would, the light rays from the sun could not enter below the inversion and, as in a light guide, be ducted from there toward the observer. The Novaya Zemlya effect would not exist and for a Fata Morgana its background would be entirely due to light scattered into the duct from above.

A convenient way to parametrize the required weakening of the inversion with distance is to introduce an attenuation factor that multiplies the temperature jump, ΔT.

$$A(x) = 1 \quad (x \le x_0) \tag{5a}$$

$$A(x) = 1/[1 + (x - x_0)^2/r^2] \quad (x > x_0). \tag{5b}$$

The strength of the inversion is kept constant from the observer at $x = 0$ till a distance x_0 away from him and beyond it decreases as a semi-lorenzian function with an *attenuation length*, r, such that it is reduced to half its strength at $x = x_0 + r$. This introduction of a dependence on distance breaks the spherical symmetry of the atmosphere. The index of refraction no longer depends solely on height above ground, but also on the distance along it. Replacing (2c), the ray curvature is now:

$$\frac{1}{r} = \frac{1}{n}\left[\frac{\partial n}{\partial h}\cos\beta - \frac{\sin\beta}{R_0 + h}\frac{\partial n}{\partial \phi}\right]. \tag{6}$$

There are several other ways in which the atmosphere may vary from place to place. For instance, the terrain may not be perfectly flat. If it is yet smooth enough, it may be a shield of constant temperature and an inversion layer superimposed on it may still guide the light rays. Another interesting way in which spherical symmetry may be broken arises from the fact that in many ways an inversion behaves as a system of two liquids: cold air below and warmer above with little mixing. The separating surface between them is able to support long running waves. In Ref. [15] this was used to bring in the dynamics by which the hafgerdingar mirage might look like three giant waves rolling in, as it is told in the Old Norse myth [3–5].

Yet another mechanism that may bring about changes in an inversion, even local changes, is wind. Especially the advection of warmer air from a direction transverse to the line of sight, may induce an additional layer. A parametrization in terms of several stacked WS- or FH4 functions might then be attempted. I will use this option further on in the simulation of Fridtjof Nansen's observation on 16 February 1894 [2] and for my simulation of the hafstramb.

All deviations from spherical symmetry that are mentioned above are parametrized in terms of distance. I find it therefore most convenient to incorporate them in a raytracing procedure that itself uses distance as the integration parameter.

3. The observations of the Dutch on Novaya Zemlya

The Novaya Zemlya phenomenon takes its name from the island where in 1597 it was observed by the crew of William Barents. Three voyages had been undertaken by the Dutch to find a trade route "north of Norway, Moscovia and Tartaria to the kingdoms of Catthay and China" as it was stated in their mission orders. The third attempt ended with the famous wintering on Novaya Zemlya. We owe the account of these voyages to Gerrit de Veer, a young man who survived the wintering episode and the return voyage in two open boats. His journal [1] was published in Dutch, Latin and German, all in 1598. A year later also in French and in Italian. An English translation appeared in 1609.

The remarkable observation on 24 January 1597 of the two weeks early sunrise became an issue of world-wide debate. As early as 1604 Johannes Kepler proposed that the Novaya Zemlya phenomenon must have been a reflection [23–26]. He was ahead of his time by almost four centuries.

There is more: the same day, in the night of 24–25 of January they witnessed a conjunction between the moon and Jupiter. Not only did it confirm their day counting, it also gave them their longitude... about twenty degrees more to the east then it had been in reality. On 27 January they saw the sun again.

Two French astronomers have written about this conjunction and Barents' longitude estimate, Pierre Charles Le Monnier [27] in 1779 and Joseph Jean Baills [28] in 1875. At the time of the conjunction also Jupiter was well below the horizon. The same extraordinary refraction by which they had seen the sun just half a day before, must have also made Jupiter visible. Baills addresses yet another observation. On 9 March De Veer's journal suggests that they might have seen a mirage of the Yamal peninsula of the Russian mainland, a distance of 170 nautical miles[1] from Ice Harbour, their wintering place on Novaya Zemlya.

I will attempt simulations for all observations. But first we give the floor to De Veer himself. The way he expresses himself may seem odd at times, but closer reading reveals details that have escaped attention for four centuries. Rather than reproducing below the existing English translation of 1609, I give my own, in which I will attempt to follow the original text as faithfully as possible.

3.1. *Waiting for the sun*

Till a week after the turn of the year 1596–1597 the weather had been boisterous and cold. Foul weather, as De Veer writes, with lots of snow. From the second week in January on, it calms down. The light of the day increases. The weather is fair and clear and at night the heavens are star-lit. They use the opportunity to determine once more their latitude by taking the height of Aldebaran. On the sixteenth, they already see "some redness in the sky". From then on, their minds are on the sun, which can return any day now, as it seems. The wish is the father of the thought and Barents tries to tone down their too high-strung expectations. More than two weeks they will have to wait.

[1]For seafaring people the nautical mile (1.852 km) is the common unit of distance. Its length corresponds to 1 arcminute as measured along a meridian. In this article I will use both miles and kilometres as convenience dictates.

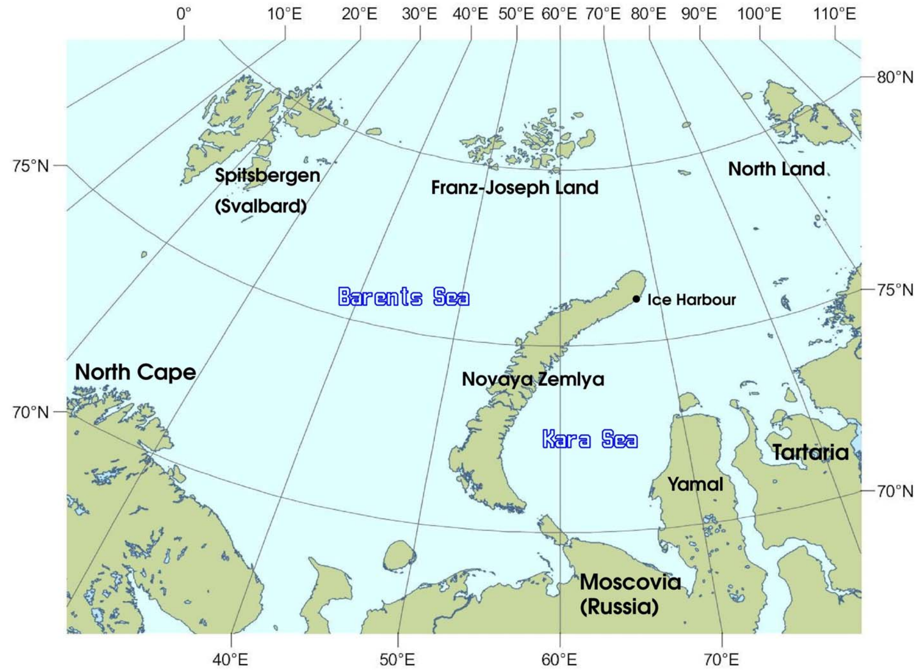


Figure 2. Map of the arctic region. Adapted from Ref. [8] with permission of the Historische Uitgeverij.

But on 24 January happens what was not to be expected. In the margin of his journal, Gerrit de Veer notes: *How the sun which they had lost the 4 of November appeared to them again on the 24 January, which is a great miracle, and which will give all learned men enough to argue about.*

And he writes: *The 24 of January it was fair clear weather, with a west wind. Then I and Jacob Heemskerck,*[2] *and another man with us, went to the sea-side on the south side of Nova Zembla, where, against our expectation, I as the first saw the upper rim of the sun; whereupon we rushed home again, to tell William Barents and the rest of our mates that joyful news. But William Barents, being a good and well experienced pilot, would not believe it and reckoned that it was still about fourteen days before the time that the sun would reappear there at our latitude. There against we firmly opposed, that we had seen the sun, and many wagers were laid about this.*

On the 25 and 26 of January it was misty and hazy weather, so that we had no sight. Then those who had laid wagers to the contrary, thought that they had won, but on the 27 it was clear and bright weather; then all of us saw the sun in her full roundness above the horizon, which made it clear enough that we had indeed seen her on the 24.

Further-on De Veer describes the days 25–27 January in more detail. Here he summarizes the most important events: they saw the sun again on the 27th and could convince their mates that they would have seen it also on the 24th.

The following text must have been added after their return to Holland. It appears to have been written in answer to Robbert Robbertsz, their former instructor on navigation, with whom De Veer and Heemskerck met in November 1597. De Veer's book was published only half a year later, in April 1598. The interval gave him the opportunity to include a defense against Robbertsz' opinion that their calendar keeping could not have been correct:

[2] On this third voyage Jacob van Heemskerck was the skipper. William Barents was the navigator.

And as a different opinion has been brought forward about this event, that it is against the understanding of all old and new writers, indeed in contradiction with the course of Nature and the roundness of the Earth and the Heavens, and about which some say that, since there had been no daylight for a long time, we had probably overslept, about which we know better: but concerning the matter in itself, since God is wonderful in his works, we want to ascribe it to his almighty power, and leave it to others to dispute of. But that no one should think that we were in any doubt ourselves, should we pass this matter over in silence, therefore we wish to give account thereof, how we made sure of our measurement and reckoning.

You must understand then, that when we first saw the sun, it was in the 5th degree and 25 min of Aquarius and it should have been, according to our first guess, in the 16th degree and 27 min of Aquarius, by the time it should have reappeared to us, at that latitude of 76°.

Barents had brought with him a book by Josephus (Giuseppe) Scala [29], an Italian physician, philosopher, mathematician and astronomer.[3] Scala's book is a compilation of pre-calculated positions of the sun, the moon and the planets for the period 1589–1600. Completely in line with Scala's terminology, De Veer writes that they would have expected to see the sun again when it would be at 16° 27′ in Aquarius: an ecliptic longitude of 316° 27′. The corresponding date is 5 February.

And he continues: *About these conflicting things we could not wonder enough and said to each other, could we not be mistaken in the time, which, however, we thought impossible since, without missing a day, we had made notes of our daily happenings, and we had always used our clock and, when that was frozen, the 12-hour sand glass.*

We discussed amongst each other how we could discern and make sure of the proper time. And having considered all that had to be considered, we thought it a good idea to look into the Ephemerides of Josephus Scala, printed in Venice for the years 1589 till 1600, and we found therein that on the 24th day of January (on which the sun first appeared to us), at the time it would be one o'clock in the night in Venice, the Moon and Jupiter were conjunct.

Thereupon we carefully estimated when the same conjunction should occur for us at the place of our house.

And having looked out sharply we found that this 24 of January was the same day on which the aforesaid conjunction happened in Venice, at one o'clock in the night, and for us in the morning when the sun was in the east: for we constantly watched the two planets as they gradually approached each other until the moon and Jupiter stood just one above the other, both in the sign of Taurus, and this at six o'clock in the morning. At that time Jupiter and the moon were conjunct, north by east[4] *on the compass at our house and the south of our compass was SSW, there was the true south,*[5] *the moon being eight days old.*

From all this it was clear that the Moon and the Sun were eight points apart.[6] *This was around six o'clock in the morning and it differs from Venice by five hours in longitude, from which one may find how much we were more easterly than Venice, namely five hours, each hour being 15°, which amounts to 75° that we were East of Venice.*

By all which it is manifestly to be seen that we had not failed in our counting, and that also we had found our right longitude by the two planets aforesaid; for the city of Venice lies at 37° and

[3] Josephus (Giuseppe) Scala and Josephus (also Giuseppe) Moleti were the two astronomers who served on the committee that shaped the Gregorian Calendar. Scala died, 29 years young, still before the publication of his magnum opus. His new-calendar style ephemerides were published under supervision and with an introduction by Moleti.

[4] North by East: 1 point east from north, or a compass bearing of 11° 15′.

[5] On the compass, the true South read as SSW, meaning that the variation of the earth-magnetic field was two points west.

[6] The moon loses to the sun just over one point each day. They stood eight points apart and the moon's phase was just past half.

25 min[7] *in longitude, and her latitude is 46 degrees and 5 minutes; whereby it follows that our place on Nova Zembla lies at 112 degrees and 25 minutes in longitude, and the high of the Pole 76 degrees. There now you have your proper longitude and latitude.*

And a bit further on he concludes: *Concerning how one can understand what has been told above, that we lost the sun at 76 degrees on the 4 of November and saw her again on the 24 of January, we will leave that to be disputed by those who have made their profession thereof. To us, it is enough that we have proven that we have not been mistaken in the time.*

The date agrees with their calendar keeping. What is more, they know now not only the latitude, 76°, of their location, they also have found its longitude as 75° east from the meridian of Venice.

It is the 25th now. During day-time the skies are overcast. The sun does not show itself. On the next day, the 26th, visibility is good again, but over the horizon is a "*bank or dark cloud, so that the sun could not be seen*". Jokes are made about the sun that nobody has seen, except then for the three men.

On the evening of that day, one of the mates dies. He had been ill for a long time. The next morning, the 27th, they dig a grave for him, taking turns in a bitter cold. Back inside the House, spirits are low. Some get visions of a doom-scenario: what if this cold stays on and they get snowed under again? Again it is Heemskerck, who pulls his men through. If everything else fails they can always escape through the chimney. And he will show them right-away how! One of the mates goes outside. He does not want to miss the spectacle of his skipper climbing out over the roof. There, an even bigger surprise awaits him: the sun. De Veer writes: *we hastily came out all together and all at the same time saw the sun in its full roundness, a little above the horizon.*

3.2. *Raytracing for the sun, 24–27 January 1597*

By their own account the House at Novaya Zemlya was at 76° north, "rather more than less". Its remnants were discovered in 1871 by the Norwegian seal hunter Elling Carlsen. The precise location is 76° 15′.4 N, 68° 18′.6 E.

Earlier we have presented a raytracing analysis [12,13] based on a Woods–Saxon type inversion of (3). Here, I give an analysis, using the hyperbolic form FH4 of (4b).

The parameters are: eye height = 14 m, ground temperature = −30 °C, ground pressure = 1040 hPa. For the inversion: $\Delta T = 8$ °C, heights $h_1 = 0$ m and $h_2 = 80$ m, diffusiveness $a = 4$ m, power $p = 3$. For the inversion length and attenuation (see (5)): $x_0 = 300$ km, $r = 200$ km. The calculation has been made for dry air.

It should be noted here that the choice of the power p is not arbitrary. For $p \leq 1$, in combination with $h_1 = 0$ m, there can be no duct. The present choice for $p = 3$, ensures that the level where the radius of a downward curved ray becomes equal to that of the Earth's surface, $1/r = -1/R_0$ in my convention, is well overhead of the observer. This is a necessary condition for a duct to exist.

On 24 January the sun was below the horizon by 5° 26′ at noon and the sun could just have been visible. At noon 27 January the sun's altitude was still −4° 42′. Within a span of an hour around mid-day, the images of the second row of Figure 3 could have been seen. They are indeed roundish, just as De Veer mentions.

On the day before, the 26th, the skies had been clear, but over the horizon there had been "a bank or dark cloud, so that the sun could not be seen". This bank or cloud was the inversion. The sun becomes visible in it when its rays are caught in the duct and manage to travel though it toward the observer, as in a light guide. When there is a mismatch, the observer will not see the sun, but the inversion will still appear to him as a dark band.

[7]As reckoned from the adopted standard meridian of Tenerife.

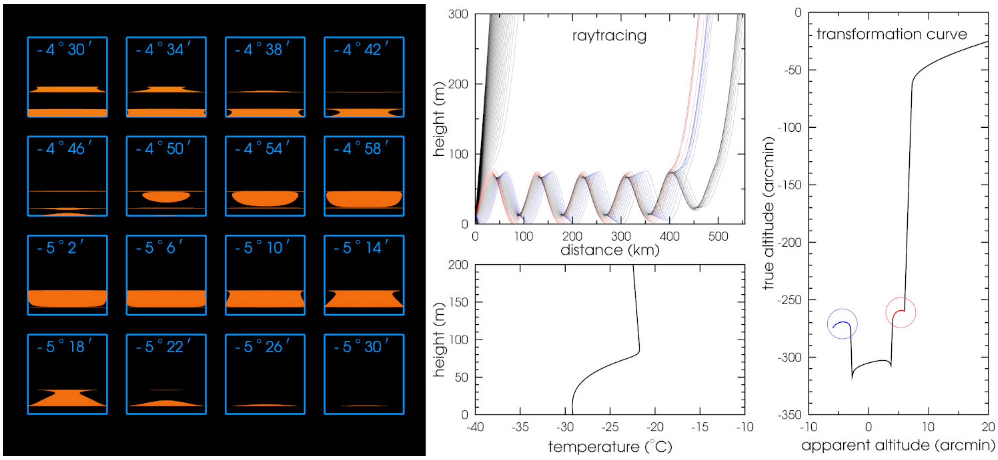


Figure 3. Raytracing of the observations of the sun on 24–27 January 1597. On the left panel, the blue boxes are $32' \times 32'$, the dimension of the sun, when not deformed. Solar altitudes are indicated in each box. The middle panels show the temperature profile (lower panel) and the ray trajectories (upper panel). Right panel: the transformation curve that gives the relation between the true angle and apparent angle as seen by the observer. The lower part features the duct, wherein rays that escape farthest out, after 5 oscillations, give the lowest apparent altitudes: the region $-3'$ to $+3.75'$, nearly symmetric around the astronomical horizon. Lower rays that escape after 4 oscillation, are shown in blue. Likewise, the rays of positive apparent altitude that also escape after 4 oscillations, are shown in red. In the transformation curve these rays produce the "wings" on both sides of the duct, which correspondingly are marked in blue and red. Rays for apparent altitudes higher than $+6'$ break through the top of the inversion right away and make no oscillation.

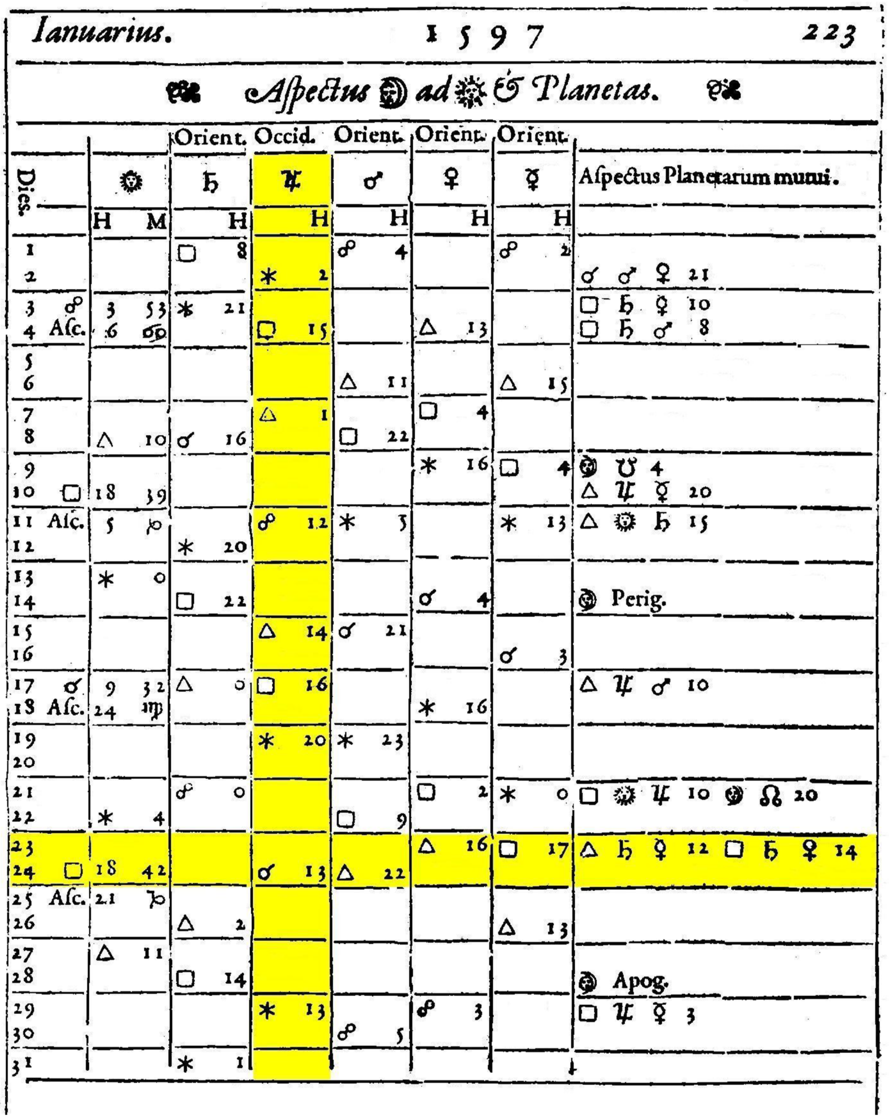
Compare this with the text of Sir Ernest Shackleton in his book *South* [30]. Almost a week before the sun was expected back, they witnessed the Novaya Zemlya effect from the ice of the Weddell Sea. On 26 July 1915 he writes:

Just before noon to-day (July 26) the top of the sun appeared by refraction for one minute, seventy-nine days after our last sunset. A few minutes earlier a small patch of the sun had been thrown up on one of the black streaks above the horizon.

3.3. *The Moon-Jupiter conjunction of 25 January 1597*

Figure 4 shows the page of Scala's ephemerides tables from which Barents learned that a conjunction of the Moon with Jupiter was predicted for 24 January at 13 h after mid-day or, in civil time: on the 25th at one hour past midnight. The time is rounded to the full hour. Yet, today we find from planetarium programs that the prediction came close within minutes. It is a conjunction in ecliptic longitude and the best criterion to recognize it would be to wait till you see Jupiter in line with the shadow edge over the moon.

Gerrit de Veer writes that they observed the conjunction in the direction north by east, one point east from north. In reality, the conjunction took place in the direction *north by west.* At the time Jupiter was below the horizon by 2° 10′ and moreover there was the mountain ridge in the line of sight. In earlier analyses [12,13] we have shown that an inversion overlying these hills, may



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Aspectus ☽ ad ☉ & Planetas.

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Figure 4. The page from the *Ephemerides Josephi Scalae*, showing the aspects of the moon to the sun and the planets for the month of January 1597.

still duct the light. Jupiter might have been visible, but lifted up by almost three degrees, not in line with the horns of the moon, but trailing behind.

It is only about two hours later, when Jupiter has gained sufficiently on the Moon, that the looked-for line-up would be seen. That would have been at around 2 h 45 m Venetian time and the direction would indeed have been north by east. In that line of sight the mountain ridge is still some 200 m in height. These times agree well with predictions from modern ephemerides programs like the one of Jet Propulsion Laboratory [31] and planetarium programs, such as SkyMap [32], Cybersky [33], StarCalc [34], that all use the same software. They give for Jupiter a true altitude of $-2° 26' 39'$ at transit through North. Three quarters of an hour later its azimuthal direction would have been 11° East and its true altitude $-2° 11' 30'$ Over the next fifteen minutes it would rise to $-2° 0' 0'$ in 14° East. These directions would have been judged as North by East on a

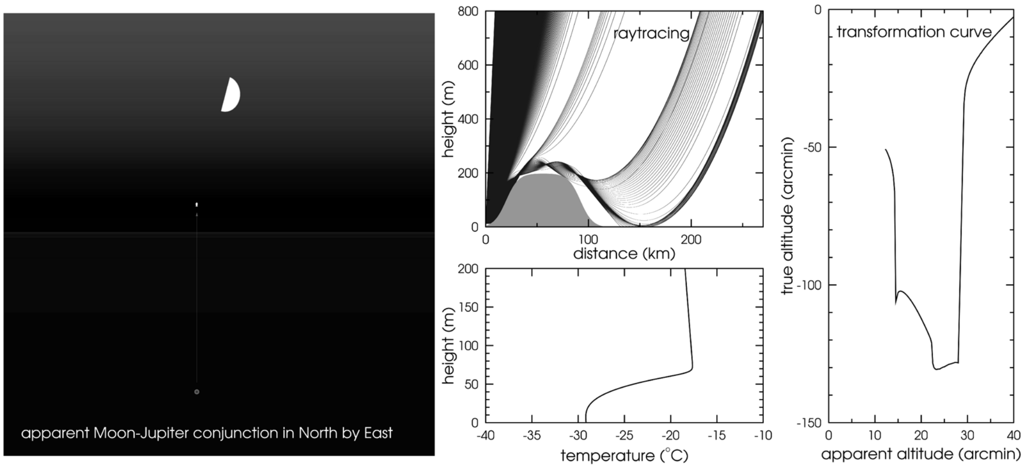

Figure 5. Simulation of the Moon-Jupiter conjunction as it was recognized by the Dutch on Novaya Zemlya in the early morning of 25 January 1597.

correctly aligned compass. De Veer mentions in a rather cryptic way that they had two compasses. Their magnetic ship's compass, taken ashore, had a misreading of two points West. The other, described as "the compass at our house", was a slab of lead with a compass rose engraved on it, mounted on a block of wood outside the house and "adjusted to the proper meridian" [8, 12, 13]. It is only on 8 February, when they see the sun rise in SSE and set in SSW, that De Veer gives these details. The leaden compass was found in the remains of the house in 1979. In the older literature about this conjunction, it appears that only Le Monnier [27] did realize the existence of this second and aligned compass.

Figure 5 gives the full analysis, not with the WS-type inversion as in [12, 13], but with the FH4 form that I used before to simulate the images of the sun. The inversion lifts Jupiter up from 2° 10′ below the horizon to 24′ above it. The moon's true altitude is 1° 26′ and refraction lifts it to 1° 52′. The nearly straight shadow edge is tilted by 18°. Jupiter's true altitude falls right on the bottom of the transformation curve, in the right hand panel. Since this bottom is rather flat, the observer sees Jupiter as a much elongated image.

Although raytracing shows that Jupiter could have been visible, yet the question remains if it was bright enough to distinguish it. The airmass, X, along its path is 690.000 kg/m^2. For the US standard atmosphere the airmass of a perfectly vertical ray is $X_{\text{unit}} = 10.350$ kg/m^2, and relative airmass is counted in these units. Adopting an extinction coefficient of $k = 0.1$, appropriate for very clean air, the apparent magnitude of Jupiter would have been raised by $kX/X_{\text{unit}} = 6.7$ from $m_0 = -2.4$ to an apparent magnitude $m = 4.3$.

Figure 6 shows how Jupiter would have been seen, rising in true altitude from −2° 31′ to −2° 30′. When higher up in the sky, Jupiter's apparent diameter is 40 arcseconds. Here, in our raytracing, Jupiter's images are up to 2.5 times vertically stretched. This magnification increases Jupiter's brightness correspondingly and reduces its magnitude from 4.3, found above, to 3.3. Against the darkness of the duct, darker yet than the sky above it, Jupiter could therefore have been visible with the brightness of a magnitude 3 star, comparable with the nearby Pleiades.

The apparent conjunction was recognized only about 1:45 h after it really took place. This translates into an error in longitude of 26°. Barents' estimate that they would find themselves at 75° east of Venice is thus reduced to about 49°, which comes much closer to its real difference of 56°.

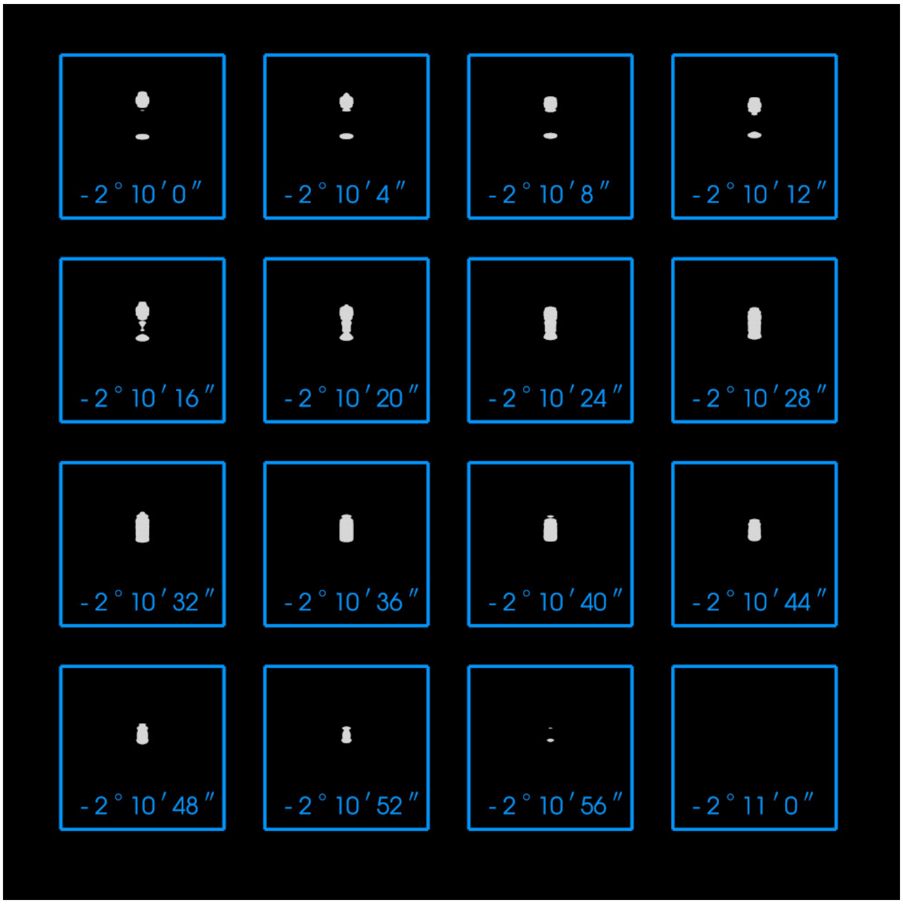

Figure 6. Images of Jupiter as they would have appeared in North by East to Barents and his crew. The blue boxes are 8 × 8 arcminutes and Jupiter's true altitude is indicated in each of the boxes.

3.4. *Seeing land across the Kara Sea*

In 1875, the French astronomer Joseph Jean Baills [28], presented a contribution to the Académie des Sciences wherein he discussed the conjunction that Barents and his men would have witnessed. The longitude was known, because the remnants of the House of Safety had recently been found. The remaining puzzle was the extreme refraction that would have lifted Jupiter from well below the horizon to above it. It prompted Baills to present a model wherein rays could suffer internal reflection against a higher region of warmer air. He does not specify the height of this boundary. Since he assumes a single reflection only, this must have been about two miles, the same as postulated earlier by Johannes Kepler when he proposed, in 1604, that the Novaya Zemlya phenomenon would have been caused by internal reflection against the boundary between the air that surrounds us and the ether above it.

Baills makes yet another suggestion: land might have been seen across the Kara Sea by the same mechanism. He refers to the diary of Gerrit de Veer, who writes:

The 9 of March the weather was foul, but not as foul as it had been the two days before and with less snow, so that we could see much farther out that the water was open in the north-east, but not from us toward Tartaria, for there we could still see ice in the Tartarian Sea, otherwise called the Ice

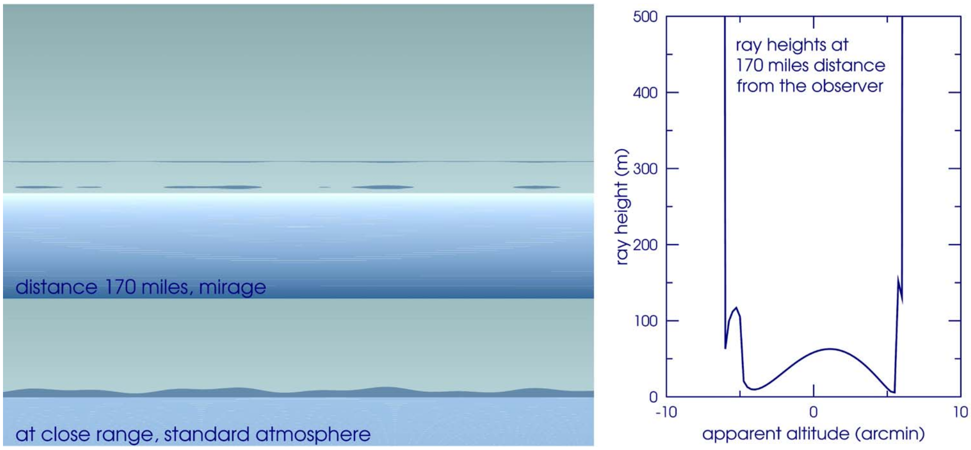

Figure 7. Simulation of the mirage by which the Dutch would have seen land across the frozen Kara Sea, as we read it in De Veer's diary on 9 March 1597. Right: the heights at which rays, traced backward from the observer, pass through a vertical plane at a distance of 170 miles (315 km) away.

Sea. We suspected that it might not be very wide in that direction, for often when the weather was clear, we had imagined to see the land, and showed it to one another, south and south-south-east from our house, like a hilly land, as land commonly looms up when it just becomes visible.

The distance to the Yamal peninsula is some 200 miles. A bit shorter is the distance to the island Ostrov Belyy to its north: about 170 miles. See the map of the arctic region in Figure 2. I attempt a simulation, this time with a WS-type inversion. Parameters are: height of the central isotherm $h_0 = 80$ m, diffuseness $a = 2$ m, temperature jump $\Delta T = 8$ °C. Choosing the way in which the inversion weakens is rather critical and this explains that the mirage of distant land would have been visible only occasionally. Keeping the inversion constant over a distance $x_0 = 250$ km with an attenuation length of 150 km works well, even though Ostrov Belyy is nowhere much higher than 10 m above sea level.

The situation differs from the early sighting of the sun in January. The sun is up now and it is broad daylight. In addition to transporting light from beyond the horizon, the duct now also carries scattered light that has entered from above. It does no longer show up as a dark band.

In the simulation the land is seen by the observer at apparent angles for which the rays are lowest at the target distance of 170 miles. From the graph in the right-hand panel of Figure 7 we read off that this occurs for rays around $-4'$ apparent angle, while a thin mirror image is seen floating around $+5'$.

4. Fridtjof Nansen's observation of the Novaya Zemlya phenomenon

Friday, 16 February 1894.

Today a pretty remarkable thing happened: around noon we saw the sun, or, to be more correct, an image of the sun, for it was only a mirage. Seeing that glowing fire light up just above the edge of the ice, made a strange impression. According to the exalted description that so many Arctic travelers give of the first return of this God of Life after the winter night, this should have been reason for stormy jubilation. But on me it did not have that effect. We had expected not to see the sun yet until several days later, so that I felt rather a tinge of disappointment: it meant that we had drifted more to the south that we knew.

Figure 8. The page of Nansen's logbook on which he describes the appearance of the sun. The text below the sketch is: "Den første luftspeiling af solen, 16 Februar 94": The first mirage of the sun. Copy from: Nansen, Fridtjof: Diaries of the *Fram* expedition, manuscript nr. 8° 2201:2, Provided by the National Library of Norway. Reused from Ref. [8], with permission of the Historische Uitgeverij.

It was with great pleasure I soon discovered that it could not be the sun itself. At first the mirage looked like a flattened red-glowing streak on the horizon. Later, two fire streaks grew out of it, one above the other with a dark space in between. By climbing up to the main-top, I got to see four or even five of such horizontal lines, one above the other and all equally long; it was as one could imagine a dull-red and square sun, with dark streaks across.

Taking the height of a star in the afternoon showed us that at mid-day, the sun must have been below the horizon by $2°22'$ *in reality. In any case, we cannot expect to see the sun's disc itself above the ice before Tuesday; this depends on refraction which is very strong in this cold air. All the same, we celebrated the sun's mirage in the evening with a tiny wee sun-feast with figs, bananas, raisins, almonds and honey cake.*

These are Fridtjof Nansen's words. The Norwegian polar expedition of 1893–1896 had for purpose to study the drift of the ice in the Arctic Sea. By the end of 1893 he let his ship *Fram* get icebound above the New-Siberian Islands. The *Fram* was no ordinary vessel: her designer, Colin Archer, had drafted her shape such that the ice would not crush her, but lift her up. As Nansen had foreseen, they slowly drifted westward. In the summer of 1896 they came free again, north of Spitsbergen. The expedition is described in Nansen's book *Fram over polhavet*, in its English edition under the title *Farthest North* [2].

On 16 February 1894 the Novaya Zemlya phenomenon was seen, at a latitude of 80° 01′ N and a longitude of about 135° E. It is the first well-documented description since Gerrit de Veer. The earliest editions of Nansen's book have a drawing made by himself of the scenery that he saw from the top of the main mast. We have reproduced it in earlier publications [9, 10]. Older yet, and almost certainly the first picture of the Novaya Zemlya phenomenon, is the sketch that he made in his logbook on the day itself.

Making a simulation of the observation that Nansen describes is not trivial. The pattern of individual stripes with dark spaces in between is best reproduced, when I assume that the inversion is not a single layer such as the WS- of FH4 form, but is instead multi-layered. As argued above, the advection of warmer air from a direction transverse to the line of sight may cause this effect. I simulate it here by stacking three WS-type inversions.

The parameters of the simulation are: ground temperature = −10 °C; ground pressure 1013.25 hPa; eye height = 15 m; diffuseness $a = 1.5$ m; and heights of the central isotherms and temperature jumps: layer 1 $h_0 = 13.3$ m, $\Delta T = 1.2$ °C; layer 2 $h_0 = 26.7$ m, $\Delta T = 1.8$ °C; layer 3 $h_0 = 40$ m, $\Delta T = 3$ °C. The length of the inversion is $x_0 = 120$ km with an attenuation length of 80 km. The full analysis is shown in Figure 9.

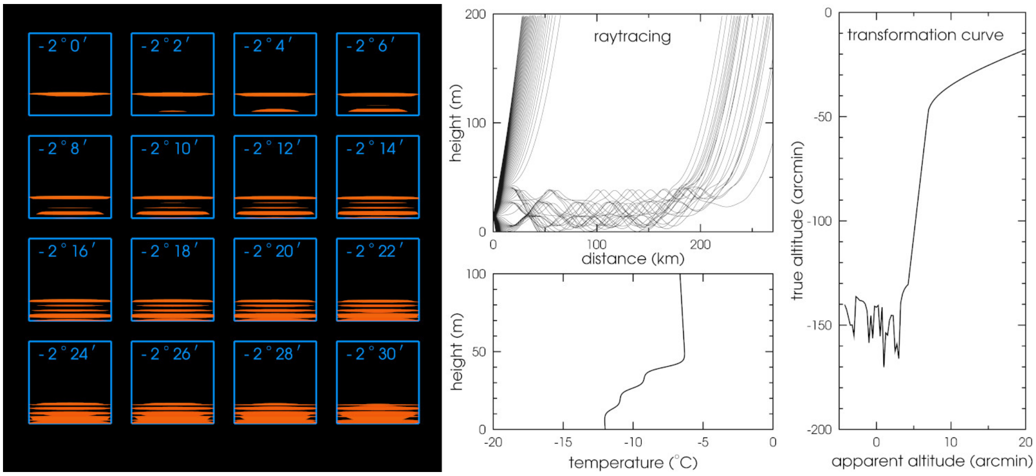

Figure 9. Simulation of Fridtjof Nansen's observation of the Novaya Zemlya phenomenon on 16 February 1894. The sighting was made from the top of the main mast, an assumed eye height of 15 m.

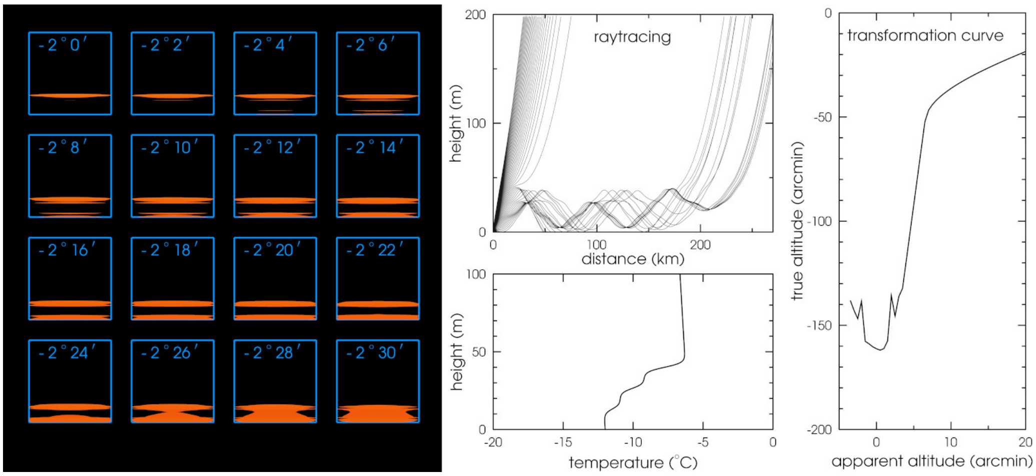

Figure 10. The images as they would have been seen from the deck of the Fram, at an assumed eye height of 5 m above the ice. *Two fire streaks grew out of it, one above the other with a dark space in between.*

It is very gratifying that this simulation reproduces the image that Nansen describes, not only in its appearance but also at the true solar altitude that he mentions: $-2°\ 22'$. The agreement is even more complete, because also his sighting from the deck of the *Fram* is reproduced. Figure 10 shows the simulation for an eye height of 5 m.

5. The hafstramb, a monster in the Greenland Sea

Konungs Skuggsjá or *Kongs Skuggsio* is the original title of a beautiful thirteenth-century manuscript in Old Norse. The writer remains anonymous by his own choice. It has been translated into Latin as *Speculum Regale* and into Danish as *Det Kongelige Speil* by Hannes Finnson and Halfdan

Einersen [3]. This is the editio princeps. A later Danish translation is *Kongespejlet* by Finnur Jónsson [4]. The English translation by Laurence Marcellus Larson is *The King's Mirror* [5]. In dialogue form the book describes a conversation between a father and a son, who sees for himself a future as a traveling merchant. The father himself has belonged to the king's entourage in earlier days, but seems now retired. He is an erudite man who has looked far beyond the borders and knows the world. Many things are discussed that a seafaring merchant should know and be able to do. On a lighter note, the father tells about the wondrous things that his son might come across in the northern seas. Some of these wonders could possibly have been mirages. Waldemar Lehn and Irmgard Schroeder [35] have suggested this for the phenomenon of the *hafgerdingar*, sea fences in English. These have the appearance of a steep wall or a giant wave on the horizon that fences in the sea all around. In reference [15] I have given a simulation of the hafgerdingar and shown that long running surface waves on the top of the inversion layer may create the illusion that these giant waves are actually moving.

There are also monsters in the Greenland Sea. Many of them are whales, but some are more mysterious. There is the *hafstramb*, translated as merman in English, and there is the *margygr*, the mermaid, more appropriately translated as the sea troll or giantess. Lehn and Schroeder have explained them as mirages [7]. The hafstramb would be a vertically stretched image of an orca, seen through a very strong and low inversion. The margygr would be a walrus.

I will give here an alternative explanation for the hafstramb: as an image of a distant iceberg.

In the Konungs Skuggsjá the father says:

About the Greenland Sea it is told that monsters live in it. I do not think they are seen frequently, but yet it is told that some people have seen them with their own eyes. The monster, which is called hafstramb, has been seen in the Greenland Sea, so it is said. It is tall in stature and long, and it rises straight up from the sea. Shoulders, neck, and head it seems to have as a human being, also mouth, nose and chin. But above the eyes and eyebrows, it looks like it's wearing a peaked helmet. It has shoulders like a man, but no arms. Downward the body grows narrower and narrower, as far as it is visible. But no one has ever been able to tell if it has a tail like a fish or if its body runs down into a point. The body looks a bit like an icicle in shape. No one has ever seen it close enough to tell if it has scales like a fish or a skin like a human. Whenever the monster was seen, a storm would invariably follow …

The hafstramb is also mentioned in the *Historia Norwegie* [6], a document in Latin, older yet that the Konungs Skuggsjá: … *a huge creature, but without a tail and without a head. It looks like a tree trunk, as it moves up and down. It shows itself only to warn seafarers of danger.*

The simulation uses two stacked WS-type inversions. The parameters are: temperature and pressure at sea level: t_0 = 10 °C, P_0 = 1013.25 hPa; eye height = 3.5 m; diffuseness a = 2.25 m; heights of the central isotherms and temperature jumps: layer 1 h_0 = 10 m, ΔT = 2 °C; layer 2 h_0 = 20 m, ΔT = 2 °C. The length of the inversion and its attenuation are important in this simulation only because they determine the background against which the icebergs are seen. I used x_0 = 110 km and r = 10 km.

Figure 11 shows how the icebergs look deformed by the mirage. In the middle panel to the left we see them from a distance of 15 miles. One may think to recognize huge humanlike silhouettes, without arms and growing slimmer from the waist down, just as is described in the Konungs Skuggsjá. This general feature is explained through the height versus apparent altitude graph for 15 miles distance, right-hand side bottom panel of Figure 11. Up to $7'$ the image is inverted and rather elongated. From the dip at $7'$ onward, the curve's rise is pretty steep and the image is compressed. Around $9'$ the curve flattens again at a height of around 20 m, elongating the images of the iceberg tops and making them look as "peaked helmets".

The upper left-hand panel shows the icebergs as seen from 10 miles. They do not have the icicle shape anymore, but instead their widths are more or less constant from top to bottom.

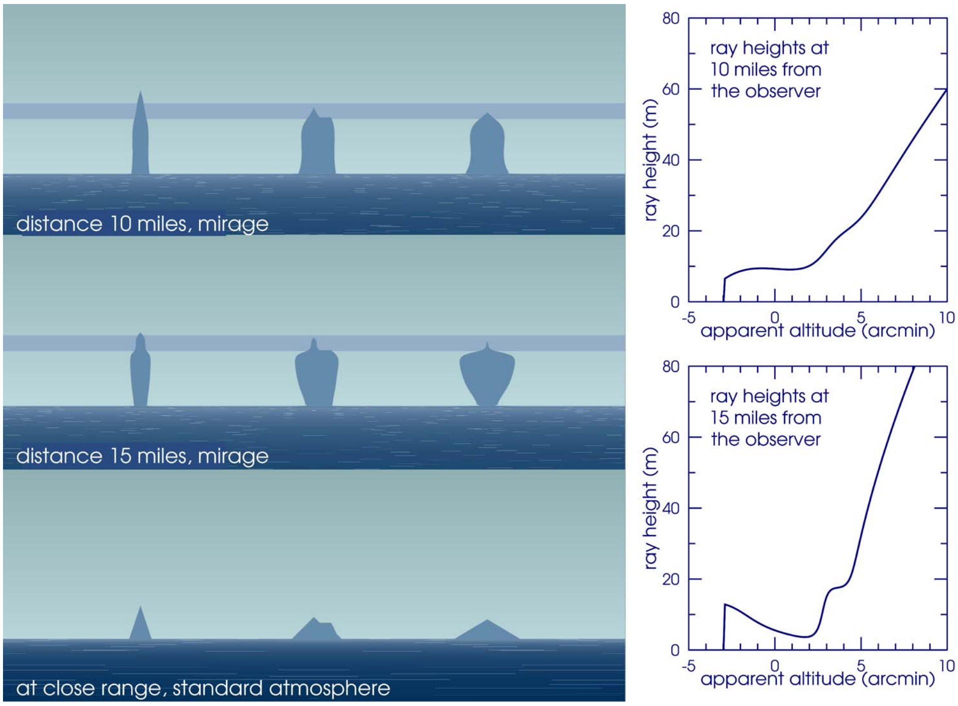

Figure 11. Left: lower panel: three icebergs of different size and shape, seen at close range. From left to right 30, 20 and 18 m in height. Middle panel: the same icebergs, as seen in the mirage at a distance of 15 nautical miles. The darker band near the top of the silhouettes is an image of the water surface about 30 miles away, as will be explained in Figure 13. Upper panel: Idem, but seen from 10 miles distance. Adapted from Ref. [9], with permission of Querido's Uitgeverij. Right: the graphs of ray height in vertical planes at 15 miles and 10 miles away from the observer, versus apparent altitude.

Indeed, the height versus apparent altitude graph is very flat in this case, as shown in the right-hand top panel. This fact is responsible for a very elongated image. The resemblance with tree trunks, as the Historia Norwegie mentions, comes closer here.

Another detail that the Historia Norwegie gives, is that they seem to move up and down. Assuming that the observer is a Viking in his longship, he will ride the waves and his eye height will go up and down with them. This translates naturally into an apparent periodical motion of the "tree trunks" relative to what appears to the Viking as the horizon and what is seen as the top of the duct. A similar change in appearance occurs when he gets up from his rowing bench to stand upon it.

6. Fata Brumosa, Fata Morgana and hafgerdingar

Over the recent years a particular type of mirage has been discussed quite a bit. It goes under several names. One is hafgerdingar, a word in Old Norse meaning sea fences. Another is Fata Brumosa. They are Fata Morganas by which one sees the surface of the sea. In the distance, it looks as if the sea were turned upright, making the impression of a "wall of water", another expression yet by which the phenomenon is known. A very nice example is a photograph by Mila Zinkova [36], shown in Figure 12.

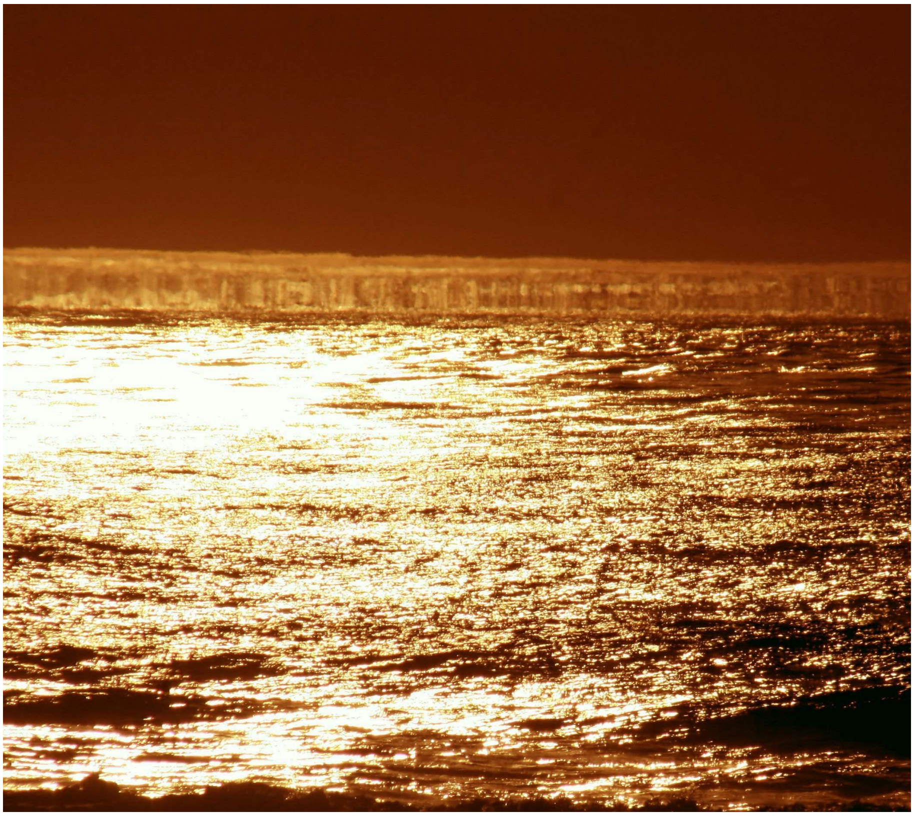

Figure 12. A wall of water, photographed by Mila Zinkova on 14 November 2008, reproduced here with her permission.

This picture features on the cover of the November issue of Weather (2009). Earlier, Lehn and Schroeder have published similar pictures [37], one photographed from Iceland, the others over the melting ice of Lake Winnipeg.

From Zinkova's article I quote: *Suddenly I saw something that was passing through the wall just as Alice passed through a mirror in Through the looking-glass. That "something" materialized as a small boat.* I present here a simulation that, I think, meets this description. It is the identical inversion that I used above to simulate the hafstramb, but now with the addition of an attenuation at the front end. The relevant parameters are: $x_{0\text{-left}}$ = 15 miles (27.8 km) and r_{left} = 10 miles (18.5 km), meaning that the inversion reaches its full strength from 27.8 km away from the observer.

The resulting ray tracings are shown in Figure 13. The hafstramb simulation (left panel) uses an attenuation at the end, so as to let the backward traced rays escape. The addition of a front end attenuation (right panel) results in bending down to the sea surface those rays that otherwise would have been caught in the duct to escape from it only at the end.

An interesting feature is that the region around 40 km is "empty", meaning that a small enough boat remains hidden for the observer as if under an invisibility cloak to use a Harry Potter phrase, until it comes close enough to reveal itself via the lowest rays.

Figure 14 illustrates this: In the lower left panel we see a small vessel. The roof of its cabin is 7 m above sea level, the short mast reaches to 10 m. We see it enter the scene at a distance of around 8 miles, just as it would appear above a horizon in a standard atmosphere. The closer it comes, the larger it gets. The horizon of the foreground is at 7 km distance, pretty much the same

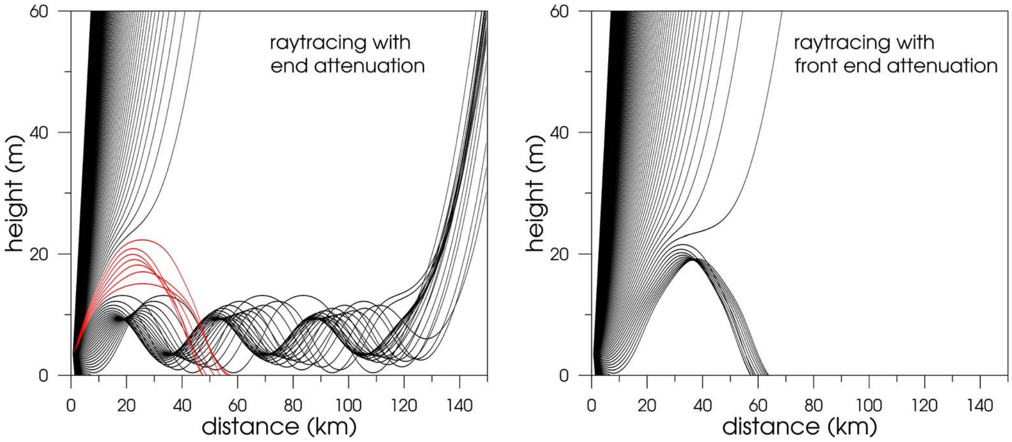

Figure 13. Left: raytracing for the hafstramb simulation, shown in Figure 11. The rays in red connect to the water surface and at the same time they mark the top of the duct. Right: for the same parameters, with the addition of a front end attenuation as described in the text.

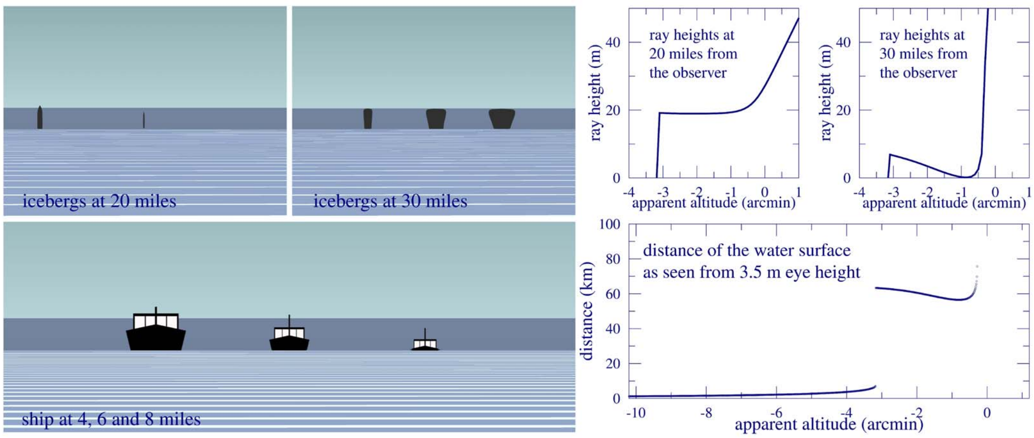

Figure 14. Left: Images from raytracing in the "hafstramb atmosphere" with additional front end attenuation, as in Figure 13. Top-left panels: the icebergs of Figure 11 at 20 miles (37 km) and at 30 miles (56 km). Lower left panel: a small boat at distances of 4, 6 and 8 miles. Right bottom: distance at which the water surface is seen versus apparent altitude. Right top panels: the height versus apparent altitude graphs for distances of 20 and 30 miles.

as it would be in a standard atmosphere from 3.5 m eye height.[8] The wall of water is seen as a background.

The top left panels of Figure 14 show once more the icebergs that I used in the hafstramb simulation. They are, left to right 30, 20 and 18 m in height. At 20 miles distance, the smallest one is not visible. The higher ones look very much vertically stretched and meet the description of tree trunks as they are described in the Historia Norwegie. At 30 miles, their

[8]For a standard atmosphere the distance of the horizon is $2.11\sqrt{h} = 3.95$ nautical miles or 7.3 km, for $h = 3.5$ m.

images are much thicker and more compact, inverted at the bottom, upright at the top and much compressed.

The right-hand panels explain the images. For a distance of 20 miles, the height-attitude graph is almost flat for heights just below 20 m. The icebergs of 30 m and 20 m height are seen, but very much stretched. The smaller one, of 18 m, is under the invisibility cloak. At 30 miles all three icebergs are seen, their lower parts inverted and elongated. Then above $-0'.5$ apparent altitude the graph rises steeply and the top parts of the images are upright and very much compressed.

The small boat appearing from below the wall of water, meets Zinkova's description that she "suddenly saw her as she appeared from behind the wall" [36].

Though inspired by similar photographic images, Lehn's simulation [37] is rather different. It produces a very strong looming of the horizon, which indeed gives one the impression of a wall, but it does not feature the "invisibility cloak". A boat, if it could be recognized at all at such great distance would first become visible *above* the edge of the water wall, not appear from underneath it.

Thus there are at least two different ways, in which a mirage may produce the wall-illusion. There may be more than two: in reference [15], I have given an interpretation of the hafgerdingar as due to a mirage which produces bands of rays that alternatingly hit the sea, or escape to the sky. Quite common is the version where a duct is seen as bounded at its top by just one band that corresponds to the sea, such as in the hafstramb simulation of Figure 11.

Young and Frappa discuss mirages seen over Lac Léman, the Lake of Geneva [38]. They also review much older sightings by Forel [39], to whom we owe the name Fata Brumosa for a mirage that appears as a foggy wall. Comparing with my hafgerdingar analysis, they conclude that *Forel's Fata Brumosa and the Norse Hafgerdingar are different names for exactly the same phenomenon: a Fata Morgana of the water surface*.

Are they basically the same or do differences exist? In my hafgerdingar simulation the duct is seen as symmetric around horizontal. Likewise so in my hafstramb simulation above, where the duct ranges symmetrically from $-3'.0$ to $+3'.0$, just as it should for a Fata Morgana in an atmosphere of spherical symmetry. The fact that farther out this symmetry is broken just to let the rays escape, does not alter this. By contrast, the wall of water in Figure 14, obtained from raytracing as in Figure 13, right panel, is not symmetric. It stretches from $-3'.17$ to $-0'.27$. In this example, spherical symmetry is broken right from the observer's location.

Does this latter case qualify as a Fata Brumosa? In Figure 14, I have given the wall a darker shade of blueish grey than the foreground. But I can well imagine that it could be lighter and foggier if the distant patch of sea that we see miraged, would reflect the glitter of the sun.

7. Summary and conclusions

Many phenomena rely on the atmosphere not being symmetric. Most obviously, when dealing with a temperature inversion, it must have a beginning and an end. Without an end, the Novaya Zemlya effect would not exist. Compare a duct with a circular light guide. Light can go round and round by internal reflection, but there is no mechanism to capture it.

For the practitioner of raytracing, modelling the interface between the duct and a standard atmosphere is a necessary tool in simulating an observation. Waldemar Lehn [35] was, to my knowledge, the first to do so in his analysis of Liljequist's observation in 1951, made at Maudheim Station, Antarctica [40]. In this article I use a two-parameter modelling for this interface by specifying a distance, x_0, away from the observer beyond which the inversion begins to weaken. An attenuation length, r, is introduced as the distance over which the temperature jump of the inversion reduces to half its strength.

For parametrizing the temperature profile, two functional forms are found particularly useful and flexible, The two parameter Woods–Saxon form and a four-parameter function that is basically the integral of the hyperbolic tangent. Both of them have been used in earlier work. For implementing the changing characteristics of the atmosphere with distance, raytracing is most easily done by choosing distance itself as the parameter for numerical integration. At the same time, this is the most convenient way to handle any other spherical symmetry breaking aspect, such as unevenness of the terrain along the line of sight. Also, when simulating a Fata Morgana of a terrestrial object, its distance from the observer must be specified. Other possible choices for the integration parameter, by height or by zenith angle, are cumbersome and not suited for raytracing in a non-symmetric atmosphere.

New simulations of the historical observations are presented. In the first place the events on Novaya Zemlya, that were seen by the Dutch in 1597. The two weeks early return of the sun and the conjunction of Jupiter and the moon that confirmed their day counting and that gave them their longitude. The visibility of Jupiter, raised by refraction to above the horizon, has been a point of concern. I find that it could have been seen with the brightness of a magnitude 3 star.

The possible sighting across the Kara Sea of the Yamal peninsula on 9 March 1597, has been another subject of debate and I give a simulation.

The other classical observation is that of Fridtjof Nansen, on 16 February 1894. He reported seeing the sun in a rectangular shape as four or five stripes with dark bands in between. A simulation seems to require a layered inversion. His illustration is found in the book wherein he describes his voyage [2]. The National Library of Norway does not have this drawing in their collection, but they do have Nansen's logbooks. The sketch he made on the day itself, immediately after the observation, is almost certainly the first-ever illustration of the Novaya Zemlya effect. It was kindly provided by the Museum and it is shown here in Figure 8.

Maybe not historical, but certainly mythical are the sightings of the hafstramb, a monster in the Greenland Sea. It is described in the Old Norse text Konungs Skuggsjá or King's Mirror [3–5] and also in the Historia Norwegie [6]. Lehn and Schroeder [7] have suggested that it might have been a mirage of an orca. I present here an alternative explanation: that it could have been mirages of a distant iceberg.

Another myth from the Konungs Skuggsjá is the hafgerdingar, or sea fences. The discussion about the different ways in which the water surface may be miraged is very vivid these days, certainly because impressive photographic material has become available. I present an analysis of how the image that one gets to see may be influenced by a front end attenuation. That is, by looking into an inversion from a location where the atmosphere is still more or less standard. It explains some details of the "wall of water", reported by Mila Zinkova [36]. Further observations and analyses may tell whether the phenomena, known as hafgerdingar, Fata Brumosa and Fata Morgana are in fact different words for the same kind of mirage, or whether and how they may be distinguished from one another.

Conflicts of interest

The author has no conflict of interest to declare.

Acknowledgements

The author thanks Luc Dettwiller, Andrew Young and Gunther Können for useful discussions and suggestions and Mila Zinkova for permitting the use of her "wall of water" photo.

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