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A perspective of High Energy Physics from precision measurements
*La physique des Hautes Energies du point de vue des mesures de précision*

# Rare kaon decays

## *Les désintégrations rares des kaons*

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**Abstract.** The article reviews the status of rare K decay physics. The explored area covers a broad program, ranging from precise tests of flavour theory and *CP*-violation to the searches for explicit violations of the standard model and the study of strong interactions at low energies.

**Résumé.** Cet article discute l'état de l'art de la physique des désintégrations rares des kaons. Tests de précision des transitions de saveurs de quarks et de la brisure de la symétrie *CP*, recherches explicites de brisure des lois de conservation du modèle standard et étude des interactions fortes à basse énergie forment le programme de cette revue.

**Keywords.** Kaon, Decay, CP symmetry violation, Rare processes, CKM tests, New physics searches.

**Mots-clés.** Kaon, Désintégration, Violation de la symétrie CP, Processus rares, Tests CKM, Recherches en Nouvelle Physique.

## 1. Introduction

The discovery of kaons and the study of their properties are seminal for the building of the Standard Model (SM). Strangeness, flavour mixing, *CP*-Violation, lack of flavour changing neutral currents (FCNC), GIM mechanism [1], all these pillars of the SM trace their origin in strange particles. We are still lacking a theory explaining why fundamental fermions appear in three families and many outstanding questions such as the baryon asymmetry of the Universe or the puzzle of dark matter remain unanswered. The interplay of symmetry violation, such as *CP*, and the mixing of quarks remains one of the most active areas of experimental research in high energy physics. To summarise the overarching strategy of flavour physics, one can say that in the SM:

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(i) The Higgs sector is the source of flavour violation. It is when the Higgs acquires a vacuum expectation value that the quarks get a physical mass and the charged-current $W^{\pm}$ interations couple to the quark interaction eigenstates with couplings given by the Cabibbo-Kobayashi and Maskawa (CKM) matrix [2,3];
(ii) So far all manifestations of *CP* violation and quark mixing are compatible with the single complex phase of the CKM matrix;
(iii) Further experimental investment in the field of flavour physics is justified by the high energy scales addressed by these studies: in many cases they exceed the direct reach achievable at colliders.

The study of kaons are not only complementary to those performed with *B* mesons but also mutually reinforcing: we need to over-constrain the CKM matrix with precise determinations of its parameters extracted independently from different quark systems. A discrepancy between the determinations of the parameters obtained from *K* and *B* would signal physics beyond the SM (BSM).

What makes rare *K* decays special is the unique combination of precise theoretical predictions and the strong suppression of the SM contributions. These two factors make the detection of BSM contribution relatively plausible. The SM contribution to rare kaon decays are tiny and well predicted. In some cases they are not only absent at tree level but also suppressed at loop levels. Final states with a neutrino - antineutrino pair are the least affected by long distance contributions.

It should be emphasised that to explore the flavour structure, it is not enough to check the unitarity of CKM, which is such almost by construction: one has to study genuine [4] electro-weak FCNC processes. Among these, the $K \to \pi\nu\bar{\nu}$ ones described in the next section are a particularly interesting combination of theoretical cleanliness and experimental challenge.

## 2. $K \to \pi\nu\bar{\nu}$: gold-plated decays to test the CKM paradigm beyond the SM

The two decay modes $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L^0 \to \pi^0\nu\bar{\nu}$ can be used to quantitatively check the description of quark mixing and *CP*-violation independently from information extracted from the *B* system. To see why this is the case it is convenient to express the formulas for the branching fractions in terms of contributions from the different loop functions. Following [5] the SM predictions can be written as:

$$\mathscr{B}(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+(1+\Delta_{\mathrm{EM}})\left[\left(\frac{\mathscr{I}m\lambda_t}{\lambda^5}X(x_t)\right)^2 + \left(\frac{\mathscr{R}e\lambda_c}{\lambda}P_c(X) + \frac{\mathscr{R}e\lambda_t}{\lambda^5}X(x_t)\right)^2\right],$$

with $\Delta_{\mathrm{EM}} = -0.003$ the electromagnetic radiative corrections, $x_t = m_t^2/M_W^2$, $\lambda = |V_{us}|$, $\lambda_i = V_{is}^* V_{id}$ the relevant combinations of CKM matrix elements, $X$ and $P_c(X)$ the loop functions for the top and charm quark respectively, and

$$\kappa_+ = (5.173 \pm 0.025) \times 10^{-11}\left[\frac{\lambda}{0.225}\right]^8$$

the parameter encoding the relevant hadronic matrix elements extracted from a suitable combination of semi-leptonic rates. As the formula shows, $\mathscr{B}(K^+ \to \pi^+\nu\bar{\nu})$ depends on the sum of the square of the imaginary part of the top loop (*CP* violating) and the square of the sum of the charm contribution and the real part of the top loop.

The corresponding formulas for $K_L^0$ are:

$$\mathscr{B}(K_L^0 \to \pi^0\nu\bar{\nu}) = \kappa_L\left(\frac{\mathscr{I}m\lambda_t}{\lambda^5}X(x_t)\right)^2,$$

and

$$\kappa_L = (2.231 \pm 0.013) \times 10^{-10} \left[\frac{\lambda}{0.225}\right]^8.$$

The $\mathscr{B}(K_L^0 \to \pi^0 \nu\bar{\nu})$ depends only on the square of the imaginary part of the top loop which is *CP*-violating. The charm contributions drop out because $K_L^0$ is mostly an odd linear combination of $K^0$ and $\bar{K}^0$. This makes the theoretical prediction for the $K_L^0$ rate even cleaner than for the $K^+$. A measurement of the $K_L^0 \to \pi^0 \nu\bar{\nu}$ mode would amount to the measurement of the square of the *CP*-violating parameter $\eta$ in the Wolfenstein parametrisation [6] and directly related to the Jarlskog invariant $J$ [7], the unique measure of the magnitude of *CP*-Violation in the SM.

Inserting the numerical factors and making the dependence to the relevant CKM parameters explicit, one obtains:

$$\mathscr{B}(K^+ \to \pi^+ \nu\bar{\nu}) = (8.39 \pm 0.30) \times 10^{-11} \left[\frac{|V_{cb}|}{40.7 \times 10^{-3}}\right]^{2.8} \left[\frac{\gamma}{73.2^\circ}\right]^{0.74},$$

$$\mathscr{B}(K_L^0 \to \pi^0 \nu\bar{\nu}) = (3.36 \pm 0.05) \times 10^{-11} \left[\frac{|V_{ub}|}{3.88 \times 10^{-3}}\right]^2 \left[\frac{|V_{cb}|}{40.7 \times 10^{-3}}\right]^2 \left[\frac{\sin\gamma}{\sin 73.2^\circ}\right]^2.$$

In the above formulas the explicit numerical errors are the theoretical ones originating from QCD and electroweak uncertainties. They amount to 3.6% and 1.5% for the charged and the neutral mode respectively.

Taking $|V_{cb}|_{\text{avg}} = (40.7 \pm 1.4) \times 10^{-3}$, $|V_{ub}|_{\text{avg}} = (3.88 \pm 0.29) \times 10^{-3}$ and $\gamma = (73.2^{+6.3}_{-7.0})^\circ$ [8], one finds:

$$\mathscr{B}(K^+ \to \pi^+ \nu\bar{\nu}) = (8.4 \pm 1.0) \times 10^{-11},$$

$$\mathscr{B}(K_L^0 \to \pi^0 \nu\bar{\nu}) = (3.4 \pm 0.6) \times 10^{-11}.$$

The predictions are currently dominated by the parametric uncertainty that will plausibly be reduced by new measurements of $|V_{ub}|$, $|V_{cb}|$ and $\gamma$ by LHCb and Belle II.

With the discovery of the Higgs boson, the particle content of the SM is complete but we know that the SM itself cannot be the full story. There are many extensions of the SM where one could expect sizeable contributions, among which we can mention: Warped extra dimensions [9]; MSSM analyses [10–12]; Simplified Z and Z′ models [13], Littlest Higgs with T-Parity [14], Lepton Flavour Universality Violation models [15] and Lepto-quarks [16]. As an example of complementarity between $K \to \pi\nu\bar{\nu}$ and $B$-meson physics one can mention the case of $B_s \to \mu^+\mu^-$: the $B$ decay is sensitive to possible new pseudoscalar (e.g. charged Higgs) interactions while the $K$ rare decay is sensitive to possible new vector ones such as a $Z'$.

The SM theoretical precision is waiting to be matched experimentally. The experiments are difficult because the three-body final state lacks a significant signature and the $\nu\bar{\nu}$ pair cannot be detected. A long series of decay-at-rest searches for $K^+ \to \pi^+ \nu\bar{\nu}$ have culminated with the final results of the E878/E949 AGS experiments which have found strong evidence for the decay and quoted as final result [17]:

$$\mathscr{B}(K^+ \to \pi^+ \nu\bar{\nu})_{\text{exp}} = (17.3^{+11.5}_{-10.5}) \times 10^{-11}.$$

Although the mean value is about twice the SM prediction, the result is still consistent with the SM prediction because of the large statistical error. In addition to the purity of the separated kaon bean, the advantages of the stopped kaon technique include good kinematic constraint to kill backgrounds from two body kaon decays and the possibility to enforce good charged particle identification following the full $K^+ \to \pi^+ \to \mu^+ \to e^+$ decay chain. Limitations of the technique are the small acceptance once acceptable levels of muon and photon rejection levels are achieved and the presence of the material of the stopping target which leads to $\pi^+$ scattering and loss of energy.

An alternative method to study the decay is to let the kaon decay in flight, as done by the NA62 experiment at CERN [18]. High momentum (75 GeV/$c$), positively charged, secondary beams from the CERN SPS are well suited to provide a high flux of kaons and good immunity to backgrounds. A drawback is that the separation at beam level of kaons from the pion and proton components is practically impossible and because of this the main difficulty of the experiment is to track each particle of the beam before it enters the decay region. Only about 6% of the beam particles are kaons and of those only about 10% are usefully decaying in the fiducial volume. So for each useful $K$ decay NA62 has to track almost 200 particles. To be able to accumulate a sufficient number of decay the beam rate is very high (≃750 MHz). The SPS beam is delivered by a slow extraction to minimise the instantaneous beam intensity. The beam is not bunched as in collider experiments because otherwise the kaon decays occuring in the same bunch would veto each other. Nevertheless, even employing a slow extraction and an overall duty cycle of about 20%, the instantaneous beam intensity is so high that accidental activity in the beam tracker leads to the wrong association between a beam track (assumed to be the kaon) and the decay product. To avoid this problem and to correctly associate the kaon minimising mistags, NA62 has developed a novel Si pixel detector tracking detector [19] dubbed Gigatracker (GTK). The time resolution achieved with three GTK stations is as good as 65 ps. First results published by NA62 indicate that the novel in-flight technique is viable for $K^+ \to \pi^+ \nu\bar{\nu}$ [20].

Based on three candidate events (with an expected background of $1.65 \pm 0.31$), collected from the 2016–2017 data sample, the NA62 Collaboration reported [21]

$$\mathscr{B}(K^+ \to \pi^+ \nu\bar{\nu})_{\text{exp}} = (4.7^{+7.2}_{-4.7}) \times 10^{-11}$$

which, if interpreted as background leads to a 90% CL upper limit of [21]:

$$\mathscr{B}(K^+ \to \pi^+ \nu\bar{\nu}) < 18.5 \times 10^{-11}.$$

The NA62 result based on 2017 data excludes in practice the BNL mean value reported above. This is the first significant result obtained with the novel in-flight technique developed by NA62 [22]. NA62 has collected more data in 2018 and results based on this data sample will significantly improve the measurement. The overall kaon flux accumulated by NA62 so far (≈$6 \times 10^{12}$ kaons) corresponds to about 50 SM $K^+ \to \pi^+ \nu\bar{\nu}$ events for a nominal overall acceptance of 10%. The acceptance of the analysis on which the result presented above was obtained is of the order of ≈1% so lot of effort is underway to increase it towards the nominal value. To achieve the goal of 100 SM events NA62 will resume data taking after the CERN Long Shutdown 2 (LS2) in 2021. So far one extra year has been granted. Data taking may be further extended depending on the progress of the analysis. The possibility to further improve the measurement towards a 5% accuracy for a SM signal is being studied within NA62. Assuming that the infrastructure could be upgraded to take a factor of four more proton intensity, the detector would need to be upgraded in order to:

(i) Improve the time resolution of the beam tracker to less than 50 ps/station

The experiment is not limited by the number of protons deliverable by the SPS, but rather by the time resolution required to resolve the tracks in the beam tracker and to correctly match the kaon one to the pion reconstructed in the downstream tracker. As stated above, the NA62 Gigatracker has a time resolution of 120 ps per station, or 65 ps for the three stations combined. The NA62 GTK success has inspired several new R&D to develop Si pixel detectors with even better timing in the view of, for example, of the Upgrade 2 of LHCb [23]. A significant improvement of the time resolution would enable NA62 to operate efficiently with increased beam intensity.

(ii) Reduce material budget of the trackers

While any amount of material in the beam leads to unwanted scattering and interactions, for this experiment it is also important to minimise the thickness of the main pion tracker because particle scattered on detector material can mimic the weak signal signature. The NA62 straw tracker has a thickness of 1.2% of a radiation length. It is made of straws of 9.8 mm diameter. It is operated in the vacuum tank to minimise scattering. The main source of material is the plastic wall of the straw which amounts to 36 μm. Reducing the radius of the straw to approx. 5 mm opens the possibility to employ much thinner walls and reduce the material budget by about a factor of two. In addition, a shorter drift time will improve the high rate capability of the detector.

(iii) Faster EM calorimetry

One of the essential aspects of NA62 is the capability to detect photons with high efficiency. Increasing considerably the beam intensity will require faster veto counters in order to avoid large signal losses due to random veto.

While the experimental situation for $K^+ \to \pi^+ \nu\bar{\nu}$ shows that we have two independent experimental techniques able to reach SM sensitivities and beyond, the situation for the neutral mode is more complex. Progress has been hampered by the lack of experimental signature, as no redundancy is available once the $\pi^0$ mass is used as constraint to define the decay vertex.

On the experimental front for the neutral mode, the KOTO experiment at JPARC has published the 90% CL limit [24]:

$$\mathscr{B}(K_L^0 \to \pi^0 \nu\bar{\nu}) < 3 \times 10^{-9}.$$

KOTO builds on the experience of the E391a predecessor experiment which was performed at KEK. It is based on the technique of letting a very well collimated “pencil” beam enter the decay region surrounded by high performance photon vetoes. By vetoing extra photons and applying a transverse momentum cut (150 MeV/$c$) to eliminate residual $\Lambda \to n\pi^0$ decays KOTO plans to reach SM sensitivities by the mid of the next decade. The first step would be to improve the model independent 90% CL Grossman-Nir limit [25], which has been recently updated incorporating the latest NA62 result:

$$\mathscr{B}(K_L^0 \to \pi^0 \nu\bar{\nu})_{\text{Grossman-Nir}} < 8.14 \times 10^{-10}.$$

KOTO is starting to consider seriously a new experiment to reach 100 SM events (KOTO Step-2) [26]. The possibility to explore the neutral decay mode at CERN once NA62 is completed has been explored in the framework of the European particle physics strategy upgrade. The experiment under study (KLEVER) [27] is based on the same technique employed by KOTO but at much higher kaon energies. Higher kaon energies are expected to simplify the task of rejecting the photons from $K_L^0 \to \pi^0\pi^0$ which is the dominant source of background from kaon decays. One should notice that with respect to the charged mode, the two-pion branching ratio is $CP$-violating and therefore suppressed by a factor of about 200. Is this fact that makes the approach to study $K_L^0 \to \pi^0\pi^0$ at all thinkable. KLEVER expects to collect 60 SM events within five years of data taking after the CERN Long Shutdown 3 (from 2026 onward). KOTO and KLEVER are holding joint meetings to address issues of mutual interest.

In conclusion, what make the case to continue the study of these rare decays compelling, is that sensitivity beyond SM is there for most of the proposed extensions. Together with the study of muon rare decays and searches for electric dipole moments of elementary particles, rare kaon decays like $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L^0 \to \pi^0 \nu\bar{\nu}$ offer a genuine window of sensitivity to access high energy scales thanks to the absence of tree level contributions (CKM Unitarity), to the absence of long distance contributions ($\nu\bar{\nu}$ pair in the final state), and the hard GIM suppression at loop level in SM.

## 3. Other rare kaon decays

### 3.1. $K_S, K_L \to \mu^+\mu^-$ and $K_{L,S} \to l_1^+ l_1^- l_2^+ l_2^-$

The study of the rare process $K_L \to \mu^+\mu^-$ has played a crucial role in the understanding of the flavour content and structure of the standard model (SM) of electroweak interactions. The $K_S, K_L \to \mu^+\mu^-$ decays are dominated in the SM by long-distance two-photons contributions (LD), while the flavour-changing-neutral-current contributions provide information on short-distance (SD) dynamics of $\Delta S = 1$ transitions [28, 29]. The SM predictions for $K_S, K_L \to \mu^+\mu^-$ are [30]:

$$\mathscr{B}(K_S \to \mu^+\mu^-)_{SM} = (5.18 \pm 1.50_{LD} \pm 0.02_{SD}) \times 10^{-12},$$
$$\mathscr{B}(K_L \to \mu^+\mu^-)_{SM} = {}^{(6.85\pm0.80_{LD}\pm0.06_{SD})\times10^{-9}(+)}_{(8.11\pm1.49_{LD}\pm0.13_{SD})\times10^{-9}(-)}$$

where the SD contribution to the uncertainty includes the uncertainty on the CKM parameter ($\bar{\eta}$ and $\bar{\rho}$ respectively); the sign ambiguity for the $\mathscr{B}(K_L \to \mu^+\mu^-)_{SM}$ depends on the (destructive or constructive) interference between short- and long-distance contributions which itself depends on the sign of an unknown low-energy constant [31]. The amplitude for the $K_L \to \mu^+\mu^-$ mode receives contributions from a complex long-distance amplitude generated by the two-photon exchange and a real amplitude induced by Z-penguin and box diagrams; the long-distance amplitude has a large absorptive (imaginary) part that provides the dominant contribution to the total rate; the uncertainty on the dispersive part of the two-photon amplitude is at present the dominant individual source of error [29]. The smallness of the total dispersive amplitude is well established thanks to precise experimental results on both $\Gamma(K_L \to \mu^+\mu^-)$ [8] and $\Gamma(K_L \to \gamma\gamma)$ [32]. The current experimental result for $K_L$ is based on over 6200 candidates from the BNL E871 Collaboration [33]:

$$\mathscr{B}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \times 10^{-9}$$

while for $K_S$ the limit from the 70 s [34] has been improved by upper limits at 90% CL by the LHCb experiment [35, 36]:

$$\mathscr{B}(K_S \to \mu^+\mu^-) < 2.1 \times 10^{-10}.$$

In the Phase-II upgrade during the High Luminosity LHC era, LHCb is expecting to collect around 300 $fb^{-1}$. If the trigger efficiency will be near 1, as can be achieved technically with the Phase-I full software trigger, LHCb will be able to explore branching fractions below $10^{-11}$ and could exclude values down towards the vicinity of the SM prediction [37].

The gap between the SM prediction and the current limit on $K_S, K_L \to \mu^+\mu^-$ leaves room for Beyond the Standard Model (BSM) contributions. The current limit already place constraints on leptoquark models [38, 39], supersymmetry [40, 41] and extensions of the SM [42]. Experiments using neutral kaon beams can address the *CP* asymmetry, which is also sensitive to BSM scenarios. Although both $K_S, K_L \to \mu^+\mu^-$ are almost *CP*-conserving decays, direct *CP* violation can be addressed by interference between $K_L$ and $K_S$ in the kaon beam, with the interference contribution affecting the $K_S$ SM predictions at the level of 60%; besides the unknown sign of the $K_L$ two-photon amplitude can be determined by a measurement of the interference [42].

The decays into 4 leptons are also of interest since the sign of the amplitude of $K_L \to \gamma\gamma$ affecting the determination of short-distance contributions to $K_L \to \mu^+\mu^-$ can be measured by studying $K_L$ and $K_S$ decays in four leptons [43]. The predicted branching ratios in the SM are of the order [44]:

$$\mathscr{B}(K_S \to e^+e^-e^+e^-) \sim 10^{-10},$$
$$\mathscr{B}(K_S \to \mu^+\mu^-e^+e^-) \sim 10^{-11},$$
$$\mathscr{B}(K_S \to \mu^+\mu^-\mu^+\mu^-) \sim 10^{-14},$$
$$\mathscr{B}(K_L \to e^+e^-e^+e^-) \sim 10^{-10},$$
$$\mathscr{B}(K_L \to \mu^+\mu^-e^+e^-) \sim 10^{-11},$$
$$\mathscr{B}(K_L \to \mu^+\mu^-\mu^+\mu^-) \sim 10^{-14}.$$

The $K_S$ modes have never been observed and there are no experimental limits available in literature. The $K_L$ modes limits are from the KTeV and NA48 experiments [45, 46].

Recently LHCb has shown prospects for the $K_S$ modes [47]. Future high-intensity kaon experiments, with intensities around 5 times the most recent ones, could address all the modes with 2 or 4 electrons.

### 3.2. $K_S, K_L \to \pi^0 l^+ l^-$

In the SM, the process $K_S \to \pi^0 l^+ l^-$ is *CP*-conserving and dominated by a single photon exchange, while the $K_L$ decay is *CP*-violating, apart from a small *CP*-conserving contribution (mainly originating from two-photon process). In Chiral Perturbation Theory expansion beyond leading order, the $K_S$ decays can be expressed as:

$$\mathscr{B}(K_S \to \pi^0 e^+e^-) = (0.01 - 0.76a_S - 0.21b_S + 46.5a_S^2 + 12.9a_Sb_S + 1.44b_S^2) \times 10^{-10},$$
$$\mathscr{B}(K_S \to \pi^0 \mu^+\mu^-) = (0.07 - 4.52a_S - 1.50b_S + 98.7a_S^2 + 57.7a_Sb_S + 8.95b_S^2) \times 10^{-11},$$

where $a_S$ and $b_S$ are free, real chiral-perturbation-theory parameters of the polynomial expansion of the EM form factor in terms of the di-lepton invariant mass $q^2$ (with $b_S$ being the coefficient of the linear term in $q^2$) [48]. The parameters $a_S$ and $b_S$ can be determined from data; avoiding assumptions from Vector Meson Dominance model is possible if a large enough statistics become available. For the $K_L$ process, *CP*-violating contributions can originate from $K^0 - \bar{K}^0$ mixing via a decay of the *CP*-even component of the $K_L$, and direct *CP*-violating contribution from short distance physics via loops sensitive to $\mathrm{Im}(\lambda_t) = \mathrm{Im}(V_{td}V_{ts}^*)$. The indirect and direct *CP*-violating contributions can interfere and the expression for the total *CP*-violating branching ratio can be written as [48]:

$$\mathscr{B}(K_L \to \pi^0 l^+ l^-)_{CPV} = \left[C_{\mathrm{MIX}} \pm C_{\mathrm{INT}} \frac{\mathrm{Im}\lambda_t}{10^{-4}} + C_{\mathrm{DIR}} \left(\frac{\mathrm{Im}\lambda_t}{10^{-4}}\right)^2\right] \times 10^{-12}$$

where $C_{\mathrm{INT}}$ is due to the interference between the direct ($C_{\mathrm{DIR}}$) and indirect ($C_{\mathrm{MIX}}$) *CP*-violating components. Here $C_{\mathrm{MIX}}$ and $C_{\mathrm{INT}}$ depend on $\mathscr{B}(K_S \to \pi^0 l^+ l^-)$ and $\sqrt{\mathscr{B}(K_S \to \pi^0 l^+ l^-)}$ respectively. The SM predictions for the branching ratios of $K_L \to \pi^0 l^+ l^-$ are:

$$\mathscr{B}(K_L \to \pi^0 e^+e^-)_{\mathrm{SM}} = (3.5 \pm 0.9, 1.6 \pm 0.6) \times 10^{-11},$$
$$B(K_L \to \pi^0 \mu^+\mu^-)_{\mathrm{SM}} = (1.4 \pm 0.3, 0.9 \pm 0.2) \times 10^{-11}$$

for constructive (destructive) interference respectively [49]. Better measurements of the $K_S \to \pi^0 l^+ l^-$ decay rate would allow to improve the quoted errors, which are currently dominated by the uncertainty due to the chiral-perturbation-theory parameters $a_S$ and $b_S$. A joint study of the Dalitz plot variables and the components of the $\mu^+$ polarisation could directly allow the separation of the indirect, direct *CP*-violating and *CP*-conserving contributions [50].

The current experimental limits for $K_S$ modes come from NA48/1 [51,52]:

$$\mathscr{B}(K_S \to \pi^0\mu^+\mu^-) = (2.9^{+1.5}_{-1.2} \pm 0.2) \times 10^{-9},$$
$$\mathscr{B}(K_S \to \pi^0 e^+ e^-) = (5.8^{+2.8}_{-2.3} \pm 0.8) \times 10^{-9},$$

together with the determination of two allowed regions for $a_S$ and $b_S$ from the combination of the two branching ratio measurements using a maximum likelihood method: $a_S = -1.6^{+2.1}_{-1.8}$ and $b_S = 10.8^{+5.4}_{-7.7}$, or $a_S = +1.9^{+1.6}_{-2.4}$ and $b_S = -11.3^{+8.8}_{-4.5}$. The 90% CL limits for the $K_L$ modes come from KTeV [53,54]:

$$\mathscr{B}(K_L \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10},$$
$$\mathscr{B}(K_L \to \pi^0\mu^+\mu^-) < 3.8 \times 10^{-10}.$$

The $K_L$ channel is sensitive to BSM models. Searches of extra-dimensions where enhancements of the branching ratio of both $K_L$ modes by a factor of about 5 are possible without violating any constraints [49]. Specific flavour structures can correlate effects in $K_S, K_L \to \pi^0 l^+ l^-$ with B-physics anomalies, for example LeptoQuark models with Rank-One Flavour violation [55, 56]. While future experiments using neutral kaon beams can address all the four channels, the LHCb experiment can significantly improve the precision on $\mathscr{B}(K_S \to \pi^0\mu^+\mu^-)$. LHCb can reach $0.25 \times 10^{-9}$ with 50 fb$^{-1}$ of integrated luminosity, assuming 100% trigger efficiency in LHC Run3; as the precision increases, the use of the $q^2$ dependence becomes a viable approach to avoid model dependence in the extraction of $a_S$ and LHCb can reach a precision of 0.10 on $a_S$ and 0.35 on $b_S$ with Phase II-Upgrade [57].

### 3.3. *Lepton Number/Flavour violation*

Rare kaon decays can be used to search for explicit violation of Lepton Number or Flavour. Both Lepton Number and Flavour are exact symmetries in the SM, and kaon decays $K \to (n\pi)\mu e$ contemplating their violation are null tests of the SM. However many extensions of the SM violate Lepton Flavour and Number. Violations can occur in BSM models, for example involving leptoquarks or the exchange of multi-TeV Gauge mediators, that can relate the observed anomalies in the B sector to observables in the kaon sector [58–63] with kaon branching ratios expected in the range $10^{-12}$–$10^{-13}$. Other models advocate Heavy Majorana Neutrinos as a source of Lepton Number violation [64,65].

Limits (at 90% CL) on the branching fractions for the $K \to (n\pi)\mu e$ modes were achieved by BNL [66], KTeV [67], E865 [68,69] in 1990–2000, and more recently by NA48/2 [70]:

$$\mathscr{B}(K_L \to e^\pm\mu^\mp) < 4.7 \times 10^{-12},$$
$$\mathscr{B}(K_L \to \pi^0 e^\pm\mu^\mp) < 7.6 \times 10^{-11},$$
$$\mathscr{B}(K^+ \to \pi^+ e^-\mu^+) < 1.3 \times 10^{-11},$$
$$\mathscr{B}(K^+ \to \pi^+ e^+\mu^-) < 5.2 \times 10^{-10},$$
$$\mathscr{B}(K^+ \to \pi^- e^+\mu^+) < 5.0 \times 10^{-10},$$
$$\mathscr{B}(K^+ \to \pi^-\mu^+\mu^+) < 8.6 \times 10^{-11},$$
$$\mathscr{B}(K^+ \to \pi^- e^+ e^+) < 6.4 \times 10^{-10}.$$

The NA62 experiment has already improved on some of these limits [71]:

$$\mathscr{B}(K^+ \to \pi^-\mu^+\mu^+) < 4.2 \times 10^{-11},$$
$$\mathscr{B}(K^+ \to \pi^- e^+ e^+) < 2.2 \times 10^{-10},$$

and has prospects to push the limits for the $K^+$ modes to the range of $10^{-12}$ to $10^{-11}$, considering the data taking foreseen after 2021. These modes can be also pursued at LHCb in the Upgrade phase, benefiting from the large strange-production cross-section and the improved efficiency for kaon decays; LHCb may be able to update the existing limits and probe a sizeable part of the parameter space suggested by the discrepancies in B physics [72].

### 3.4. $K^+ \to \pi^+ l^+ l^-$

The decays $K^+ \to \pi^+ l^+ l^-$ ($l = \mu, e$) are Flavour-Changing-Neutral-Current processes; their short-distance contribution is described by $Z, \gamma$ penguins and box diagram, with the amplitude depending on the logarithm-like GIM mechanism.

The decays $K^+ \to \pi^+ l^+ l^-$ are dominated by long-distance contributions involving one photon exchange ($K^+ \to \pi^+ \gamma \to \pi^+ l^+ l^-$), and their branching fraction can be derived within the framework of Chiral Perturbation Theory in terms of a vector-interaction form factor, which describes the single-photon exchange and characterises the di-muon invariant-mass spectrum. The form factor includes a small contribution from the two-pion-loop intermediate state and is dominated by a term phenomenologically described as a first-order polynomial ($a_+ + b_+ z$), where $z = (m_{\mu\mu}/M_K)^2$ and $a_+$ and $b_+$ are free parameters, used to describe the non-perturbative QCD effects in the chiral expansion [73, 74]. In order to obtain both the parameters and the corresponding branching fraction, the differential decay rate spectrum must be reconstructed from experimental data.

Similarly to $B \to K l^+ l^-$, this process can be described by an effective Lagrangian with nozero Wilson coefficients for the semi-leptonic operators $Q_{7V}$ and $Q_{7A}$ [75], and new physics processes can be interpreted as deviations from the Standard Model Wilson coefficients $C_{7V}$, $C_{7A}$. In particular, the Wilson coefficient $C_{7A}$ can be related to $a_+$, making the form factor measurement a test of beyond-the-SM effects [76]. Beside, Lepton Flavour Universality implies the free parameters to be the same for both the electron and muon channels. Their comparison would provide a test of Lepton Flavour Universality, with any deviation being a sign of short-distance new physics dynamics [77].

The current best experimental measurements of the $K^+ \to \pi^+ l^+ l^-$ branching ratios are from the NA48/2 collaboration [78, 79]:

$$\mathscr{B}(K^+ \to \pi^+ e^+ e^-) = (3.11 \pm 0.04_{\text{stat}} \pm 0.12_{\text{syst}}) \times 10^{-7},$$
$$\mathscr{B}(K^+ \to \pi^+ \mu^+ \mu^-) = (9.62 \pm 0.21_{\text{stat}} \pm 0.13_{\text{syst}}) \times 10^{-7}.$$

Both the NA48/2 and E865 [80, 81] have extracted the free parameters $a_+$ and $b_+$ for muon and electron channels, placing limits on Lepton Flavour Universality violation. However, such test is at present limited by the uncertainties of the measurements, especially in the muon channel.

At NA62 both larger and significantly cleaner samples of both channels are expected to be collected over the lifetime of the experiment because of vast increases in instantaneous rate, improved tracking and larger field-integrals. The LHCb mass resolution is sufficient to separate the muon decay from the kinematically similar three-pion decay; the experiment can collect of order $10^4$ decays in the muon channel per year of upgraded-LHCb data taking. Similar considerations apply to the electron channel, where a reduced reconstruction efficiency is somehow compensated by the larger branching fraction [82].

## 4. HNL and exotics

The long lifetime of kaons opens the interesting possibility to investigate with good sensitivity decays of kaons in exotic final states including heavy neutral leptons (HNL) or exotics such as $K^+ \to \pi^+ X$ where X is a long-lived boson. As by-product of the $K^+ \to \pi^+ \nu\bar{\nu}$ analysis, one can search for new stable neutral bosons in two body decays of the type: $K^+ \to \pi^+ X$ and $K_L^0 \to \pi^0 X$.

A generic possibility of $k$ new sterile neutrino mass states can be written as

$$\nu_\alpha = \sum_{i=1}^{3+k} U_{\alpha i}\nu_i, \quad (\alpha = e, \mu, \tau).$$

On general grounds, the extension of the neutrino sector is motivated by its relation to the neutrino mass generation mechanism. The $\nu$MSM [83, 84] is the most economical theory accounting for neutrino masses and oscillations, baryogenesis, and dark matter. Three heavy neutral leptons (HNLs) are posited to provide a Dark Matter candidate ($m_1 \approx 10$ keV/$c^2$) while two more massive neutrinos could exist with $m_{2,3} \approx 1$ GeV/$c^2$.

The production of Heavy Neutral Leptons [84–86] can be searched in $K^+ \to l^+\nu$ as a peak search over a well know, well modelled background, independently of the HNL decay mode. While pion decays allow one to explore the mass region between 60 and 135 MeV/$c^2$ [87], kaons decays enable us to extend a very sensitive search up ≈450 MeV/$c^2$. In particular, both the rare decay $K^+ \to e^+\nu$ and the abundant $K^+ \to \mu^+\nu$ have been successfully used to set limits in the mass range up to about 450 MeV/$c^2$.

Limits at 90% CL on the square of the mixing angle extend down to about $10^{-8}$ for $K^+ \to \mu^+ N$ [88] and close to $10^{-9}$ for $K^+ \to e^+ N$ [89]. NA62 foresees to reach a sensitivity of order $10^{-9}$ and $10^{-8}$ on the electron coupling and muon coupling respectively, with the existing full data set.

This is part of a broader programme covering searches for feebly-interacting long-lived particles at LHC experiments and possible future facilities [90], with an interplay between the exploration of large masses and relatively strong couplings at colliders, and masses in the MeV–GeV region and low couplings in meson decays and at future beam-dump facilities.

## 5. Outlook

What makes the case to continue the study of rare kaon decays compelling, is their sensitivity beyond SM of most of the proposed extensions, offering a genuine window of sensitivity to access high energy scales. The NA62 experiment at CERN will resume data taking after LS2 to complete its physics programme for $K^+ \to \pi^+\nu\nu$ and a variety of other rare kaon decays and exotic searches. The possibility of high-intensity kaon beams, both charged and neutral, at CERN after LS3 is being explored, with a broad physics case covering the most interesting kaon decays as well as the golden channels $K \to \pi\nu\nu$. In Japan, the KOTO experiment plans to reach the SM sensitivity for $K_L \to \pi^0\nu\nu$ by the middle of the next decade, and is considering a new experiment to reach 100 SM events (KOTO Step-2). The LHCb experiment at CERN has a programme of rare kaon decays; in the Phase-II upgrade during the High Luminosity LHC era, the experiment will be able to explore branching ratios below $10^{-11}$ thanks to a large strange-production cross-section and an improved efficiency for kaon decays.

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