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Experiments on double diffusive convection

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Expériences sur la convection doublement diffusive

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Abstract. Double diffusive convection is a form of convection that was studied in the past mostly for its application in oceanography and which receives increasing attention in the context of planetary cores. This review will focus on measurements of transport properties of double diffusive convection which were performed in two types of experiments: the experiments either start from two layers separated by a sharp interface, or they investigate convection within uniform gradients. The most recent experiments were done on the finger regime in which the slowly diffusing component drives the motion while the rapidly diffusing component stabilizes the fluid and this review is restricted to this configuration.

Résumé. La convection doublement diffusive est une forme de convection qui a été étudiée dans le passé surtout pour son application océanographique et qui reçoit de plus en plus d'attention dans le contexte de la convection dans les noyaux planétaires. Cette revue est consacrée aux mesures des propriétés de transport de la convection doublement diffusive qui ont été effectuées dans deux types d'expériences: soit l'expérience démarre de deux couches séparées par une discontinuité, soit elle étudie la convection à l'intérieur de gradients uniformes. Les expériences les plus récentes ont été réalisées dans le régime des doigts de salinité dans lequel la composante qui diffuse lentement entraîne le mouvement alors que la composante qui diffuse rapidement stabilise le fluide et cette revue se concentre sur cette configuration.

1. Introduction

Double diffusive convection is concerned with convection in fluids whose density depends on two ingredients or components. The most prominent example is ocean water with a density determined by both temperature and salinity. This combination is at the origin of the term "thermohaline convection" as synonym for "double diffusive convection" or "two component convection". Peculiar effects arise in ocean water convection because of the widely different diffusion constants for heat and salt ions, which explains the emphasis on double diffusion in

the name "double diffusive convection". Similar effects must be expected in the Earth's core if both temperature and chemical composition contribute to density variations in the outer core. Other applications of double diffusive convection appear in geology such as in magma chambers.

The most spectacular effects arise when one of the components contributes to a stable stratification of the medium while the other component on its own leads to unstable stratification. The discrepancy in diffusion rates allows for a mechanism which permits convective motion despite a stable stratification of total density. The form of this motion depends on whether the fast or the slowly diffusing component is driving the motion. The latter case leads to convection in narrow elongated structures as for example in Figures 1 and 2, called "finger convection", while the former case shows less distinctive structures and is usually called the "diffusive regime", except that astrophysicists tend to call it "semiconvection". Both types of double diffusive convection lead to layer formation, so called "staircases". They consist in the finger regime of vertically stacked layers with every second layer filled with fingers, while large scale overturning convection occurs in the intervening layers as demonstrated for example by the visualizations in Ref. [1].

Numerical computations struggle to simulate double diffusive convection in the parameter regimes one is generally interested in. It helps numerical simulations that the important phenomena occur both in 2D and in 3D so that much can be learned from 2D simulations. But it remains challenging to simulate staircases with their coexisting convection rolls, which extend over a large width, and fingers, which require a high resolution. Numerical methods also struggle to cope with a large ratio of diffusivity constants. State of the art methods use two different grids, one for every component of the double diffusive system. Presumably the most advanced numerical method and simulations at the most extreme parameters accessible today are presented in Refs. [2, 3].

This review will regard double diffusive convection as an extension of the more familiar Rayleigh–Bénard convection problem which inquires the vertical heat transport through a layer in between horizontal boundaries with controlled temperatures. This does not do full justice to the richness of double diffusive effects studied in the laboratory. For example, a simple rising plume is a non trivial object of surprising shape in a double diffusive environment [4, 5] and the phenomenon of lateral intrusion [6, 7] does not exist in single component convection. However, the vertical flux through a stratified layer is the most fundamental problem and deserves special attention.

The existence of staircases motivates experiments in which one attempts to isolate a single finger layer between two convecting layers. This is done by starting from two layers initially separated by a sharp interface as described in Section 3. The experiments on transport between fixed horizontal boundaries more akin to Rayleigh–Bénard convection will appear in Section 5.

A key observable in all types of convection is the flux of heat or salinity across the convective layer. It is of interest to compare the advances made in this respect for double diffusive convection versus Rayleigh–Bénard convection. Experiments in the 1960's throughout the 1980's on double diffusive convection were contemporaneous with experiments on Rayleigh–Bénard convection using very much the same techniques, frequently performed by the same researchers. They used cells a few tens in centimeters in size filled with standard working fluids such as water or air. The activity on Rayleigh–Bénard convection gained new momentum in later decades through innovative set ups using more exotic working fluids such as cryogenic gas [8] or compressed gas [9]. In addition, there was a race towards ever larger facilities [10, 11]. One of the rewards of all this activity was a large database for the heat flux in turbulent convection in a gas in dependence of the Rayleigh number, in which the Rayleigh number spans many orders of magnitude. These data showed that the reasoning leading to the celebrated 4/3-law of double diffusive convection to be discussed below is not valid for all Rayleigh numbers in Rayleigh–Bénard convection. It is still not known with certainty how the heat flux scales with Rayleigh number at

extreme parameters, which motivates further and further experiments [12].

Experimental research on double diffusive convection stagnated in comparison and advanced at a much slower pace, at least in as far as quantitative flux measurements are concerned on which this review will focus. If the example of Rayleigh–Bénard convection is any indication, we must expect that measurements over an extended range of parameters in double diffusive convection will also give new impetus to the subject and call into question current theories. This paper will review the experimental techniques employed so far.

The behaviour of double diffusive convection depends very much on the sign of the gradients of the two components involved, leading to either finger convection or the diffusive regime. However, any experimental apparatus can usually study both regimes and easily switch from one type of double diffusive convection to the other. It is therefore sufficient for the purpose of this review to focus on one of the two regimes. Only the literature on finger convection will be cited because the most recent experiments were restricted to this regime. Many reviews of double diffusive convection have appeared in the past, the most complete of which is certainly the book by Radko [13]. The present paper will review aspects that are too technical or too experimentally minded to have been mentioned before. There will also be a special focus on fluxes as they are the most important and most accurately measured quantities, and visualizations or other less quantitative results will receive less attention.
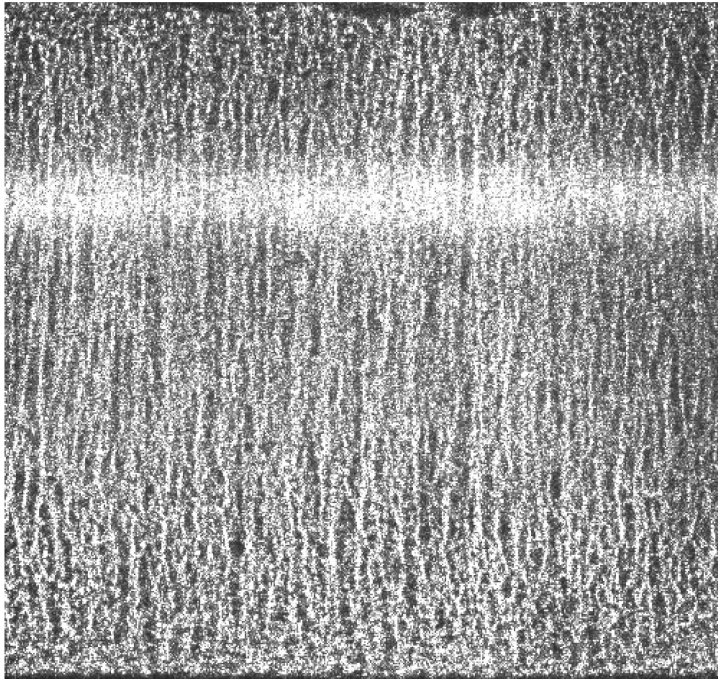

Figure 1. Shadowgraph picture of finger convection for $Ra_c = 1.15 \times 10^{11}$ and $Ra_T = -9.70 \times 10^7$. The bright horizontal bar is an artefact of the illumination. This picture is taken from Ref. [14].
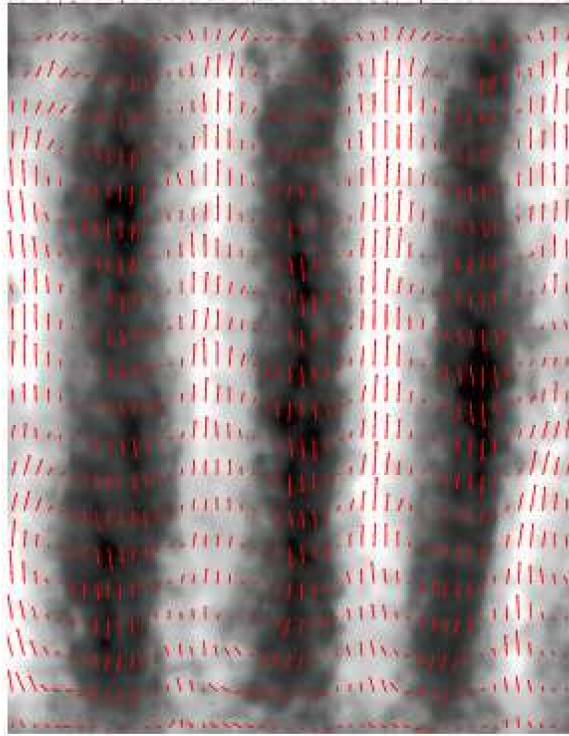
Figure 2. Velocity field obtained from particle image velocimetry at $Ra_c = 1.05 \times 10^9$ and $Ra_T = -3.81 \times 10^5$. The red lines indicate direction and velocity of the flow, the shade of gray depends on the vertical component of velocity. This figure is taken from Ref. [14].

2. Finger convection basics

2.1. *Fingers and layers*

The two components governing the density of the convecting fluid are commonly called temperature and salinity even if two other physical quantities are responsible for the density variations. Temperature just stands for the rapidly diffusing component, and salinity for the slowly diffusing one. For instance, it is experimentally convenient to use sugar and salt dissolved in water, in which case the salt concentration as the more rapidly diffusing variable plays the role of temperature and "salinity" stands for sugar concentration.

For a start, let us think of a water volume consisting initially of two layers with a sharp interface, with a layer of cold fresh water underlying a layer of warm salty water. Let us assume that the combinations of temperature and salinity are such that the top layer is less dense than the bottom layer so that there is no reason for a large scale overturning motion. How will this system evolve? The interface will of course thicken by diffusion. A more efficient vertical transport arises from a possible instability. Assume that a particle from the top layer is displaced into the bottom layer. If there was no diffusion, this particle would return to the top layer because of its small density. In the presence of diffusion, however, the particle exchanges heat with its environment, and more so than it exchanges salinity because of the different diffusion constants. In a thought experiment going to the extremes, imagine temperature diffusing instantaneously and salinity not diffusing at all. The lowered particle then immediately loses its temperature contrast with the other fluid at the same height while retaining its salinity so that this particle is now more dense than the

other fluid in the bottom layer and it will continue to sink. The identical reasoning holds for a particle lifted from the bottom to the top layer that continues to rise. The two layer system is therefore prone to a convection instability. The horizontal wavelength of the instability must be small enough so that the temperature in for example every down flow quickly equilibrates by diffusion with the neighboring up flows. The convection rolls will tend to be slender and look like fingers. At the same time, fingers cannot be too narrow either to prevent a too rapid exchange of salinity between neighboring fingers, and also of momentum by viscous friction. This instability mechanism is obviously not limited to an initially sharp interface but may operate in smooth gradients as well.

The fingering instability must saturate at some point. The fingers can only grow to some finite length before they will become unstable themselves, just like the plume of smoke rising from a cigarette. It is at present not clear what the most potent instability mechanism for fingers is. A much debated candidate is a collective instability [15, 16] in which an internal gravity wave is excited by the finger flow and which prevents any further lengthening of the fingers. Once the fingers have reached their maximum length, they transport salty water from the top layer to the base of the fingers within the bottom layer. This increases the density there, so that the bottom layer below the fingers is unstable to global overturning. Likewise, the up going fingers pump fresh water to the bottom of the top layer which creates an unstable density stratification within the top layer, too. The finger instability thus leads from top to bottom to overturning convection in relatively warm salty water in the top layer, a finger layer in the middle, and overturning convection in relatively cold fresh water in the bottom layer. This state will last if temperature and salinity at some top and bottom boundaries can be controlled and maintained. Otherwise, the motion will end once temperature and salinity differences are used up.

A modification of this system helps to understand why layering frequently occurs spontaneously in double diffusive systems, at least transiently. Replace in the two layer system considered so far the bottom layer with fresh water that is itself stably stratified because its temperature increases with height, but the density of the top layer is still smaller than the density of the topmost fluid in the bottom layer. How is the system going to evolve from these modified initial conditions? The ideas exposed above still apply in the beginning: Fingers develop because of the rapid lateral diffusion of heat compared with the slow lateral diffusion of salinity, the fingers stop growing in length because of some instability, the fingers pump salty water to their base and fresh water to the top. The difference with the previous system is that now the bottom layer is not globally unstable, but only down to the depth at which the bottom layer has the same density as the fluid that accumulates at the bottom of the fingers. This starts an overturning motion of finite depth which mixes salinity and temperature within large scale convection rolls. The bottom of these rolls qualitatively recreates the same conditions that characterized the interface we started from: Relatively warm and salty water overlying stably stratified fresh water. We therefore expect a second layer of fingers to form at the bottom of the motion considered so far, followed by a second overturning layer. If only the water layer extends deep enough, we can expect more and more layers to form as deeper and deeper strata start to move.

The subsequent fate of the these layers is less clear. For these layers to survive in a stationary state, one needs of course to supply fluxes of salinity and heat at the top and bottom of the system, but in addition, the flow needs to be able to adjust so that the fluxes through each finger layer equals the fluxes through each overturning layer. Whether such a state is reached, and what the flow characteristics in such a state are, is part of the questions laboratory experiments seek to answer.

2.2. *Equation and parameters*

We need to introduce the relevant parameters which allow us to compare different experiments among each other and to compare experiments with situations in nature. The experiments are always parametrized within the Boussinesq approximation which linearizes the dependence of density ρ on temperature T and salinity or ion concentration c as

$$\rho(T,c) = \rho_0 \left\{ 1 - \alpha(T - T_0) + \beta(c - c_0) \right\} \tag{1}$$

where T_0 and c_0 are the temperature and ion concentration of a reference state at which the density is ρ_0, so that $\rho(T_0, c_0) = \rho_0$. The signs in eq. (1) are chosen such that the expansion coefficients α and β are both positive. The Boussinesq approximation assumes that density variations need only be retained in the buoyancy term in the momentum equation and that all other material properties are constant. This leads to the following equations of evolution for the temperature and concentration fields $T(\boldsymbol{r}, t)$ and $c(\boldsymbol{r}, t)$, the velocity $\boldsymbol{v}(\boldsymbol{r}, t)$ and the pressure $p(\boldsymbol{r}, t)$:

$$\frac{\partial}{\partial t} \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \boldsymbol{v} + g\alpha T \hat{\boldsymbol{z}} - g\beta c \hat{\boldsymbol{z}} \tag{2}$$

$$\nabla \cdot \boldsymbol{v} = 0 \tag{3}$$

$$\frac{\partial}{\partial t} T + \boldsymbol{v} \cdot \nabla T = \kappa \nabla^2 T \tag{4}$$

$$\frac{\partial}{\partial t} c + \boldsymbol{v} \cdot \nabla c = D \nabla^2 c \tag{5}$$

where $\hat{\boldsymbol{z}}$ denotes a unit vector in the vertical direction. The diffusivities for heat and salt are κ and D, the kinematic viscosity of the fluid is ν, and the gravitational acceleration is g.

To find the parameters that help comparing different situations, we remove dimensions from these equations. We first select a length L, typically the total height of an experimental cell, and express the position vector $\boldsymbol{r}$ in terms of the non dimensional vector $\boldsymbol{r}'$ as $\boldsymbol{r} = \boldsymbol{r}'L$. Likewise, we may rewrite time as $t = t'L^2/D$ and the various fields as $\boldsymbol{v} = \boldsymbol{v}'D/L$, $p = p'\rho D^2/L^2$, $c = c'\Delta c$, and $T = T'\Delta T$ where Δc and ΔT are typical concentration and temperature differences occurring in the system. These might be the differences between the top and bottom layers in the experiment started from two layers as described above, or it could be the difference between top and bottom boundaries imposed by controlled boundary conditions as in Sections 5.2 and 5.3. With these choices, one arrives at equations of evolution containing four non dimensional parameters. Two of them are combinations of material properties, namely the Schmidt and Lewis numbers Sc and Le defined as $Sc = \nu/D$ and $Le = \kappa/D$. The other two parameters quantify the buoyancy forces and are the thermal and chemical Rayleigh numbers Ra_T and Ra_c defined by

$$Ra_T = \frac{g\alpha \Delta T L^3}{\kappa \nu}, \quad Ra_c = \frac{g\beta \Delta c L^3}{D\nu}. \tag{6}$$

There are therefore four independent control parameters, but there is no universal agreement on which parameters exactly are the most revealing. Apart from trivial changes of notation (the chemical Rayleigh number is frequently called the salinity Rayleigh number Ra_S), a reliable source of confusion resides in the sign conventions. Most papers study either finger convection or the diffusive regime and define the Rayleigh numbers such that they are positive for the system under consideration. A more systematic notation uses negative Rayleigh numbers to signal the stabilizing component. Finger convection then occurs in the quadrant $Ra_T < 0$, $Ra_c > 0$. This convention requires Δc and ΔT to be defined as

$$\Delta T = T_{\text{bottom}} - T_{\text{top}}, \quad \Delta c = c_{\text{top}} - c_{\text{bottom}}. \tag{7}$$

Certain combinations and variations of the four control parameters above are in widespread use. For example, one often sees the "diffusivity ratio" τ which is nothing but the inverse Lewis number, $\tau = 1/Le$. Another popular combination of material properties is the Prandtl number Pr with $Pr = \nu/\kappa = Sc/Le$. All the experiments discussed in this review were done in aqueous solution. With the help of various solutes and temperatures, Pr can then be varied from approximately 5 to 10. If the two components taking part in the double diffusive convection are heat and ion concentration, then $Le \approx 100$. The exact value depends on the ion species and temperature. If on the other hand the two components are concentrations of salt and sugar, then $Le \approx 3$.

The two Rayleigh numbers are frequently combined to the so called density ratio, denoted by R_ρ or Λ and defined as

$$\Lambda = \frac{Ra_T}{Ra_c}\frac{Sc}{Pr} = \frac{\alpha\Delta T}{\beta\Delta c}. \tag{8}$$

Sign conventions again matter in this definition. With the convention adopted in this review, $\Lambda < 0$ for finger convection. However, most papers adapt their definitions in order to obtain a positive Λ for their specific application. The diffusivity ratio is an intuitive parameter because it compares the buoyancy due to temperature variations with the buoyancy due to salinity variations. Double diffusive effects are necessary for any motion in the finger regime if $|\Lambda| > 1$ so that the stabilizing thermal buoyancy overwhelms the destabilizing chemical buoyancy and the layer is density stable. Global overturning is naively expected for $|\Lambda| < 1$. If $|\Lambda|$ is too large, the fluid becomes linearly stable and the stabilizing thermal gradient is strong enough to prevent any motion, even finger convection. In the model of uniform gradients in an infinitely extended fluid, $|\Lambda|$ needs to be less than Le for finger convection to occur according to linear stability analysis. The density ratio directly measures the distance to the intuitive limits of the finger regime.

The quantities one desires to measure are chiefly the thermal and salinity fluxes, $\boldsymbol{j}_T$ and $\boldsymbol{j}_c$, defined by rewriting eqs. (4), (5) as $\frac{\partial}{\partial t}T + \nabla \cdot \boldsymbol{j}_T = 0$ and $\frac{\partial}{\partial t}c + \nabla \cdot \boldsymbol{j}_c = 0$ which implies $\boldsymbol{j}_T = \boldsymbol{v}T - \kappa\nabla T$ and $\boldsymbol{j}_c = \boldsymbol{v}c - D\nabla c$. Only averaged fluxes are usually accessible experimentally. Let us denote the average over time and the entire fluid volume by angular brackets. The ratio of the averaged heat flux divided by the heat transport by diffusion alone in a state of rest is called the Nusselt number Nu. If the temperature difference is imposed between two horizontal planes the distance L apart and if z denotes an axis pointing upwards, Nu is given by (note the definition of ΔT above):

$$Nu = \frac{F_T}{\kappa(-\Delta T)/L} \tag{9}$$

where $F_T = \langle j_{T,z}\rangle$ is the global vertical thermal flux. The analogous quantity for salinity, based on the vertical ion flux $F_c = \langle j_{c,z}\rangle$, is called the Sherwood number Sh in the chemical literature and we will adopt this name here:

$$Sh = \frac{F_c}{D\Delta c/L}. \tag{10}$$

Another quantity determined in many experimental papers is the flux ratio γ, which is defined as $\gamma = (\alpha F_T)/(\beta F_c)$, with the usual precautions concerning sign conventions.

There is a simple idea almost every paper compares it results to which is the 4/3-law. Let us try to determine the Sherwood number as a function of four control parameters which we choose to be Ra_c, Λ, Pr and Sc and let us assume that Sh depends as a power law on Ra_c so that $Sh \propto Ra_c^q f(\Lambda, Pr, Sc)$ with a possibly complicated function f. How can the exponent q be determined? It is plausible to assume that the salinity flux should be independent of L because fingers evolve according to the heat and salinity exchange between neighboring fingers which are spaced by a distance much less than L. One may also argue that the thickness of the concentration boundary layer near one plate should not be influenced by the presence of another

distant boundary, which again implies that the salinity flux must be independent of L and which leads to $Sh \propto Ra_c^{1/3} f(\Lambda, Pr, Sc)$ and in dimensional variables to $F_c \propto \Delta c^{4/3}$.

3. The two layer system

A whole series of experiments studied the system described in Section 2.1. Start from two layers separated by a sharp interface with warm salty water lying over cold fresh water (or any other combination of slow and fast diffusers) and let the system evolve. These experiments face two major challenges: How to set up the initial conditions and how to diagnose the evolution. This section will review some milestones in chronological order.

An early experiment dates back to Turner [17] who actually used the combination of heat and salt. The experiment was started by filling a $25 \times 25 \times 45\,cm^3$ tank halfway with light fluid and by then adding dense fluid through a pipe at the bottom of the tank. The fluid volume is thereby stably stratified at all times and tolerates some mechanical disturbance without immediately mixing both layers. Nonetheless, this filling procedure is the more reliable the stronger the density contrast between the two layers, so that the density ratio cannot be made too small (it ranged from 2 to 10 in these experiments). One also wants to prevent fingers from already forming during the filling. This is why the tank was initially filled with warm and cold fresh water, and salt was added to the top only once the tank was filled. Two rectangular grids were mounted in the tank, one in each layer. The two layers were initially homogenized by oscillating the grids up and down. The stirring must be gentle enough in order not to destroy the interface, yet rigorous enough to homogenize the layers. After a few minutes, the stirring was stopped and the state thus obtained constituted the starting point for the double diffusive convection.

The vertical temperature profile in the tank was monitored with moveable thermistors. The main idea of the experiment was to set the grids into motion anew after a certain time to mix each layer again, withdraw samples from each layer and determine their salinity with a conductivity probe together with their temperature. This technique allowed to determine heat and salinity fluxes. The experiment can of course not be run longer than the time it takes to deplete the salinity and temperature contrast between the two initial layers.

Stern and Turner [18] chose to investigate the salt-sugar system. In this case, salt is the fast diffuser. The large sugar molecules have a diffusivity smaller than that of salt by a factor of 3. The choice of salt and sugar dissolved in water as opposed to the more oceanographically minded salinity in hot or cold water is mainly motivated by the concern that lateral heat losses might influence the measurements. Stern and Turner used two different filling procedures. They either first filled half of the tank with dense salty water and carefully poured less dense sugary water on top by pouring on a thin piece of wood floating on the water, or they used what they called the dam break method. With this method, a vertical wall is introduced in the tank and each side of the tank is filled with one or the other of the two types of fluid. The wall is then removed and the lighter fluid flows on top of the denser fluid. This sounds like a rather brutal method but the authors report that internal waves generated by the dam break method died out quickly and that the resulting two layers were a useful initial condition for the subsequent finger formation. The observations relied on visualizations with dye and aluminum flakes.

Linden [19] used the combination of heat and sugar to drive double diffusive convection. The main reason for this choice was the desire to measure velocity with the thymol blue technique [20]. To use this technique, one dissolves the pH-indicator thymol blue at concentrations of typically 10^{-4} per weight in the water and titrates the solution with HCl and $NaOH$ just to the acid side of the end point of the indicator so that the solution appears orange yellow. If a current is sent through two electrodes sticking into the fluid, H^+ ions are drawn towards the negative electrode where they react to H_2 which locally increases the pH. If the increase is large enough,

the color switches to dark blue. The darkened fluid gets carried away from the electrode by the flow and can be followed for as long as it takes for the pH of the darkened drop to return to its original value by diffusion with the environment. This technique works well if there are no other positive ions than H^+ to transport current, and this is why Linden [19] avoided a salty solution. In the modern days of particle image velocimetry, there is presumably no space left for this elegant visualization technique.

Schmitt [21] returned to the heat-salt system but in a tank larger than those used previously with a height of $98 \, cm$ and a cross section of $19 \times 19 \, cm^2$. The convection in a larger tank leads to more vigorous flow which can be relied upon to well mix the fluid outside the finger layer and which obviates the need for a stirring device as used by Turner [17]. A larger tank also accommodates larger flow structures so that more bulky probes can be inserted into the flow without disturbing it. The experiment was equipped not only with a moveable thermistor, but also with a salinity probe. Apart from the rig that held the probe, the mechanically obstructing part of the salinity probe was a platinum electrode of radius $0.25 \, mm$ which sampled conductivity in a volume approximately ten times this size. Another in situ salinity probe appeared in Ref. [22].

The filling procedure used by Schmitt [21], or later by McDougall and Taylor [23] involved a mechanical barrier between the two layers. This was a propylene sheet in the case of McDougall and Taylor [23] which was carefully removed once the two layers were set up to allow finger formation to start.

A very convenient visual detection of fingers is by a shadowgraph either with a vertical or a horizontal plane of view. The experiments in this section do not require top and bottom boundaries with special properties, so that they can be made transparent. One can then take shadowgraph pictures along a vertical line of sight as was done by Shirtcliffe and Turner [24] who observed fingers with square cross section in the sugar-salt system.

Let us now turn to the results of these experiments. There are three issues that nearly every experiment on the two layer system addresses. These are the salinity flux, the heat flux or equivalently the flux ratio, and the thickness of the finger zone. The results concerning the last of these topics is usually framed in terms of a Stern number A defined as

$$A = \frac{\alpha F_T - \beta F_c}{\nu \left(\alpha \frac{\partial}{\partial z} \overline{T} - \beta \frac{\partial}{\partial z} \overline{c} \right)} \tag{11}$$

where F_T and F_c are the thermal and ion fluxes introduced in the previous section and $\overline{T}$ and $\overline{c}$ are the horizontal averages of temperature and ion concentration. The collective instability of fingers [15, 16] sets in if the Stern number exceeds a threshold value of 1. While experiments with the heat-salt system are broadly in agreement with this criterion, Ref. [25] found a Stern number on the order of 10^{-3} in the saturated fingers in the sugar-salt system, so that the Stern number does not seem to yield a universally valid measure for finger lengths.

All the flux measurements are compatible with the 4/3-law. However, the data within each data set extends over barely one order of magnitude in Ra_c so that the exponent in the flux law is poorly constrained and the results are not a definitive confirmation of the 4/3-law. Concerns have been voiced over variations of the prefactor in the 4/3-law from one experiment to another which might call into question the validity of the 4/3-law [26]. Considering the data compilations provided by Refs. [22] and [27] which show variations of up to almost a factor of 2 in either this prefactor or the flux ratio and a comparable scatter in the data obtained from every individual apparatus, one simply has to admit that the data on the two layer system lack the accuracy for definitive conclusions. The experiments on the two layer system investigate the transient of a system whose initial conditions are not well controlled. It would be rather surprising to find perfect agreement between the results obtained from one filling procedure and another. It is presumably impossible to run measurements on the two layer system of the same quality

as we grew accustomed to from other fluid dynamic experiments in the past two decades or so. The experiments described in this section had the merit of launching the subject and of providing indications of relevant mechanisms, but other set ups must provide the stringent tests for quantitative theories.

4. Educational experiments

The experiments of the previous section use low cost equipment, offer pleasing visualization and demonstrate basic physics. They are ideally suited for student labs. Ref. [28] assembles a gratifying combination of methods and parameters of a two layer system for a student exercise. They employ a method not mentioned so far in this review which speaks in favor of sugar as one of the two components: Sugar is optically active and rotates the direction of polarization of polarized light traversing a sugar solution. This effect yields a non invasive method for measuring sugar concentration profiles.

A collection of material properties useful for double diffusive systems involving water, temperature, salt and sugar, including optical activity, is compiled in Ref. [29].

Another set of experimental instructions worth mentioning realizes a salt fountain which is a concept introduced in Ref. [30] that predates the discovery of salt fingering (albeit one may argue about when exactly salt fingers were discovered. For a nice account of the early history of salt finger experiments, see [31].). A salt fountain is essentially a single finger. Imagine a vertically oriented pipe whose walls are made of a good thermal conductor lowered into a part of the ocean in which warm salty water lies over comparatively cold and fresh water with a small length of the pipe protruding out of the water. If the water inside the pipe is given a push upwards, it will continue to rise for the reasons given in Section 2.1 to explain the existence of fingers. In the present case, the pipe walls prohibit any lateral salt diffusion whatsoever. One obtains a perpetual outflow at the upper end of the pipe which looks like a fountain. Ref. [30] contains a succinct description of a laboratory demonstration of such a fountain, and Heavers [32] gives more detailed instructions.

5. Gradient systems

We noted in Section 3 that the two layer system is sensitive to initial conditions which vary from apparatus to apparatus because different filling procedures start the experiment from one or the other non zero velocity field. In addition, the interface between the two layers has thickened by diffusion and mixing by the time the experiment properly speaking starts. The two layer system is an unsatisfactory system from the experimental point of view and it seems preferable to start finger formation from uniform temperature and salinity gradients, or even better, maintain these gradients as long as desired so that the fingers can reach a statistically stationary state.

This section will discuss three types of experiments. The first type sets up uniform gradients, lets fingers evolve, and runs until the gradients are consumed by convection. These experiments again investigate a transient and share in principle the same problem as encountered in the two layer system, but in alleviated form because the initial conditions do not contain a sharp gradient. The second type of experiments maintains solute gradients with semi permeable membranes, while the third type achieves the same goal in an electrochemical cell. The last two classes of experiments are therefore of a similar design as Rayleigh–Bénard convection, in the sense that a gradient is imposed across a cell and the transport through the cell can be accurately measured in a statistically stationary state.

5.1. *Run down experiments*

A variation of the two layer experiment by Turner [17] was used by Taylor [33] to produce a uniform gradient. Two layers with warm salt water over cold fresh water were prepared at first. Fingers started to form at the interface. Then, a grid was dropped through the tank which destroyed the fingers and partially mixed the two layers to create a transition zone with a continuous gradient in which new fingers formed. Dye was mixed into the water in the initial upper layer to visualize motions, and the tank was equipped with thermistors and conductivity probes moveable both in vertical and horizontal directions. The main result of this investigation was that individual fingers did not extend throughout the entire gradient zone. The finger layer rather looked like a disorganized ensemble of elongated blobs. According to the spatial spectra deduced from the probes moving across the tank, the fingers were only approximately twice as long as wide for density ratios between 2 and 5.

Stern and Turner [18] report about two different run down experiments with preconfigured gradients. The first one realizes one of the initial conditions considered in Section 2.1 with a uniform stabilizing temperature gradient in fresh water with a layer of warm and salty water on top. For this experiment, the tank was first filled with stratified water with the commonly used two bucket method: The tank is filled through a tube with its outlet at the bottom of the tank. This tube is connected to a reservoir (or bucket) full of warm water, which itself is connected to a second reservoir full of cold water. The second connection ensures that the water levels in both reservoirs is at the same height at all times. As the first reservoir empties into the tank, water flows from the second reservoir into the first where it is mixed with the water already present. The temperature of the water exiting the first reservoir is therefore decreasing linearly as a function of the delivered volume which results in the stably stratified layer in the tank.

For the configuration discussed in Section 2.1, one needs in addition to prepare some warm salty water of density less than that of the water at the top of the stably stratified layer and carefully pour that water into the tank from the top. Following this procedure, Stern and Turner [18] observed fingers and an increasing number of layers if the stabilizing gradient was increased, as expected from the discussion in Section 2.1. In a second experiment, Stern and Turner filled the two buckets with either a salt or a sugar solution of identical temperature to fill the tank with a uniform layer with a stabilizing salt and a destabilizing sugar gradient. Finger formation already started during filling with this procedure so that measurements on the subsequent transients are again sensitive to details of how the experiment was started. Stern and Turner used this feature to make an interesting observation: If the tank was filled slowly and carefully, fingers developed that extended through the entire tank. Rapid filling or filling at an increasing rate led to a layer with large scale convection patterns that was not replaced by fingers later on, even though it was sandwiched between two finger layers. If on the contrary the authors locally stirred the tank full of extended fingers, the flow quickly returned to vertical fingers. Later experiments explored in greater detail the robustness of salt fingers created by the two bucket method against mechanical perturbations such as periodic shear [34], localized stirring [35] or intermittent turbulence [36].

5.2. *Membranes as boundaries*

Krishnamurti [1] was the first to build an experiment in which differences in temperature and salinity between top and bottom boundaries could be maintained over long periods of time for the sugar-salt system. The convection cell itself had a cross section of $30\,cm \times 10\,cm$ and heights varying from $30\,cm$ to $183\,cm$. The central idea of the apparatus was to place a reservoir filled with sugar solution above and another reservoir filled with salt solution below the convection cell, and

to introduce between the reservoirs and the cell a membrane that lets salt and sugar pass from one fluid volume to the next, but which is impermeable to fluid flow. Each reservoir was $30cm$ deep.

The feasibility of the experiment depends on whether one finds membranes with the desired properties. In a preliminary experiment, Krishnamurti and Zhu [37] test the properties of their membranes by comparing measurements to results of linear stability calculations and other analytical solutions and find good agreement. The combination of membrane and reservoir acts like a Dirichlet boundary condition by which salt or sugar concentrations at the top and bottom boundaries are fixed. However, there will be flow through the membranes if enough pressure head is present so that certain precautions were necessary during the operation of the apparatus as described now.

The top and bottom reservoirs were mechanically mixed to guarantee homogeneous concentration within each tank. With the help of additional tanks, each reservoir could be flushed and its contents be replaced with freshly prepared solution with the prescribed concentrations. The outflow of each reservoir was collected and its composition determined which allowed to measure the vertical fluxes of salt and sugar. The concentrations in each tank were kept at nearly constant levels by flushing the two reservoirs in turn and at regular intervals, but never simultaneaously. All the connections to one reservoir need to be closed while the other is flushed to prevent possible pressure differences from pushing water through the membranes. It is also important that there be no air bubbles within the working fluid, which is why several bubble traps were part of the set up.

A later experiment [38] simplified the apparatus by forgoing the flushing, so that the concentrations in each reservoir changed, but hopefully slowly enough compared with the dynamics in the convection cell. The measurements of the fluxes must then be done by taking samples from the reservoirs to determine their concentrations.

In addition to flux measurements, the double diffusive convection was diagnosed by particle image velocimetry and particle tracking velocimetry. Vertical profiles of sugar and salt concentrations were determined by sending a horizontal beam of polarized light through the cell and by measuring both the angle by which the polarization direction was rotated and the deflection angle of the beam (from which the refractive index was deduced).

As one could have expected, Krishnamurti's idea of a novel experimental system led to a whole series of new observations. These concerned first of all qualitative descriptions inspired by visualizations. In addition to straight fingers, Krishnamurti [1] mentions "lumpy" and "wavy" fingers. The largest convection tank could accommodate a record number of up to 15 layers (8 finger layers and 7 overturning layers). A remark in [1, Section 4.3] reads: "The junction between the convection layer and the finger layer fluctuated so rapidly that the image of the laser beams made the fluid appear to be "boiling" and reliable measurements were difficult to obtain here." Fast fluctuations of this type were never reported before. In addition, the overturning could have an unusual structure and consist of many disorganized small scale plumes but lacking a large scale circulation [38].

On the quantitative side, the flux measurements point at a difficulty if one attempts to express the fluxes in terms of power laws or a sum of several power laws as we are used to from Rayleigh–Bénard convection, because every time a new convection or finger layer appears, the flux through the cell decreases discontinuously. Thanks to the measurements of the vertical profile of salt and sugar concentrations, it was also possible to compare the flux law through a finger layer in a staircase with the flux laws obtained from the two layer system in Section 3. The fingers within the staircase did not fit well the 4/3-law but instead followed the relation $F_c \propto \Delta c^a$ with a between 0.18 and 0.19.

5.3. *Electrochemical cells*

The second system we know of that allows us to study double diffusive convection in a statistically stationary state is an electrochemical cell. This system is a variation of the one introduced in Ref. [39] as a model system to study Rayleigh–Bénard convection at high Prandtl number. Imagine a cell bounded by two horizontal copper electrodes filled with aqueous solution of $CuSO_4$. If a current is sent through this cell, copper ions detach from one electrode and deposit on the other. The behaviour of such an electrolytic cell is textbook material in electrochemistry [40]. A non uniform spatial distribution of copper ions develops which may lead to convection instability. The variable copper ion concentration is responsible for the variations of density. However, the electrolytic cell really becomes useful for convection studies only after one adds a supporting electrolyte like H_2SO_4 to the solution. The sulfuric acid does not participate in any reaction at the electrodes, but the electric field due to the potential difference applied between the two electrodes separates the ions of the sulfuric acid which accumulate near the electrodes within layers whose thickness is on the order of a Debye length. These microscopically thin layers screen the bulk of the cell from the electric field. For a sufficient concentration of H_2SO_4, the bulk is nearly field free and the copper ions diffuse and are advected by macroscopic fluid flow, but they do not drift in any electric field. If c denotes the copper ion concentration, then its equation of evolution is given by eq. (5).

The next step is to find a way to control the Rayleigh number. This is done with the help of the limiting current, a phenomenon common to all electrolytic cells [40]. The cell is initially filled with a solution of known copper concentration c_0. Let us consider the variations of the electric current through the cell as a function of the applied voltage. The current increases with voltage as long as the voltage is small. A copper concentration gradient must build up near each electrode because only diffusion can transport ions in the proximity of the boundaries. With increasing voltage, the current and the gradients increase until the copper concentration near the cathode drops to zero. This is when the limiting current is reached, because a further increase of the voltage cannot increase the current any more. Since the bulk concentration is c_0 and the concentration at the cathode is zero, the only way to increase the concentration gradient at the cathode is to reduce the thickness of the concentration boundary layer. This would require a higher advection velocity in the bulk to carry away the larger current, which is only possible if the buoyancy forces increase, which necessitates larger concentration differences. However, the concentration drop at the cathode is already at its maximum value, so that a further increase of the current is impossible. Because the average concentration is c_0 and the concentration at the cathode is down to zero, we must by symmetry have a concentration of $2c_0$ at the anode. With the applied voltage set such that the limiting current flows through the cell, we know the concentration difference between top and bottom boundaries and can thus compute the chemical Rayleigh number.

Such an electrochemical cell was used in Ref. [39] to study Rayleigh–Bénard convection (with further technical details given in Ref. [41]) and Hage and Tilgner [14] modified it simply by regulating the temperature of the two copper electrodes so that double diffusive convection could be investigated. The ion flux is very conveniently determined in this system from the electric current flowing through the cell, and particle image velocimetry allows to measure both convection velocities and the size of salinity fingers.

The most remarkable finding in Ref. [14] was that finger convection occurs for $|\Lambda| < 1$ when the density stratification is top heavy and large scale overturning convection could occur according to linear stability analysis (even though it is in competition with narrow fingers for $0.9 < |\Lambda| < 1$, see Ref. [42]), but finger convection nonetheless prevails. Almost all measurements in Ref. [14] were taken for $|\Lambda| < 1$ to explore this unexpected form of convection. The main observables were
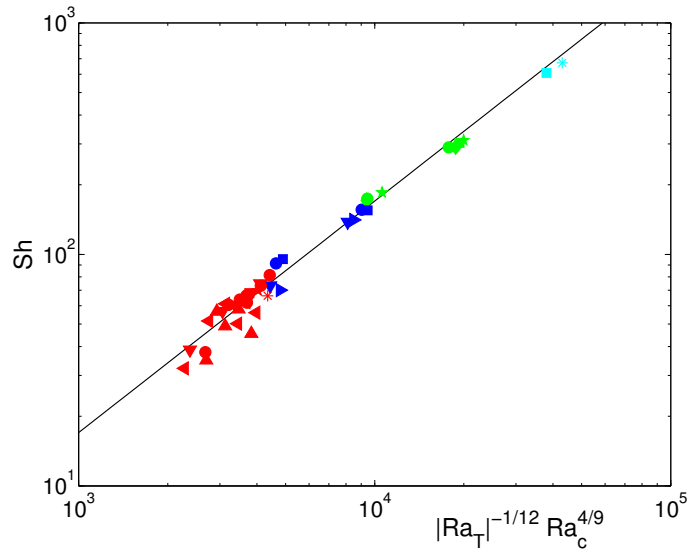
Figure 3. The Sherwood number as a function of $|Ra_T|^{-1/12} Ra_c^{4/9}$ obtained from measurements in an electrochemical cell. The symbol shapes vary according to the magnitude of the adverse temperature gradient and the symbol color according to the cell height. The original figure appeared in Ref. [14].

found to obey scaling laws in terms of the two Rayleigh numbers, and the exponents obtained from the best least square fits agreed with the following simple picture: The dynamics of long and narrow fingers should not be affected by distant boundaries, so that the height L of the cell should not matter. The fluid inside the fingers retains the ion concentration of the boundary they started from with little loss due to diffusion during the journey from one electrode to the other, while temperature quickly diffuses horizontally so that the fluid in the bulk only probes the local temperature gradient but is insensitive to the temperature at the boundaries. The dynamics of the fingers should therefore depend on Δc and $\Delta T/L$, but not on L. Scaling laws for the finger thickness d, the convection velocity v and the thickness of the concentration boundary layer λ may involve powers of Δc and $\Delta T/L$, but none of L alone. The Sherwood number is given by $Sh = L/(2\lambda)$. The 4/3-law is of course compatible with this reasoning as explained in Section 2.2 and reads $Sh \propto Ra_c^{1/3} f(\Lambda, Pr, Sc)$ if expressed for Sh. The best fit in Ref. [14] actually yielded $Sh \propto |Ra_T|^{-1/12} Ra_c^{4/9}$. However, this is identical to $Sh \propto Ra_c^{1/3} |\Lambda|^{-1/12} Ra_c^{1/36}$, so that apart from a virtually undetectable factor $Ra_c^{1/36}$, this expression conforms with the 4/3-law. These experiments used rectangular cells of cross section $20 \times 20\,cm^2$ and heights varying from $2\,cm$ to $20\,cm$, so that Ra_c varied by more than three orders of magnitude. These measurements (shown in Figure 3) are as of today the most precise test of the 4/3-law.

A later experiment with the same cell focussed on the transition from finger convection to overturning convection [44]. This study showed that the ion flux can actually be increased by an adverse temperature gradient. The maximal ion transport at a given Ra_c is observed at the Ra_T at which the transition from fingers to convection rolls occurs. The fingers are slower than the convection rolls, but they fill the entire fluid volume with a velocity field with a large vertical component, whereas convection rolls contain large cores which contribute little to vertical transport.

Kellner and Tilgner [44] identified as a possible criterion for the transition from fingers to convection rolls the condition that the time it takes fluid in fingers to travel across the cell may
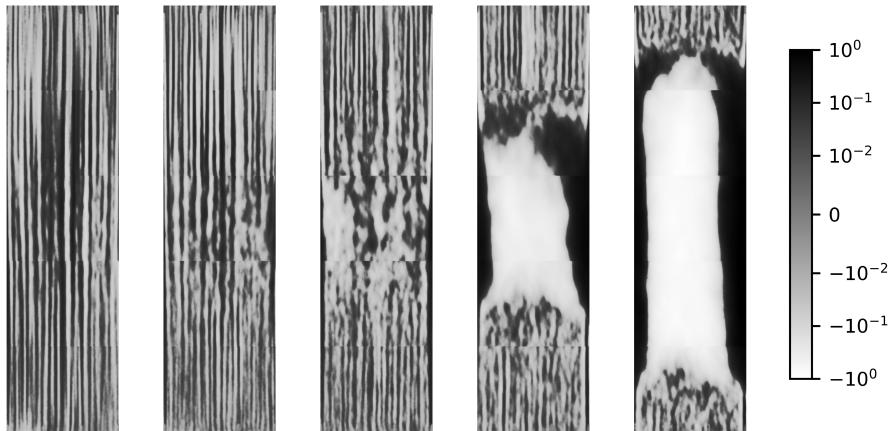
Figure 4. Grey scale maps of the vertical velocity normalized by the maximal absolute value of vertical velocity for $Ra_c = 1.12 \times 10^{13}$ and, from left to right, $Ra_T = -1.22 \times 10^{11}$, -1.01×10^{11}, -4.97×10^{10}, -2.44×10^{10} and -1.23×10^{10}. This figure is taken from Ref. [43].

not be shorter than the time it takes heat to diffuse across a finger, because the heat diffusion is central to the finger forming mechanism explained in Section 2.1. This criterion was confirmed by later experiments [43].

Ref. [43] used a cell of up to $80\,cm$ height in the hope of finding staircases made of several layers. An overturning layer in between two finger layers could indeed be observed for $Ra_c > 10^{12}$ and Ra_T in the right range. For $|Ra_T|$ too small, there simply is a single roll layer as in Rayleigh–Bénard convection, and for $|Ra_T|$ too large, fingers extend straight from top to bottom. At intermediate $|Ra_T|$, one observes fingers which are thicker in the center than near the boundaries, or overturning layers within a staircase whose thickness increases with decreasing $|Ra_T|$ as shown in Figure 4.

6. Conclusion and outlook

Single component or Rayleigh–Bénard convection has been studied for well over a century. The fact that this problem still attracts interest is certainly for the most part due to the charming simplicity of the boundary conditions, the governing equations and the underlying physics which nonetheless give rise to very complex behavior. Double diffusion adds another layer of complexity to the flow. The experimental study of the double diffusive version of the problem is very immature compared with the single component case. The data coverage is limited, as shown in Figure 5. What is more, the results from the different experiments do not combine well to a coherent picture. For example, Stern and Turner [18] mention a dependence on initial conditions in that convective layers, once formed, are not easily replaced by fingers. Other experiments on gradient systems do not report a similar effect, even though the authors of Ref. [43] made an effort to find a dependence on initial conditions or a hysteresis and did not find any.

Another point concerns the short fingers discovered by Taylor [33]. This feature is spectacular enough to be easily detectable in any visualization, and yet, it did not show in the experiments of Ref. [14] at nearby parameters. Flux measurements also raise questions. Krishnamurti [1] found from best fits to her measurement of fluxes through finger layers within staircases a relation different from the 4/3-law, while the results of Ref. [43] were compatible with the 4/3-law.
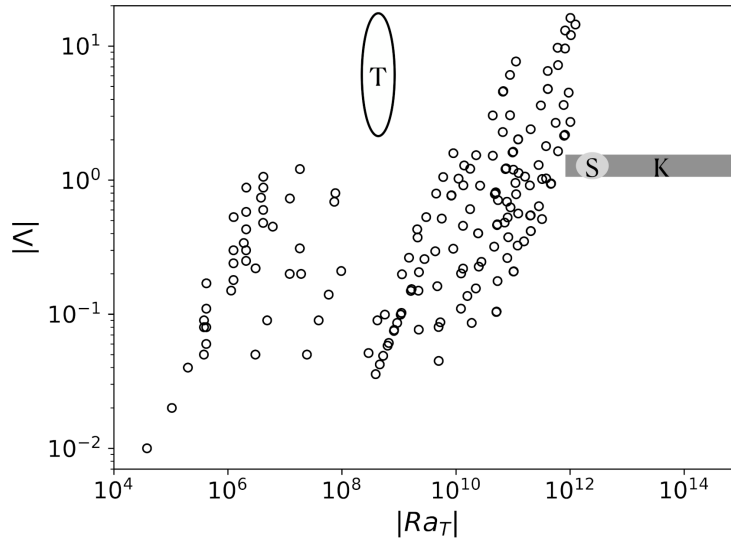
Figure 5. Overview of the parameters of the experiments on gradient systems. The many circles show data points obtained with the electrochemical cells, and the oval labeled T shows the experiment by Taylor [33]. The grey shaded regions correspond to experiments on the sugar-salt system. The light shaded region labeled S is covered by Stern and Turner [18] and the dark shaded region labeled K by Krishnamurti [1].

None of these observations constitutes an outright contradiction for the moment, either because the measured quantities are not exactly comparable, or because the control parameters in the different experiments do not overlap, or because experimental uncertainties do not allow firm conclusions. Nonetheless, the collection of these observations raises questions and these points certainly need closer inspection as part of a more complete investigation of the parameter space.

The limited survey in Figure 5 also implies that we presently do not know well the dependence of salinity and heat transport on the control parameters. The situation could be notably improved by fully exploiting the tools we already have. But new ideas for experimental systems capable of controlling an applied salinity and temperature gradient would certainly also be of great benefit to the field. No matter which system is used, there remains a lot of data collection to be done.

Declaration of interests

The authors do not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and have declared no affiliations other than their research organizations.

References

[1]   R. Krishnamurti, "Double-diffusive transport in laboratory thermohaline staircases", *J. Fluid Mech.* **483** (2003), pp. 287–314.

[2]   Y. Yang, R. Verzicco and D. Lohse, "Scaling laws and flow structures of double diffusive convection in the finger regime", *J. Fluid Mech.* **802** (2016), pp. 667–689.

[3]   J. Li and Y. Yang, "On the wall-bounded model of fingering double diffusive convection", *J. Fluid Mech.* **973** (2023), article no. A37.

[4] A. B. Tsinober, Y. Yahalom and D. J. Shlien, "A point source of heat in a stable salinity gradient", *J. Fluid Mech.* **135** (1983), pp. 199–217.

[5] R. W. Griffiths and A. A. Bidokhti, "Interleaving intrusions produced by internal waves: a laboratory experiment", *J. Fluid Mech.* **602** (2008), pp. 219–239.

[6] B. R. Ruddick and J. S. Turner, "The vertical length scale of double-diffusive intrusions", *Deep-Sea Res., Part A* **26** (1979), no. 8, pp. 903–913.

[7] R. Krishnamurti, "Double-diffusive interleaving on horizontal gradients", *J. Fluid Mech.* **558** (2006), pp. 113–131.

[8] F. Heslot, B. Castaing and A. Libchaber, "Transitions to turbulence in helium gas", *Phys. Rev. A* **36** (1987), no. 12, pp. 5870–5873.

[9] A. Belmonte, A. Tilgner and A. Libchaber, "Boundary layer length scales in thermal turbulence", *Phys. Rev. Lett.* **70** (1993), pp. 4067–4070.

[10] J. J. Niemela, L. Skrbek, K. R. Sreenivasan and R. J. Donnelly, "Turbulent convection at very high Rayleigh numbers", *Nature* **404** (2000), pp. 837–840.

[11] R. DuPuits, C. Resagk, A. Tilgner, F. H. Busse and A. Thess, "Structure of thermal boundary layers in turbulent Rayleigh–Bénard convection", *J. Fluid Mech.* **572** (2007), pp. 231–254.

[12] P.-E. Roche, "The ultimate state of convection: a unifying picture of very high Rayleigh numbers experiments", *New J. Phys.* **22** (2020), no. 7, article no. 073056.

[13] T. Radko, *Double-Diffusive Convection*, Cambridge University Press, 2013.

[14] E. Hage and A. Tilgner, "High Rayleigh number convection with double diffusive fingers", *Phys. Fluids* **22** (2010), article no. 076603.

[15] M. E. Stern, "Collective instability of salt fingers", *J. Fluid Mech.* **35** (1969), pp. 209–218.

[16] J. Y. Holyer, "On the collective instability of salt fingers", *J. Fluid Mech.* **110** (1981), pp. 195–207.

[17] J. S. Turner, "Salt fingers across a density interface", *Deep-Sea Res.* **14** (1967), pp. 499–611.

[18] M. E. Stern and J. S. Turner, "Salt fingers and convecting layers", *Deep-Sea Res., Part A* **16** (1969), no. 5, pp. 497–511.

[19] P. F. Linden, "On the structure of salt fingers", *Deep-Sea Res.* **20** (1973), pp. 325–340.

[20] D. J. Baker, "A technique for the precise measurement of small fluid velocities", *J. Fluid Mech.* **26** (1966), pp. 573–575.

[21] R. W. Schmitt, "Flux measurements on salt fingers at an interface", *J. Mar. Res.* **37** (1979), no. 3, pp. 419–436.

[22] J. Taylor and P. Bucens, "Laboratory experiments on the structure of salt fingers", *Deep-Sea Res.* **36** (1989), no. 11, pp. 1675–1704.

[23] T. J. McDougall and J. R. Taylor, "Flux measurements across a finger interface at low values of the stability ratio", *J. Mar. Res.* **42** (1984), pp. 1–14.

[24] T. G. L. Shirtcliffe and J. S. Turner, "Observations of the cell structure of salt fingers", *J. Fluid Mech.* **41** (1970), pp. 707–719.

[25] R. B. Lambert and J. W. Demenkow, "On the vertical transport due to fingers in double diffusive convection", *J. Fluid Mech.* **54** (1972), no. 4, pp. 627–640.

[26] J. R. Taylor and G. Veronis, "Experiments on double-diffusive sugar-salt fingers at high stability ratio", *J. Fluid Mech.* **321** (1996), pp. 315–333.

[27] C. Y. Shen, "Heat-salt finger fluxes across a density interface", *Phys. Fluids* **5** (1993), no. 11, pp. 2633–2643.

[28] R. M. Heavers and L. A. Colucci, "Sugar Fingers and Double-Diffusive Convection", *J. Chem. Educ.* **86** (2009), no. 11, article no. 1326.

[29] B. R. Ruddick and T. G. L. Shirtcliffe, "Data for double diffusers: Physical properties of aqueous salt-sugar solutions", *Deep-Sea Res., Part A* **26** (1979), no. 7, pp. 775–787.

[30] H. Stommel, A. B. Arons and D. Blanchard, "An oceanographical curiosity: the perpetual salt fountain", *Deep-Sea Res.* **3** (1956), no. 2, pp. 152–153.

[31] R. W. Schmitt, "The Salt Finger Experiments of Jevons (1857) and Rayleigh (1880)", *J. Phys. Oceanogr.* **25** (1995), no. 1, pp. 8–17.

[32] R. M. Heavers, "The salt fountain", *Phys. Teach.* **32** (1994), no. 9, pp. 524–525.

[33] J. R. Taylor, "Anisotropy of Salt Fingers", *J. Phys. Oceanogr.* **23** (1993), no. 3, pp. 554–565.

[34] M. G. Wells, R. W. Griffiths and J. S. Turner, "Generation of density fine structure by salt fingers in a spatially periodic shear", *J. Geophys. Res. Oceans* **106** (2001), no. C4, pp. 7027–7036.

[35] M. G. Wells and R. W. Griffiths, "Localized stirring in a field of salt-fingers", *Dyn. Atmos. Oceans* **35** (2002), no. 4, pp. 327–350.

[36] M. G. Wells and R. W. Griffiths, "Interaction of salt finger convection with intermittent turbulence", *J. Geophys. Res. Oceans* **108** (2003), no. C3, article no. 25.

[37] R. Krishnamurti and Y. Zhu, "Double-diffusive convection with imposed vertical mass flux", *J. Mar. Res.* **48** (1990), pp. 89–108.

[38]   R. Krishnamurti, "Heat, salt and momentum transfer in a laboratory thermohaline staircase", *J. Fluid Mech.* **638** (2009), pp. 491–506.

[39]   R. J. Goldstein, H. D. Chiang and D. L. See, "High-Rayleigh-number convection in a horizontal enclosure", *J. Fluid Mech.* **213** (1990), pp. 111–126.

[40]   R. F. Probstein, *Physicochemical Hydrodynamics. An introduction*, John Wiley & Sons, 1994.

[41]   H. D. Chiang and R. J. Goldstein, "Application of the Electrochemical Mass Transfer Technique to the study of Bouyancy-driven Flows", in *Transport Phenomena in Heat and Mass Transfer* (J. A. Reizes, ed.), Elsevier, 1992, pp. 1–25.

[42]   R. W. Schmitt, "Thermohaline convection at density ratios below one: A new regime for salt fingers", *J. Mar. Res.* **69** (2011), pp. 779–795.

[43]   A. Rosenthal, K. Lüdemann and A. Tilgner, "Staircase formation in unstably stratified double diffusive finger convection", *Phys. Fluids* **34** (2022), article no. 116605.

[44]   M. Kellner and A. Tilgner, "Transition to finger convection in double-diffusive convection", *Phys. Fluids* **26** (2014), article no. 094103.