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History of Sciences and Ideas / *Histoire des sciences et des idées*

Adventures in physics

Aventures en physique

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To the memory of my friend Gérard Toulouse

Abstract. In the 1970s Gérard Toulouse and myself we wrote three papers on how the homotopy groups of the manifold of internal states influenced the defects of ordered media. Specifically, we showed how the algebra of homotopy groups acted on the dynamics of defects.

Résumé. Dans les années 1970, Gérard Toulouse et moi-même avons écrit trois articles sur la façon dont les groupes d'homotopie de la variété des états internes influençaient les défauts des milieux ordonnés. Plus précisément, nous avons montré comment l'algèbre des groupes d'homotopie agissait sur la dynamique des défauts.

Keywords. Symmetry breaking, Non-commutativity, Bouligand rings.

Mots-clés. Brisure de symétrie, Non-commutativité, Anneaux de Bouligand.

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I am a mathematician, and my personal field of research is geometric topology, more precisely low-dimensional manifolds and geometric group theory. In my life I have worked a lot on the Poincaré conjecture and related topics.

Over the years, however, my mathematical interests were quite varied. I also worked on the singularities of mappings, under the effect of a symmetry group, and I happened to stumble on the Higgs potential, although I did not understand its meaning at the time. But there, I quite naturally met the topic of symmetry breaking, something which fits perfecly with what I am about to tell you.

In the early 1970s, I was a professor in the mathematics department of the Paris-Orsay University; our neighbors and friends, the Williamses, an Anglo-Belgian couple, worked in the condensed matter physics department of the same university. Claudine Williams was organizing monthly lectures on general scientific topics for the members of her department. And in about 1973, I gave there an introductory lecture on topology.

As soon as I finished my lecture, two members of the audience approached me, saying that the kind of things I had talked about was exactly what they needed. The two were Gérard Toulouse and Maurice Kléman. I understood from them that topology was discovered to help understand the defects of ordered media. At that time, only two groups in the world had started to work on this topic. One in Paris, more precisely in Orsay and Bures-sur-Yvette, with Toulouse, Kléman and Louis Michel from IHES, and the other in Moscow, with Volovik and Mineev who worked in the team of Dzialoshinsky. Incidentally, a younger physicist, Misha Monastyrsky joined them a bit later. And after some years, Misha started coming to Bures-Orsay, we met and we became friends.

This was how things were in those old days. But, about thirty years later, the situation changed drastically. With the work of Ed Witten, Barry Simon (who, incidentally, had been my student in a topology course at Harvard, in the early 1960s) and others, that led to topological quantum field theory, a topic that fascinates me, and with other things like that, topology became a big hot topic in physics.

But let us go back to the early 1970s. Gérard and I started talking a lot together, he taught me physics and I taught him topology. And since I was having some problems with my own mathematical work at the time, I welcomed a temporary change of horizon. Gérard was a charming personality, full of enthusiasm, inspiration, and clever ideas. He was also very good at communicating these things. Of the many physicists I have met in my life, he was the one with whom I could interact best.

Concerning our work together, it was already known at the time that defects of ordered media were connected with the homotopy groups of the manifold of internal states, a space that somehow encodes symmetry breaking. In particular, the existence of defect lines was connected with the fundamental group of that manifold of internal states, and similarly, the defect points had to do with the second homotopy group.

But Gérard and I looked at the dynamics of defects, and we discovered that the non-commutativity of that fundamental group, when it exists, prevented the defect lines from crossing [1]. Many physicists outside the topic of condensed matter became interested in our result, partly I think, because non-commutativity is so fundamental in quantum theory.

Actually, the famous Russian field theorist Sasha Polyakov suggested that we look at what happens to our little theory when one tries to extend it to higher dimensions; that was clearly outside of condensed matter physics.

And there we found a beautiful surprise. The totality of homotopy groups, when endowed with the Whitehead product, becomes a supersymmetric algebra. And we found out that the non-commutativity of the Whitehead product is the obstruction to the crossing of higher-dimensional defects [2]. So, somehow, supersymmetry is now involved. Gérard told me that he had never experienced such joy at a discovery.

I will introduce here a little anecdote, which I find amusing. In the early 1970s, Jean Dieudonné, the famous French mathematician, wrote a book on the mathematics of Bourbaki. And when it came to topology, he stated that this topic had no connection with physics. When he published a second edition a few years later, he corrected that statement, and he also quoted Gérard, myself, and Louis Michel.

Our third and last work together, a piece done by us two, together with Yves Pomeau, Bernard Derrida and Yves Bouligand, concerned the following puzzling issue. Bouligand, who was a biologist but also a distinguished physicist, had discovered what are now called Bouligand rings in his study of biological tissues. These were defect rings, existing in a situation when the potential obstruction to crossing was living at the center of the fundamental group of the manifold of internal states, and which nevertheless stayed hooked together, refusing to cross. But non-commutativity was not the explanation.

What we discovered was that the explanation for this paradoxical situation was the emergence of a new player in the game, the third homotopy group of the manifold of internal states. The deep reason for the appearance of this totally new ingredient, was that we were now in the context of textures, where long-range effects also become important [3]. And I think that the mathematics involved here is not unrelated to the still conjectural magnetic monopoles of t'Hooft and Polyakov.

We have now reached the late 1970s. I had managed to overcome the difficulties which were bogging me down earlier, so I went back to my mathematical research. Actually, I had another, shorter incursion into physics, when I helped Geoff Chew from Berkeley to develop a topological bootstrap theory.

However, those years of collaboration with Gérard Toulouse and the others were a very exciting and interesting period in my scientific career. Gérard was a very important figure in my scientific life.

A short appendix on Bouligand rings

Yves Bouligand developed an experimental technique for studying the cholesteric order. He found defect lines, called now the Bouligand rings, which stayed linked, although the homotopical obstruction to their crossing lived in the center of the fundamental group of the manifold of internal states. Non-commutativity was therefore not what kept them linked.

We are actually here in the presence of textures, which are ordered media where the order parameter is defined everywhere, and the region where it is not constant plays the role of defects. It is now a completely new actor, the third homotopy group of the manifold of internal states, which is the obstruction to crossing. For more information, see [4].

Declaration of interests

The author does not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and has declared no affiliations other than his research institution.

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