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History of Sciences and Ideas / *Histoire des sciences et des idées*

An explanation for Aristarchus' measurement of the sun's distance

Une explication à la mesure de la distance du soleil par Aristarque

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Abstract. Aristarchus of Samos determined the angular distance between moon and sun at half-moon as 87°, from which he deduced that the sun's distance is "between 18 and 20 times that of the moon". In this study, we show that his result can be reproduced and understood when it is assumed that the observation was made when, apart from being perceived as straight, the shadow edge over the moon's surface was vertical. A measurement could then have been done with instruments that were available at the time: the gnomon and the quadrant. An example that might have been observed by Aristarchus himself is presented and analyzed.

Résumé. Aristarque de Samos a déterminé que la distance angulaire entre la lune et le soleil à la demi-lune était de 87°, ce qui lui a permis de déduire que la distance du soleil est « entre 18 et 20 fois celle de la lune ». Son résultat peut être reproduit et compris si l'on suppose que l'observation a été faite lorsque le bord de l'ombre sur la surface de la lune, en plus d'être perçu comme rectiligne, était vertical. Une mesure aurait alors pu être effectuée avec les instruments disponibles à l'époque : le gnomon et le quadrant. Un exemple que Aristarque aurait pu observer lui-même est présenté et analysé.

Keywords. Aristarchus, Ancient astronomy, Distance of the sun.

Mots-clés. Aristarque, Astronomie ancienne, Distance du soleil.

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1. Introduction

Aristarchus of Samos (circa 310–230 BC) is known as the first astronomer to propose a heliocentric model of the universe. His own written work about his ideas on the universe has not survived. Fortunately, his book Περὶ μεγεθῶν καὶ ἀποστημάτων ἡλίου καὶ σελήνης (*On the Sizes and Distances of the Sun and the Moon*) has been preserved and it has been translated into many languages. The list of references [1–7] is far from complete, but it contains sources that are readily accessible. The first edition to include the Greek original is that of 1688 by John Wallis [1].

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The oldest preserved copy of Aristarchus' treatise is the 10th-century manuscript with catalog number Vat.gr.204 in the Vatican Apostolic Library. It has been digitized and is easily accessible. This seems to be the version that most other surviving copies are based upon, and it is the primary source that Thomas Heath used in his *Aristarchus of Samos, the ancient Copernicus* [2]. In the present work, we rely on the Greek text as we find it in Heath's work.

In his treatise, Aristarchus first presents a number of working hypotheses. Numbers 1–4 pertain to the distances of the sun and the moon, while 5 and 6 deal with their sizes. In this work, we will only consider distances.

Hypothesis 1: The moon receives its light from the sun.

Hypothesis 2: The earth may be considered as a point and also as the center of the moon's sphere.

Hypothesis 3: When we see the moon as half, the great circle that separates the dark side from the illuminated one is oriented along our line of vision.

Hypothesis 4: When we see the moon as half, its [angular] distance from the sun is less than a quadrant by one thirtieth of a quadrant.

Having presented his hypotheses, Aristarchus then proceeds by clarifying them and discussing their consequences in a number of propositions, eighteen in total. He is careful to remark in proposition 2 that the sun is bigger than the moon and that the part of the moon's surface that is illuminated must therefore be larger than a hemisphere. But in proposition 4, he states that the effect is small: at half-moon, the terminator circle and the "maximum circle" are less than 1/3960 of a quadrant (1°/44) apart. The circle over the moon's surface that separates the dark and the illuminated parts does not sufficiently differ from the maximum circle to be perceptible to our eyes. This is the a posteriori justification for neglecting, in hypothesis 3, the subtle difference between the shadow edge over the moon's spherical surface and the great circle (or "maximum circle") parallel to it. In addition, the statement that this dividing circle is oriented along our line of vision is more precisely phrased in proposition 5, where it is stated that this circle and the connecting line between its center and our eye are in one and the same plane.

It is important to note that Aristarchus uses here the visual acuity of the human eye: lines apart by $1°/44 = 1.36'$ cannot be distinguished. In today's optometry "normal" 20/20 vision (10/10 vision in France) is defined just a bit sharper as an angular perception limit of $1'$. In his hypothesis 6, Aristarchus had originally adopted a semidiameter of 1° for the moon, which is far too big. Archimedes [8–10] tells us in his *Psammites*, also known as *Arenarius*, or *The Sand Reckoner*, that Aristarchus corrected this in hindsight to $15'$. Since we see the moon's semidiameter as $15'$, a $1'$ displacement of the middle of the terminator from the half-moon situation, corresponds to a change of the phase angle, from 90° to $[90° \pm \sin^{-1}(1/15)] = 93.8°$ or 86.2° and hence a phase of $[1 + \cos(93.8°)]/2 = 0.467$ or $[1 + \cos(86.2°)]/2 = 0.533$. For all phases closer to 0.5, the moon will be judged as half.

Hypothesis 4 still puzzles us today. It states that the angular distance of the moon from the sun, its *elongation*, would measure 87° when the moon is seen as half. The situation is illustrated in Figure 1. The ratio of the sun's distance to that of the moon would be $1/\cos(87°) = 19.1$. In Aristarchus' time, trigonometric tables were not yet available. Instead, he established an upper limit of 20 and a lower limit of 18. Our today's knowledge is that this sun/moon distance ratio is much larger: 389.18, based on time-averaged distances. The correct value for the elongation of the moon is therefore 89.85°. Atmospheric refraction makes us see the sun higher than its true altitude. For the moon, the combined effect of parallax and refraction is to lower its apparent altitude. As a result, the apparent elongation may be smaller than the true one, but this effect is not enough to bring it below 89.5°. The puzzling question remains: how could Aristarchus have been so far off?

A widespread opinion is that Aristarchus and his Alexandrian contemporaries did consider the argumentation and the mathematics as more important than the actual measurement. In his

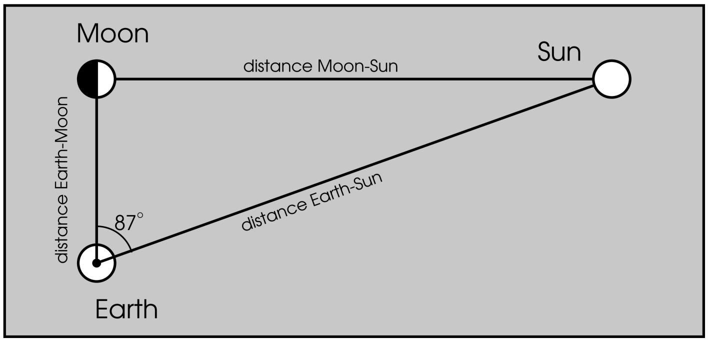

Figure 1. The Earth–Moon–Sun plane. Seen from the earth, the elongation would be "a quadrant minus one thirtieth part of it", or 90° − 3° = 87°; see hypothesis 4.

monumental work *A History of Ancient Mathematical Astronomy*, Otto Neugebauer writes [11, p. 271]: "… there is much in the astronomy of Eudoxus, Aristarchus, and Archimedes (i.e. in the period just preceding Apollonius) that shows a lack of interest in empirical numerical data in contrast to the emphasis on the purely mathematical structure". About Aristarchus, he writes "I think this [i.e. Neugebauer's] analysis leads to the conclusion that Aristarchus' treatise on the sizes and distances is a purely mathematical exercise which has little to do with practical astronomy" [11, p. 643].

Earlier, Paul Tannery in *Recherches sur l'histoire de l'astronomie ancienne* was of the same opinion: "il semble que tous les auteurs aient volontairement méconnu que la science des astres repose essentiellement sur l'observation" [12, p. 43–44].

Different yet is the critique of Reviel Netz in his *Ludic Proof*: Alexandrian scientists would primarily have been interested in storytelling and Aristarchus and Archimedes would have made "the deliberate choice to mystify their audiences" [13, p. 42].

It has been suggested, by Evans [14, p. 72] that, instead of measuring the elongation, Aristarchus could have estimated it from the time it takes the moon to go from its third quarter to its first quarter. Assuming that the moon moves at a constant velocity along its orbit, the finite distance to the sun would cause the path from third quarter to first quarter via new moon to be shorter than the remaining path from first quarter back to third quarter via full moon. This argument is also given by Carman and Buzón [7, p. 191–192] and by Papathomas [15]. The idea is, however, easily rebutted: the time from third to first quarter is not constant at all, as Aristarchus must undoubtedly have been aware of. Instead, it oscillates between 13.5 and 16 days, around half a synodic month, which is 14 days, 18 h and 22 min on average. This is illustrated in Figure 2, where this difference is plotted for the years 2025 and 2026.

Below, we will analyze a first quarter observation that Aristarchus might have witnessed on 16 June 255 BC. The preceding third quarter fell 15 days and 2 h and 40 min earlier, on 1 June 255 BC, when both sun and moon were below the horizon. But even reckoned from the time of moonrise, just over four hours later, there still remained 14 days and 22 h and 30 min to the first quarter on 16 June, *longer—not shorter*—than half a synodic month.

It has also been suggested that Aristarchus may not have performed any measurement at all [14,16].

In our view, however, there is a sufficient number of indications in Aristarchus' text to suggest that he does refer to an actual measurement of the distance to the sun. For example, this is the case in the wording of propositions 2 and 4 above and in the missing part of a quadrant

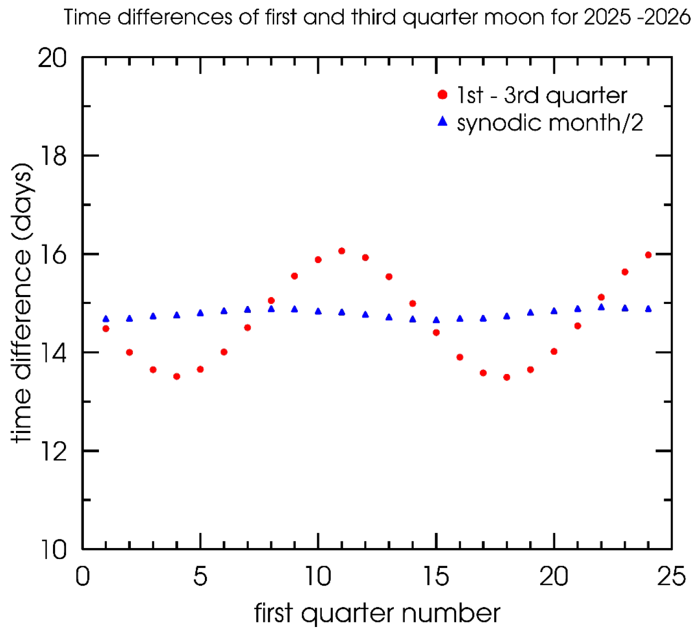

Figure 2. Time differences of first and third quarter moon for 2025–2026. Red dots: the times from third quarter to first quarter via new moon. Blue triangles: half of the synodic month, evaluated as the time between two successive first quarters.

in Hypothesis 4. Therefore, in this article we wish to assume that he did intend to perform a measurement as precisely as possible. We show that his result can be reproduced and understood if we extend his hypothesis 4 with the further specification that the observation was made at an instant that the shadow edge over the moon's surface was seen as vertical. An accurate measurement would then have been possible by using the gnomon and the quadrant. Both instruments did exist in Aristarchus' time and must have been available to him.

In fact, it is hard to imagine that he could have made a measurement with a backward tilting half-moon, and when the half-moon is tilting forward, the sun is already below the horizon.

2. Elongation and tilt angle

As an example, Figure 3 illustrates the situation as it occurred in Alexandria on 16 June 255 BC at 16:43:25 UT, about a quarter of an hour before sunset. In reproducing the ephemerides, we use the planetary program SkyMap Pro 9, of Chris Marriott [17]. Other professional planetary programs [18–20] use the same routines and give identical results.

The zenith for the observer is in P and he sees the sun along PS at a zenith angle of b = $(90° - H_S)$, or, at height H_S above the horizon. Similarly, the moon is seen along PM at a zenith angle of a = $(90° - H_M)$, or, at height H_M. The angle $\gamma = \Delta AZ = AZ_S - AZ_M$ is the difference in azimuth between sun and moon. The moon is illuminated along SM. Since here $\beta = 90°$, the moon is seen upright. The side c = SM represents the elongation ψ, or *lunar distance* as it is known in nautical astronomy. The well-known cosine rule [21] gives:

$$\cos(\psi) = \sin(H_S)\sin(H_M) + \cos(H_S)\cos(H_M)\cos(\Delta AZ). \quad (1)$$

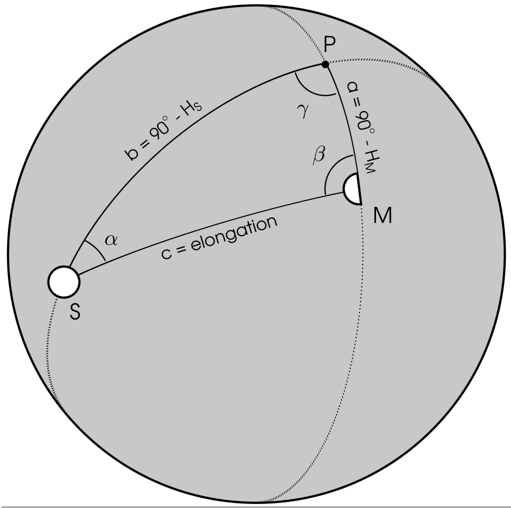

Figure 3. Celestial sphere, illustrating the situation as seen from Alexandria (in P) on 16 June 255 BC, just before sunset.

The equation applies equally to the apparent elongation ψ', which is expressed in terms of the apparent heights H'_S and H'_M, wherein the effects of refraction and parallax have been included [22, and older references quoted therein]. Both are vertical corrections and they do not affect the azimuths. Therefore:

$$\cos(\psi') = \sin(H'_S)\sin(H'_M) + \cos(H'_S)\cos(H'_M)\cos(\Delta AZ). \quad (2)$$

The moon's tilt angle, T, is found from [23–25]:

$$\tan(T) = [\sin(H_S)\cos(H_M) - \cos(H_S)\sin(H_M)\cos(\Delta AZ)]/[\cos(H_S)\sin(\Delta AZ)]. \quad (3)$$

This equation is less well known and not easily searchable. We therefore repeat its derivation [25] in the Appendix of the present work, together with the slightly different form given by Myers and Myers [26].

Figure 4a presents a panoramic view of the moon over the day, in steps of one hour. Horizontally, the azimuth angles are shown as compass directions. Vertically, the moon's height is plotted as its tangent, with tilt angles calculated from Equation (3). The dashed curve represents the celestial equator. In a panoramic view, half of it is above the horizon, stretching from east to west, via south. The part from west via north back to east is below the horizon.

The phase of the moon changes from 44.3% at its rise to 50.5% at moon set. Just under 0.5% in an hour. For an observer without modern equipment to zoom in, and relying only on his naked eye vision, it will be impossible to tell whether the moon's phase is precisely half or whether it is off by a few percent [14]. Also, the elongation changes slowly, by about half a degree in one hour. By the time the moon's phase reaches precisely 50%, the sun is already below the horizon.

In contrast to the phase and the elongation, the moon's tilt angle changes rapidly. One hour before its shadow edge is upright (which happens at 16:43:25 UT), it is tilted backwards by 27.7°. One hour after, its forward tilts is +18.3, hence a change of 46° in two hours. This represents a change of a full degree in just over two and a half minutes.

Figure 4b shows the situation when the shadow edge of the moon is precisely vertical. This happens at 16:43:25 UT, when the moon's phase is 47.9%. We propose that this is the configuration described by Aristarchus. Table 1 lists the heights of the moon and the sun, their azimuths, and the elongation for this instant in time. In addition to the true values, the apparent values

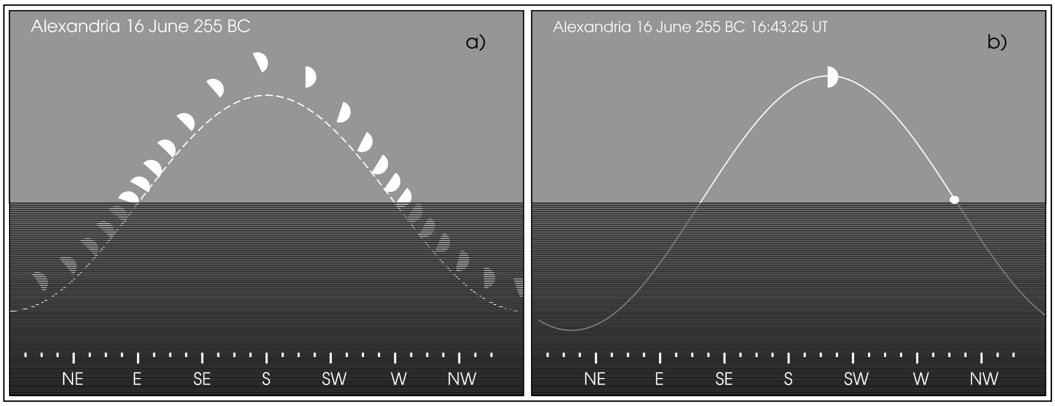

Figure 4. Panoramic views of: (a) The moon, as observed on 16 June 255 BC from Alexandria, in steps of 1 h. At meridian transit (15:52:47 UT), the moon's declination is 6°30′8″ and its altitude 64°53′4″. The dashed curve indicates the celestial equator. (b) Moon and sun at 16:43:25 UT, when the moon is upright. The sinusoidal curve is the great circle through the positions of the moon and the sun.

Table 1. Alexandria, 31°12′N, 29°55′E, anno 255 BC, at UT = 16 h 43 m 25 s

	True height	Apparent height	Azimuth
Sun	2°18.7′	2°34.4′	296°2.8′
Moon	62°38.2′	62°11.1′	207°14.6′
Elongation	True 87°23.8′	Apparent 87°9.9′	Phase = 0.4786

for the heights and for the elongation are also shown. The apparent elongation is almost exactly equal to Aristarchus' value of 87°, in agreement with hypothesis 4.

3. How Aristarchus could have measured the moon's elongation from the sun

Aristarchus is said to have invented the scaphè (σκάφη), a hollow half or quarter sphere, equipped with a shadow pin. For measuring the moon–sun distance it would, however, not have been practical, since the moon does not cast a shadow, at least not one that is perceptible in daylight. For Aristarchus, the zero-tilt moment would have been an attractive criterion to base his measurement upon, since, in addition, the moon was already indistinguishably close to being half full.

A gnomon, a vertical stick in its simplest form, on a horizontal sundial would then have been very much suited. The moon's azimuth can be found with great precision by aligning the straight shadow edge over its surface with the gnomon. In the horizontal plane, the direction that is at right angles with this moon azimuth gives a point on the horizon, which we denote by Q (of "quadrant"). This point marks precisely the 90° difference from the moon's position in both azimuth and elongation. This observation may also be understood from Figure 4b, where this great circle is shown in a panoramic view. Half of it is above the horizon while the other half is below, as is the case for any great circle. Its maximum is halfway the "above horizon" part. When in this point, the moon is seen upright. It is illuminated by the sun (hypothesis 1) and the connecting path to the sun is locally horizontal in our 2D panoramic rendering.

Figure 5 shows a detail of Figure 4b, as seen in the direction of the sun.

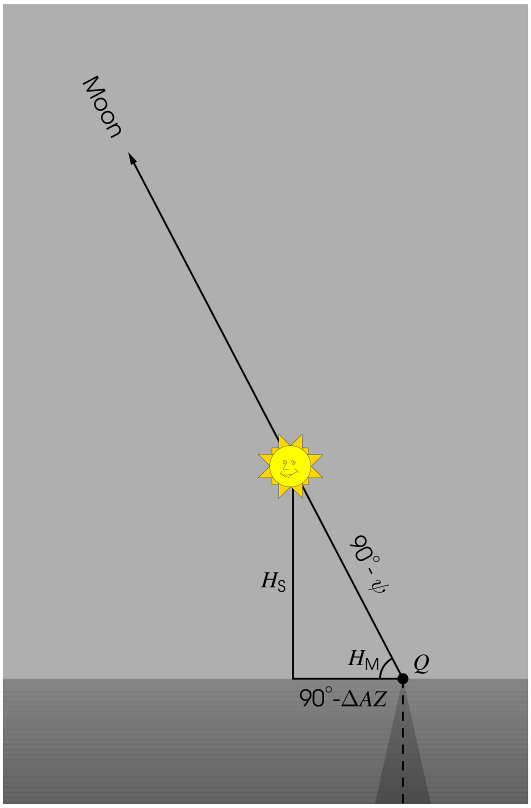

Figure 5. A close-up of Figure 4b in the direction at right angles with the azimuth of the moon. The “triangle of missing fractions”, with Q-Sun as its hypotenuse, is really a spherical triangle with the foot of the gnomon as its center, but because of it being very small, it is a flat triangle to good approximation. The sides and angles are related as in formula (4).

For the elongation and the sun–moon azimuth difference, their “missing parts” of filling a full quadrant are (90° – ψ) and (90° – ΔAZ), and these angles are related by:

$$\cos(H_M) = \cot(\Delta AZ)/\cot(\psi) \approx (90° - \Delta AZ)/(90° - \psi). \tag{4}$$

Equation (4) follows from Napier’s mnemonic rules [21] by taking $\beta = 90°$ in the spherical triangle PSM of Figure 3.

Not having recourse to spherical trigonometry, yet Aristarchus could have come to the same result by reasoning as follows: it is of course trivially true that 90° in azimuth is a quadrant of a full circle along the horizon. He would likely have found the missing fraction of the elongation by noting that from the upright moon to the point Q spans exactly one quarter of the moon–sun great circle. The sun’s azimuth difference from the moon falls short of the 90° point Q by 1.2° (see Table 1), and it would have been quite easy to measure this. For Aristarchus, the procedure would then have been to evaluate the corresponding missing fraction in elongation by producing a line from Q via the sun in the direction of the moon and measuring it against the azimuth deficit that he had found, or directly, by using a handheld quadrant. In doing so, he would, of course, have used the sun’s elevation as he saw it. Refraction had lifted the sun by an amount of 15.7′, about equal to its semi-diameter, and for Aristarchus the missing fraction would have been 2°50′,

Table 2. Elongation and phase at other places at the local time of upright moon, 16 June 255 BC

Place	Coordinates	Time (UT)	True elong.	App. elong.	H_{Sun}	Phase
Basrah	30°31′N, 47°46′E	15:27:08	86°42.1′	86°29.8′	2°56.4′	0.4726
Alexandria	31°12′N, 29°55′E	16:43:25	87°23.8′	87°09.9′	2°18.7′	0.4786
Syracuse	37°04′N, 15°11′E	18:01:01	88°06.3′	87°50.9′	1°33.9′	0.4848
Carthage	36°47′N, 10°12′E	18:21:01	88°17.3′	88°01.2′	1°25.1′	0.4864
Massilia	43°16′N, 05°24′E	19:02:26	88°39.9′	88°23.9′	1°00.0′	0.4897
Cadiz	36°29′N, 06°15′W	19:28:56	88°54.4′	88°36.1′	0°54.5′	0.4918
Ilha do Faial	38°34′N, 28°42′W	21:09:09	89°49.4′	89°27.4′	0°08.6′	0.4998

almost precisely "one thirtieth of a quadrant". The resulting apparent elongation of 87°10′ is the same as that obtained from a direct calculation, using Equation (2), see Table 1.

4. A *Gedankenexperiment*: the moon's elongation and longitude

One might ask if the result of 87° is an artefact of the method that we describe: finding the elongation at a time when the half-moon's terminator is seen vertical. Let us therefore imagine that a similar observation was not only conducted in Alexandria, but also in other important centra of the ancient world, Basrah in Mesopotamia, the Greek and Phoenician settlements in the Mediterranean, Syracuse, Carthage, Massilia and Cadiz. And, just for the sake of argument, let us include the most westerly island of the Azores, Ilha do Faial.

The results are given in Table 2. The third column gives the time when the moon's shadow edge is locally seen as vertical. Over a time of five and a half hours, the true elongation increases from 86°42.1′ for the Basrah observation to 89°49.4′ on the Azores. For each later observation, the height of the sun is lower, almost 3° for Basrah and only just above the horizon for the Ilha do Faial. At the same time, the phase gets closer and closer to 0.5000. According to Aristarchus' criterion for visual acuity, formulated above, all phases are close enough to 0.5000 to make the moon look like half.

This *Gedankenexperiment* illustrates that the elongation of 87°, observed by Aristarchus, is not an artefact of the method. Had he done the experiment in a more westerly location, he would have found a somewhat larger elongation.

5. Discussion and conclusion

As shown above, Aristarchus' fourth hypothesis that, at half-moon, its angular distance to the sun is less than a full quadrant by one thirtieth of it can be understood if we assume that the measurements were made at a moment when the shadow edge over the moon's surface was seen as vertical. This moment can be timed to within a few minutes. By contrast, the naked eye cannot pinpoint the precise first or last quarter phase of the moon within several hours.

Shadow-casting instruments available in Aristarchus' time, would not have been practical, and the visor-equipped armillary sphere had not yet been developed in his days. The additional requirement that the moon should be seen as upright would, however, make it feasible to do the observation using a gnomon. Indeed, it allows one to accurately find a point on the horizon, which we name Q, that marks a full quadrant's distance from the moon, both in azimuth and along the great circle that connects moon and sun. The problem is then simplified from finding the elongation itself to finding its missing fraction from 90°, and the gnomon would be an almost ideal instrument.

Table 3. Elongation and phase seen from Alexandria for upright moon on different dates

Date	Time (UT)	True elong.	App. elong.	H_{Sun}	Phase
20 January 255 BC	15:17:42	89°06.3′	88°45.8′	00°48.1′	0.4935
25 October 267 BC	15:16:44	88°23.8′	88°14.4′	00°50.5′	0.4873
16 June 255 BC	16:43:25	87°23.4′	87°11.1′	2°18.7′	0.4786
11 August 271 BC	16:32:52	86°22.7′	86°12.2′	2°35.8′	0.4697

As an example, we have analyzed the situation that Aristarchus may have observed himself in Alexandria on 16 June 255 BC. Above, we have shown how the observed elongation changes when measured on the same day, but from different longitudes.

A similar experiment could also have been performed at the same location, Alexandria, but on different dates. Table 3 gives some examples of the resulting elongation, which ranges from 89°6.3′ to 86°22.7′. This range is biased towards values smaller than 90° by the trivial requirement that the sun must be above the horizon to be visible. On the low side, it is restricted by the fact that for phases smaller than about 0.467, the moon is perceived as not yet half, as shown above in our discussion on visual acuity. The corresponding lower limit on the apparent elongation is around 86°. Aristarchus' value of 87° falls near the middle of this range.

From our analysis, one may conclude that Aristarchus' intention to perform measurements as accurately as possible should not be underestimated.

Declaration of interests

The authors do not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and have declared no affiliations other than their research organizations.

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Appendix A. The moon tilt

In Figure A1, the moon is in its last quarter and the observer sees it west from the sun. The sun illuminates the moon along the direction SM and the shadow line over the moon is perpendicular to it. The observer in P sees the moon along the direction PM. If we denote the angle between PM and SM as α, then it follows immediately that the inclination angle, which we will call T, is given by $(90° - \alpha)$. Calculating α requires some spherical trigonometry [21].

Use the following two relationships:

$$\sin(\alpha)/\sin(a) = \sin(\gamma)/\sin(c) \quad \text{(sine rule)} \tag{A.1}$$

$$\sin(c)\cos(\alpha) = \cos(a)\sin(b) - \sin(a)\cos(b)\cos(\gamma) \quad \text{(sine-cosine rule of the first kind).} \tag{A.2}$$

Eliminating $\sin(c)$ gives:

$$\cot(\alpha) = \cos(\alpha)/\sin(\alpha) = [\cos(a)\sin(b) - \sin(a)\cos(b)\cos(\gamma)]/[\sin(a)\sin(\gamma)]. \tag{A.3}$$

The observer measures or calculates the moon's and the sun's altitudes above the horizon, H_{M} and H_{S}, and also their azimuthal angles, AZ_{M} en AZ_{S}. The connection with the symbols, used above, is:

$$a = 90° - H_{\mathrm{S}}; \quad b = 90° - H_{\mathrm{M}}; \quad \gamma = \Delta AZ = AZ_{\mathrm{S}} - AZ_{\mathrm{M}}.$$

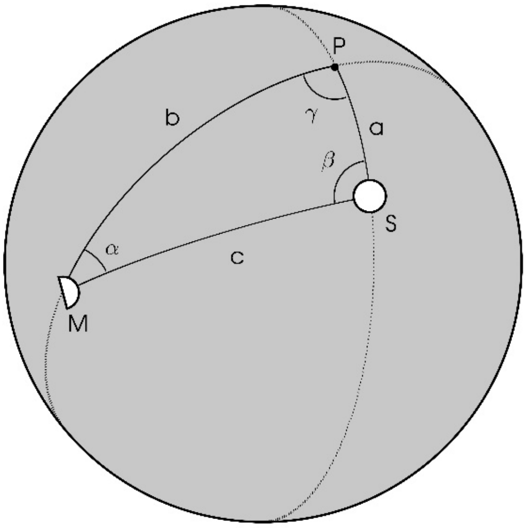

Figure A1. The zenith for the observer is in P. Sun and moon are shown as S and M, the foot points of their projections onto the celestial sphere.

The moon's tilt angle is $T = 90° - \alpha$ and it is found from:

$$\tan(T) = [\sin(H_S)\cos(H_M) - \cos(H_S)\sin(H_M)\cos(\Delta AZ)]/[\cos(H_S)\sin(\Delta AZ)]. \tag{A.4}$$

This is the formula as we have given it in [23,24] without proof and with proof in [25].

Its physical interpretation is that the nominator and the denominator are the sideways and the normal projections, respectively, of the vector product $\hat{\boldsymbol{r}}_S \times \hat{\boldsymbol{r}}_M$, the cross product of the unit vectors in the directions to the sun and the moon, as they are seen in a cartesian frame with the y-axis pointing directly to the moon, the x-axis in the observer's horizontal plane and the z-axis completing a right-handed frame.

The observer may instead wish to define his coordinate frame as a horizontal x–y plane and his own zenith direction as the z-axis. This involves a rotation about the x-axis over an angle H_M which leaves the sideways x-component unchanged, but multiplies the normal component by $\cos(H_M)$. The result is then:

$$\tan(T') = [\sin(H_S)\cos(H_M) - \cos(H_S)\sin(H_M)\cos(\Delta AZ)]/[\cos(H_M)\cos(H_S)\sin(\Delta AZ)]. \tag{A.5}$$

In this form, the equation has recently also been derived by Myers and Myers [26].

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