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Review article

Astrophysical invariance of the fine-structure constant: observation and theory

Patrick de Laverny[*, a] **and Gilbert Reinisch**[a]

[a] Université Côte d'Azur, Observatoire de la Côte d'Azur, CNRS, Lab. Lagrange, Nice, France

E-mails: laverny@oca.eu, reinisch@oca.eu

Abstract. A theoretical result based on nonlinear quantum electrodynamics could suggest the invariance of the fine-structure constant everywhere in the Universe and at any epochs of its history. The present paper updates while summarizing this theoretical framework, and provides a critical survey of recent astrophysical observations related to this constant's value. We conclude that, from an observational point of view, this constant could indeed be invariant over cosmic space and epochs, in line with the theoretical study.

Keywords. Fine structure constant, quantum electrodynamics, astrophysics.

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1. Introduction

Ten years ago, Ref. [1] proposed a dual semi-classical description of a stationary nonlinear quantum system, using both its lowest-order Hartree–Schrödinger nonlinear definition and, consequently, its first-order quantum electrodynamics (QED) equivalent. This theory defined the numerical value of the fine-structure constant $\alpha = e^2/\hbar c = 7.297\ldots 10^{-3}$ with an error of about $5\,10^{-4}$, mainly due to the tricky numerical resolution of its corresponding implicit analytical integro-differential system: see Figure 1 below. Ref. [1] concluded that "the fine-structure constant... would not be expected to vary on cosmological timescales". Ten years later, it is remarkable that this prediction has been verified repeatedly by an increasing number of elegant and precise astrophysical observations.

In [2] for example, the authors were the first to search for a possible variation of a constant of Nature around a black hole and in a high gravitational potential. They made measurements of the difference between distinct absorption lines with different sensitivity to α from late-type evolved giant stars belonging to the short-period star cluster orbiting the supermassive black hole in our Galactic Center. Using near-infrared spectroscopic measurements of five stars which are among the closest late-type stars to the black hole, they obtained a constraint on the relative variation of α below 10^{-5}. Therefore "constants [are] still constant near black holes" [3].

Although the calculations in [1] were already published, their summary and update are relevant and necessary for understanding the present article whose main goal is to confront their predictions to current observational measurements of α.

* Corresponding author

2. Overview of quantum nonlinearity

"We cannot solve a problem with the same thinking that created it." Einstein's statement should appear as the cornerstone of [1]. Indeed, two physical enigmas, namely:

— the astrophysical invariance of the fine-structure constant [3];
— the infrared (IR) divergence of long-wavelength "soft" photons in quantum electrodynamics (QED) [4];

can both be solved by quantum nonlinearity, which, moreover, allows the former to be explained by the solution of the latter. Nonlinearity is actually a complementary new thinking in quantum mechanics. It adds a first essential property, namely non-orthogonality (or overlap) of its eigenstates. The latter in turn leads to a second essential consequence due to Born's postulate: a conservative nonlinear quantum system has a non-zero probability of being simultaneously in several eigenstates. Such a property is of course impossible for a conservative linear quantum system where the eigenstates are orthogonal and therefore where the Hilbert projection of one eigenstate onto the next — which measures the probability amplitude of the first overlapping the second — is zero. In contrast, a nonlinear and conservative quantum system yields eigenstate overlap and therefore intrinsic transitions between its non-orthogonal eigenstates. These transitions can be described in terms of emission/absorption of QED photons provided — and this is where "one of the greatest damn mysteries of physics" arises [5] — that the probability of such photon exchanges between, say, interacting electrons be equal to the numerical value of the fine-structure constant $\alpha = e^2/\hbar c \sim 1/137$. This assumption must be postulated a priori in order to allow all classical Coulomb properties to be recovered by QED's lowest-order long-wavelength semi-classical limit [5,6]. Hence the fundamental (astro)physical interest in the investigation of the origin and the properties of the numerical value of α.

2.1. *Cancellation of QED's IR divergence*

Let us consider IR divergence in 3D Hartree's simplest semiclassical nonlinear quantum system constituted by two parabolically-confined Coulomb-interacting electrons ("quantum-dot Helium" [7–9]). We wish to analytically demonstrate that quantum nonlinearity cancels QED's IR divergence out in this system. It suffices to limit ourselves to radial isotropic s states (no angular momentum). The nonlinearity is ab-initio provided by the source term of Poisson equation in Hartree's semiclassical Schrödinger–Poisson (SP) quantum description. Remember that IR divergence occurs in QED's lowest-order semiclassical limit of long-wavelength photons [4,10]: hence the corresponding choice of the semiclassical SP system to study it.

In the 3D radial parabolic potential defined by its angular frequency Ω, the discrete real-valued steady-state electron-electron scattering solutions $\psi_{a,b}(X)$ corresponding to the two lowest-energy $S=0$ opposite-spin eigenstates ("s" states) — namely, ground state $|a)$ and 1rst excited state $|b)$ (we use parentheses instead of brackets in order to emphasize eigenstate nonlinearity) — are defined by Hartree's following semiclassical SP differential system in appropriate dimensionless units [1]:

$$\left[\nabla^2 + C_{a,b}(X) - \frac{1}{4}X^2\right]\psi_{a,b}(X) = 0, \tag{1}$$

$$\nabla^2 C_{a,b}(X) = \psi^2_{a,b}(X), \qquad C_{a,b}(X) = \mu_{a,b} - e\Phi_{a,b}(X). \tag{2}$$

Potentials $C_{a,b}(X)$ are defined by the eigenvalues (or chemical potentials) $\mu_{a,b}$ of the nonlinear eigenstates $\psi_{a,b}$ while $\Phi_{a,b}(X)$ are the related Coulomb potentials (in units of $\hbar\Omega$) between the two interacting electrons. The initial conditions are $\psi_{a,b}(0) = u_{a,b_0}$, $[\mathrm{d}\psi_{a,b}/\mathrm{d}X]_{X=0} = 0$,

$C_{a,b}(0) = C_{a,b0}$, $[\mathrm{d}C_{a,b}/\mathrm{d}X]_{X=0} = 0$, while the eigenstates' regular boundary condition are $\lim_{x\to\infty}\psi_{a,b}(X) = 0$. Both eigenstates are normalized in our reduced units according to [1]:

$$\int_0^\infty \psi_{a,b}^2(X)X^2\,\mathrm{d}X = \mathbf{N} = \frac{e^2/L}{\hbar\Omega} \propto \frac{1}{\sqrt{\Omega}}, \tag{3}$$

where the characteristic length $L = \sqrt{\hbar/2m\Omega}$ is the "harmonic length" of the oscillator. It defines the reduced radial coordinate $X = r/L$. Eigenstate non-orthogonality — or overlap — which is the hallmark of nonlinearity in this system is defined by the inner product in agreement with normalization (3). It reads:

$$(a\,|\,b) = \frac{1}{\mathbf{N}}\int_0^\infty \psi_a\psi_b X^2\,\mathrm{d}X = \frac{W_{ab}}{\mu_b - \mu_a}. \tag{4}$$

Indeed, it is equivalently defined by the matrix elements $W_{ab} = (a|W|b) = (b|W|a) = \mathbf{N}^{-1}\int_0^\infty \psi_a W \psi_b X^2\,\mathrm{d}X$ of Coulomb potential difference $W(X) = \Phi_b(X) - \Phi_a(X)$ [1]. Boundary conditions $X \to \infty$ in (2) respectively yield eigenvalues $\mu_{a,b} = \lim_{X\to\infty}\big[C_{a,b}(X) + \mathbf{N}/X\big] = \lim_{X\to\infty} C_{a,b}(X)$.

Poisson equation defining potential difference $W(X)$ yields the corresponding charge density of the latter:

$$\rho(X) = \frac{e}{4\pi\mathbf{N}L^3}\big[\psi_b^2(X) - \psi_a^2(X)\big]. \tag{5}$$

Now consider (in gaussian units) the following semiclassical QED potential operator [6]:

$$H_W = \int \mathbf{j}\mathbf{A}\,\mathrm{d}^3x = \left(\frac{2\pi}{L}\right)^{3/2}\sqrt{2\pi\hbar c}\int \big[\mathbf{a}_{k'}\rho_{k'}^* + \mathbf{a}_{k'}^+\rho_{k'}\big]\frac{\mathrm{d}^3k'}{\sqrt{k'}}, \tag{6}$$

corresponding to charge density (5) and thence to potential difference $W(X)$. In (6), $\mathbf{j}$ is the 4-vector particle current density and $\mathbf{A}$ the related potential operator; $\mathbf{a}_{k'}^+$ and $\mathbf{a}_{k'}$ are respectively the creation and the annihilation operators of photons with wave vector k' and frequency $\omega_{k'} = ck'$ [6]. We use the stationary configuration where $\mathbf{j}$ reduces to its 4th charge density component (5) while $\rho_{k'}$ is the 3D radial Fourier component of ρ. Probability amplitude A_k to create a soft long-wavelength photon $|k\rangle$ of wavenumber k out of photon vacuum $|0\rangle$ is [1]:

$$A_k = \frac{\langle k|H_w|0\rangle}{-\hbar ck} = -\left(\frac{2\pi}{L}\right)^{3/2}\sqrt{2\pi\alpha}\,\frac{n_k}{k^{3/2}}, \tag{7}$$

where $n = \rho/e$ is the particle density defined by (5) and we have used the quantum properties of operators $\mathbf{a}^+$ and $\mathbf{a}$. Its 3D Fourier component (as a function of the reduced wave number $\kappa = Lk$ where $\kappa X = kr$) is $n_\kappa = g(\kappa)/(\mathbf{N}\sqrt{8\pi^3})$ with:

$$g(\kappa) = \int_0^\infty \big[\psi_b^2(X) - \psi_a^2(X)\big]\frac{\sin\kappa X}{\kappa X}X^2\,\mathrm{d}X. \tag{8}$$

Note the crucial property $g(0) = 0$ due to (3).

We obtain the photon number per eigenstate:

$$N_{\mathrm{qed}} = \sum_k |A_k|^2 = \left(\frac{L}{2\pi}\right)^3\int_0^\infty |A_k|^2 4\pi k^2\,\mathrm{d}k = \frac{\alpha}{\pi\mathbf{N}^2}\int_0^\infty g^2(\kappa)\frac{\mathrm{d}\kappa}{\kappa}, \tag{9}$$

which has no IR singularity in the long-wavelength limit $\kappa \to 0$ since $g(0) = 0$ as already emphasized. This simple example analytically demonstrates the cancellation of IR divergence of the photon number N_{qed} in semiclassical QED, due to the disappearance of its logarithmic divergence at $k \to 0$ as a result of nonlinear eigenstate overlap defined by (5) and (8). Since these eigenstates can exhibit mixed-state statistical properties [11,12], we anticipate a possible generalization of this theory to the quantum physics of mixed states, obtained by statistical combination of eigenstates equipped with appropriate normalized probabilities.

2.2. *SP vs QED mathematical definition of the fine-structure constant*

In accordance with (3), we update [1] using renormalization $\widetilde{A}_k = \mathbf{N} A_k$ of probability amplitude A_k by taking into account all plasmons $\hbar\Omega$ included in the characteristic Coulomb energy e^2/L of the system. It yields by use of (9) the following renormalized photon number of the corresponding Coulomb interaction:

$$\widetilde{N}_{\rm qed} = \sum_k |\widetilde{A}_k|^2 = \left(\frac{L}{2\pi}\right)^3 \int_0^\infty |\widetilde{A}_k|^2 4\pi k^2\, \mathrm{d}k = \frac{\alpha}{\pi}\int_0^\infty g^2(\kappa)\frac{\mathrm{d}\kappa}{\kappa}, \tag{10}$$

which is of course also regular with respect to $\kappa \to 0$ (no IR divergence since $g(0) = 0$). This number of photons $\widetilde{N}_{\rm qed}$ leads to the analytical definition — albeit implicit — of α [1]:

$$\alpha(\mathbf{N}_*) = \frac{\pi (a\,|\,b)^2_{\mathbf{N}_*}}{\int_0^\infty \kappa^{-1} g^2_{\mathbf{N}_*}(\kappa)\,\mathrm{d}\kappa}, \tag{11}$$

where $\mathbf{N}_*$ is the solution of the following energy conservation (in units of $\hbar\Omega$):

$$E_{\rm qed}(\mathbf{N}_*) = \Delta_{ab}(\mathbf{N}_*), \tag{12}$$

between QED's photon energy:

$$E_{\rm qed}(\mathbf{N}) = \sum_k |\widetilde{A}_k|^2 \hbar\omega_k = \frac{\mathbf{N}}{\pi}\int_0^\infty g^2_{\mathbf{N}}(\kappa)\,\mathrm{d}\kappa, \tag{13}$$

and SP's energy gap $\Delta_{ab}(\mathbf{N}) = 2 + \mu_b(\mathbf{N}) - \mu_a(\mathbf{N})$. The numerical simulation of the solution (11)–(13) is displayed by the $\alpha_{\rm try}$ blue stars in Figure 1. They are obtained by more and more accurate numerical definition (from right to left) of the two nonlinear eigenstates $\psi_{a,b}$ of SP differential system (1)–(3). While their respective norms should ideally be equal in agreement with (3), their numerical simulation yields an inevitable relative norm difference $\Delta(\mathbf{N})/\mathbf{N} \ll 1$. The asymptotic decrease in the difference, as numerical precision increases, between the blue stars $\alpha_{\rm try}$ and the experimental value $\alpha = 7.29735\ldots 10^{-3}$ represented by the red cross-circles in Figure 1 demonstrates the remarkable convergence of the numerical solution $\alpha_{\rm try}$ defined by (11)–(13) to $\alpha = 7.29735\ldots 10^{-3}$ and, consequently, the relevance of the current mathematical definition of the fine-structure constant within the nonlinear theoretical quantum framework SP vs QED.

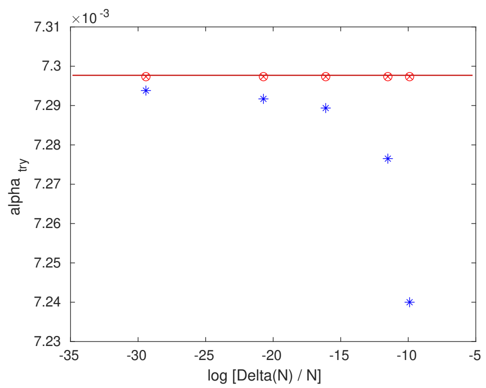

Figure 1. The blue stars display the $\alpha_{\rm try}$ values of α obtained numerically from (11)–(13) with increasing precision from right to left. The red cross-circles corresponding to the exact experimental value $\alpha = 0.00729735\ldots$ allow comparison of $\alpha_{\rm try}$ with the latter. The numerical convergence of $\alpha_{\rm try}$ to α confirms the theory. The precision parameter $\Delta(\mathbf{N})/\mathbf{N}$ defines the norm difference $\Delta(\mathbf{N})$ between the two numerical nonlinear eigenstates of the SP quantum differential system which should be ideally zero: cf. (3).

Remark.

— Unlike the number of photons (10), photon energy (13) is independent of α [13]; that is, it does not depend on the speed of light $c = e^2/\hbar\alpha$, as expected since QED's lowest-order semiclassical limit should restore the classical non-relativistic Coulomb electrostatic field.

— Without the cancellation of the infrared logarithmic divergence of the number of photons $\widetilde{N}_{\text{qed}}$ in QED, described in Section 2.1, the solution (11)–(13) of α would not exist because it comes from the probabilistic equality, at $N = N_*$, between this number of photons and the square of eigenstate overlap (4) [1].

A crucial point concerns the fundamental difference between current theory and numerology. The latter, widely present in the literature, starts from the known experimental value $\alpha = 7.297\ldots 10^{-3}$ and reconstructs it using arbitrary numerical estimates without any physical explanation. These estimates can be very precise (in fact, much more precise than the current error of 0.05% illustrated by Figure 1), but they all fail to answer this fundamental question: why choose such a numerical definition? No physical explanation is provided. Conversely, the current theoretical definition of α is, to the authors' best knowledge at least, the first to mathematically define this number without knowing its value a priori, solely through the exclusive use of nonlinearity combined with the fundamental principles of quantum physics. This could suggest that the numerical value of the fine-structure constant is an elementary transcendental mathematical number, and therefore invariant across cosmic space and timescales as shown below.

3. Search for variations of the fine-structure constant by using atomic clocks

Configuration interaction in many-electron atoms may cause anomalies in their fine structure which make the frequency intervals small and very sensitive to variation of the fine-structure constant. Repeated precision laboratory measurements of these intervals over long periods of time can put strong constraint on possible time variation of α. Specifically, it has been proposed to measure the low frequency between two close states with different dependence on α because small change of α may lead to orders of magnitude larger relative change in frequency [14]. Therefore atomic clocks in the search for variations of α have driven the development of novel atomic clock technologies. In laboratory experiments, the α variation is constrained by measuring the relative change in the frequency ratio of two atomic clocks. Comparisons of atomic clocks based on transitions show indeed a different dependence of their frequency on the value of α. Consequently, a potential variation of α would become observable as a change in the frequency ratio of the clocks. Using the annual variation of the Sun's gravitational potential at Earth due to the ellipticity of the Earth orbit, these atomic clock laboratory measurements infer a potential temporal variation $\Delta\alpha/\alpha \sim 10^{-18}$/yr compatible with zero [15]. While these studies on the α variation in atomic clocks have typically focused purely either on a nuclear or electronic transition, [16] proposed to use the hyperfine electronic bridge (HEB) transitions to improve the detection of α variation. HEB transitions involve simultaneous changes in both nuclear and electronic structures. Then, the sensitivity factors are significantly enhanced by factors of 2.7 to 3.5, making the HEB approach promising for improving the precision of α variation detection.

4. Astronomical measurements of the fine structure constant

Following Section 2, the fine structure constant could remain invariant with time and in any astrophysical environment. Over the years, several local (i.e. present time) or early universe experiments have been proposed to examine the hypothetical α non-constancy. They can be classified into geological, atomic clocks, astrophysical or cosmological categories (see, for a review, [17,18]). We remind that Earth-based (and thus local) experiments that compare atomic clocks based on different transition frequencies over a few years have derived a very accurate and strong constrain for a time variation of α that should be smaller than $\sim 10^{-18}\,\mathrm{yr}^{-1}$ [15]. We review below some of the recent astronomical observations that have looked for a possible varying fine structure constant elsewhere in the Universe.

Among the astronomical observations performed so far,[1] one of the most adopted method relies on the analysis of spectra of extremely distant astrophysical sources (quasars or Quasi-Stellar Objects, QSOs) partially absorbed by intervening intergalactic gas clouds. This allows to span very long timescales and distant cosmic regions. Other methods analysing absorption or emission-line spectra (see below for some examples) rely on a similar strategy: since atomic line transition energies vary with α, a possible variation of this constant would slightly shift the line position, hence leading to a velocity shift ΔV. This shift is related to the variation of the fine structure constant by: $\Delta\alpha/\alpha \approx K \times \Delta V/c$ where K is a constant specific to the studied transition and c the light speed. High-resolution and high-SNR spectra are therefore mandatory to measure the tiny expected ΔV. By choosing adequate targets and spectral lines, one can thus expect to unveil a hypothetical change in the α 's numerical value at different epochs (and position, if one looks for any spatial variations). We point out that these procedures actually require to address severe and complex calibration problems together with systematics effects.

A few groups managed to deeply analyse several QSO or damped Ly-α absorption system spectra over the last two decades. Conflicting results were first obtained but the situation appears now clearer partly because the spectra quality has increased and the analysis methodologies were refined over time. For instance, an ambitious program using the VLT/UVES, Keck/HIRES and Subaru/HDS spectrographs (i.e. the largest telescopes available) has been devoted to fine structure constant measurements, developing very accurate calibration technics. The main conclusion of this study is that the measurements of $\Delta\alpha/\alpha$ are all consistent with a null value at a 2σ-level [20]. Interestingly, this result has also been independently confirmed from new Keck/HIRES spectra by [21]. More recently, [22] analysed high-quality VLT/UVES + ESO3.6m/HARPS spectra of a single QSO with an absorption system located at $z \sim 1.1$, focussing deeply on different sources of systematics. Their measurement is consistent with no α variations at a 10^{-6} precision level, comparable with the precision reached by other studies devoted to the analysis of hundreds of QSO spectra. Similarly, [23] analysed new HARPS spectra of the same QSO system, carefully focussing on their wavelength calibration. They reached again similar conclusions and precision. All of this was then confirmed by [24] who focussed on the same target but based on VLT/ESPRESSO spectra. Combining measurements from previous studies, these authors even reached a limit value for $\Delta\alpha/\alpha$ slightly lower than 10^{-6}. Quite similar results and accuracies were also obtained from mm-band and radio spectra [25,26].

In contrast to the absorption line spectra analysed by the above-mentioned studies, [27] explored emission-line (mostly the [O III] doublet) spectra of 46 QSO and 40 Lyα galaxies covering a redshift range from $z \sim 1$ to ~ 3.7. These authors again report no α variation over the explored time interval at a 10^{-5} level. Similarly, [28] analysed more than 10^5 galaxy spectra collected thanks to the Dark Energy Spectroscopic Instrument and covering half of all cosmic time ($z < 1$) and did not find any obvious, temporal variation or large-scale structures in the spatial distribution of $\Delta\alpha/\alpha$ with a $\sim 10^{-4}$ uncertainty. Finally, the same group reported very recently James Webb Space Telescope observations of 578 even more distant (z up to 9.5) emission-lines galaxies [29]. Their analysis is consistent with no temporal or spatial variations of the fine structure constant above a similar level of $\sim 10^{-4}$.

[1] For extensive reviews on the different methods developed to derive α values from observed astronomical spectra, see [17–19].

Moreover, new astrophysical experiments have been conducted by considering other target types and they all do agree with the above prediction of an invariant α. First, [2] have considered evolved giant stars orbiting the supermassive black hole at the Galactic Center. They report a $\Delta\alpha/\alpha$ lower than 10^{-5} in this extreme gravitational potential. Attempts to measure the fine structure constant close to a white-dwarf surface (i.e. again in a strong gravitational potential although less than close to a black hole) have also been tried but no convincing results are reported because of complex sources of uncertainties (as continuum placement, among others) have to be addressed [30].

Then, [31] analysed high S/N and high-resolution HARPS spectra of 17 Solar twins (hence stars with similar spectra as the Sun) and report that α would vary by less than ~50 parts-per-billion within 50 parsecs from Earth. Moreover, they convincingly show that their adopted methodology would have detected any variation at a 10^{-7} level.

Finally, one could also cite the analysis of cosmic microwave background measurements that led to a non-variation of α constrained at the level of 10^{-3} [32].

In summary, the astrophysical quest for a hypothetical varying α over cosmic scales and epochs has failed so far. The reported relative variation of the fine structure constant is found to be below 10^{-5} or 10^{-6}, depending on the experiments. It is also likely that some of these works were able to detect a variation at a 10^{-7} level. It is important to note that most of these works are totally independent between each other and suffer from very different systematic errors, but they all reach the same conclusion: α could indeed behave as a real constant.

Unfortunately, this observational situation will not be rapidly improved. For instance, one will have to wait for future very high-resolution and very stable spectrographs such as those in construction for the European-Extremely Large Telescope (see, for instance, [33]). These new instruments could allow to perform during the next decade even more accurate velocity calibrations over several nights and better solve all the possible systematics in the line shift determinations. This will be essential to further look for any hypothetical cosmic space and/or time variations of α.

5. Conclusion

The astrophysical quest for detecting a possible variation of the fine structure constant value over cosmic space and time scales has failed so far. All the reported observations are indeed totally compatible with an invariant α. This is fully in agreement with the SP-vs-QED numerical results presented in Section 2 and Figure 1, and with an indication that the fine structure constant would be an elementary transcendent mathematical number. As a consequence, α should remain invariant in any astrophysical environment, at any epochs of the Universe history. Moreover, e, $\hbar$ and c would not have independent values.

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The authors do not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and have declared no affiliations other than their research organizations.

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