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The trans-Planckian problem and gravitational interactions

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The trans-Planckian problem and gravitational interactions

Le problème trans-planckien et interactions gravitationnelles

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Abstract. The strong gravitational interaction of counter-propagating quantum field modes is of foundational importance to the trans-Planckian problem of black hole horizons. This article, in memory of Renaud Parentani, is primarily an exposition of Renaud's attempt, using perturbation theory, a large N approximation, and spherical reduction, to capture a mechanism by which those interactions might quench the trans-Planckian near-horizon quantum field correlations. Before giving a detailed account of Renaud's calculation, the trans-Planckian problem is briefly introduced, and the paper concludes with a discussion of related issues and questions.

Résumé. La forte interaction gravitationnelle entre des modes de champ quantique se propageant en sens opposés revêt une importance fondamentale pour le problème trans-planckien lié aux horizons des trous noirs. Cet article, écrit en mémoire de Renaud Parentani, expose principalement la tentative de ce dernier — fondée sur la théorie des perturbations, une approximation de type « grand N » et une réduction à symétrie sphérique — de mettre en évidence un mécanisme par lequel ces interactions pourraient inhiber les corrélations du champ quantique trans-planckien au voisinage de l'horizon. Avant de présenter en détail le calcul de Renaud, le problème trans-planckien est brièvement exposé, et l'article se conclut par une discussion sur des questions connexes.

Keywords. Black hole, trans-Planckian, gravitation, quantum gravity.

Mots-clés. Trou noir, trans-planckien, gravitation, gravité quantique.

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1. Introduction

Renaud Parentani was one of the most insightful and creative physicists I have known. His methodology was to take a question — whether it be foundational or phenomenological — and, with the help of simplifications, approximations, and astute technical devices, craft it into a formulation that he then studied with precise analysis from which he drew sharp results and lessons. I treasured his instruction, collaboration and friendship, both as a physics colleague and on a personal level. Time spent with him was always illuminating, whether on the subject of physics or any of his many other passions. His passing was a terrible loss for me personally and for science.

Renaud and I first met in 1995, thanks to a shared interest in the trans-Planckian problem of black hole horizons. Although we wrote only one paper together on that topic [1], it was a major theme underlying both of our lines of research, and the subject of countless discussions. For this contribution in his memory, I focus on his efforts to go beyond the semiclassical approximation in order to understand the role of quantum gravitational interactions in “taming the trans-Planckian monster” [2]. This involves an area of his research that remained quite tentative and incomplete, so is rather unlike most of his work. It is also, I suspect, relatively unknown, perhaps in part because his papers on the subject are not so easy to follow. I wanted to understand it better myself, and to spread the word about it, with the hope that it will bear fruit.

2. The trans-Planckian problem

Traced backward in time toward the horizon, an outgoing field mode blueshifts exponentially, as measured by freely falling observers. Near the horizon this mode frequency increases as $e^{\kappa t}$, where κ is the surface gravity and t is the backwards-elapsing Killing time. For a solar mass black hole $\kappa \sim 10^5$ inverse seconds, so over a Killing time of one second the frequency increases by $\sim e^{10^5}$, quickly surpassing exponentially far beyond the Planck frequency. These modes are populated at the Hawking temperature $\hbar\kappa/2\pi$ in the free-fall vacuum state, but they also play a classical role; for example an outgoing mode of the electromagnetic field could be excited by an oscillating electric dipole outside the black hole.

To account for the physics of outgoing modes, relativistic field theory on a fixed black hole spacetime background thus relies on an exponentially trans-Planckian (as measured in any given free fall frame) reservoir of modes at the horizon.¹ Since the theory possesses local Lorentz invariance there is *prima facie* nothing wrong with this. However, it has the dubious effect of forcing us to assume local Lorentz invariance of the theory to unbounded boosts, far beyond any that could ever be measured experimentally. This doesn’t pass the smell test. Moreover, that trans-Planckian reservoir would carry an infinite entanglement entropy, contradicting the finiteness of the Bekenstein–Hawking black hole entropy $A/4G\hbar$. One expects that quantum gravity somehow eliminates the apparent role of trans-Planckian degrees of freedom, and the question is how does that work?

Another facet of the question is presented by exponential (or even sub-exponential) redshifts in cosmology, which require either a trans-Planckian reservoir of modes in the early universe, or some sort of mode creation, to account for the presence of the highest observed frequency modes today, even if inflation never took place. Mode creation, that is, creation of new degrees of freedom as the universe expands, does not seem to me implausible. It might resolve the cosmological trans-Planckian problem, but it is presumably not relevant for the black hole one, since a (stationary) black hole exterior spacetime has time translation symmetry. For a review of both of these puzzles and related work, as of the turn of the century, see [4].

¹The earliest use of the term “trans-Planckian” in this context that I am aware of was by ’t Hooft [3].

Renaud gave much thought to the trans-Planckian problem. Most of his work in this domain involved exploration of Hawking radiation and the related inflationary particle creation, using field theory with modified dispersion and/or dissipation. This original motivation also led to his extensive investigation of the potential impact of UV physics on cosmological observables, as well as of classical and quantum properties of physical realizations of analog black holes. A brief account of how he viewed these issues and results, as of 2002, can be found in [5]. He spent the next eighteen years further developing these ideas.

On the fundamental side, Renaud pursued the idea that quantum gravitational interactions and metric fluctuations must “tame” the theory, i.e., eliminate the apparent role of trans-Planckian field modes in Hawking radiation and render finite the entanglement entropy. Although this is the least complete aspect of his work, I believe it is the most significant with respect to addressing the fundamental puzzles of horizon physics.

3. Beyond the semiclassical description

The conclusion section of [2] offers a colorful description of why and how quantum gravitational interactions might be expected to intervene and resolve the trans-Planckian problem. Together with Barrabes and Frolov, Renaud explored in [6] the impact of a prescribed ensemble of black hole metric fluctuations on the trans-Planckian problem, and in several papers [7–11] he attempted to implement this idea in quantum gravity. These papers are all closely related. They form a sequence of refinements of the initial analysis in [7]. I will base my review on the final one [11], which exists only as an arXiv submission. The title is the same as that of [10], but carries an asterisk that reads: “An earlier version of this work was published in Proceedings [21] in 2002. We postponed submitting it to the arXiv in the hope of improving the evaluation of radiative corrections. We have recently modified the text, corrected some mistakes, and added remarks on this difficult point which still needs work — that hopefully someone will take over.”

3.1. *The role of gravitational interactions*

A near-horizon outgoing mode, which is the exponentially blue-shifted ancestor of a Hawking quantum, has strong gravitational interactions with ingoing low frequency modes once the invariant mass of the outgoing-ingoing pair reaches the Planck mass. The idea Renaud pursues is that this interaction — with both modes in their ground states — obliterates the free-field description of the ancestor.² That is, traced backwards in time, an outgoing mode dissipates into the strongly interacting quantum gravitational vacuum, eliminating the role of trans-Planckian frequencies in the origin of the outgoing modes and the Hawking quanta they carry. To exhibit this phenomenon Renaud aims to compute the two-point correlation function, with one point far from the horizon and the other on the past light cone of that point, close to the horizon. Within an approximation and truncation scheme to be described below, he finds that the free-field light cone singularity is replaced by a finite correlation that decays away exponentially as the point near the horizon recedes to the past.³

²The earliest discussion of the essential importance of gravitational interactions between ingoing and outgoing black hole modes that I am aware of was by ‘t Hooft [12,13], who considered the effects of an ingoing “particle”, i.e. excitation, on the outgoing emitted radiation.

³In de Sitter spacetime an eternal inertial observer is surrounded by a spherical horizon which behaves in many respects like a black hole horizon [14], the key differences being that there is no asymptotically flat region and that future null infinity is spacelike rather than null. But a de Sitter horizon presents a trans-Planckian problem as well, and it would be interesting to adapt Renaud’s model to that case.

The reason for restricting attention to *low frequency* ingoing modes, i.e., modes with wavelengths that are not much shorter than the Schwarzschild radius, is that only for them could the presence of the black hole plausibly make a difference. That is, the effect Renaud is trying to capture must arise from the *difference* between the flat space interactions and those that occur in the neighborhood of the black hole horizon. In flat space, Planckian collisions with vacuum modes must be occurring everywhere and all the time. Evidently, that does not prevent, for example, an electromagnetic wave from propagating freely through flat spacetime, since such a wave corresponds to an excitation *above* the interacting quantum gravitational vacuum.

The physics is somewhat analogous to that of the Lamb shift: as the electric charge coupling is turned on, a free electron in the Minkowski vacuum develops an infinite self-energy due to “radiative corrections”. But a physical one-electron state is an excitation above the interacting QED vacuum, and its mass is a parameter we determine by observation, so we do not observe the divergent effect of the radiative corrections. In a hydrogen atom, however, the electron is subject to slightly modified radiative corrections, which depend on the electron orbital. It is the *difference* between these and those for a free electron that brings about the Lamb shift, raising the energy of the $2s_{1/2}$ state above that of the $2p_{1/2}$ state by about one part in 10^{11} of the electron mass.⁴ Hans Bethe famously estimated the Lamb shift using a nonrelativistic calculation, with an ad hoc high energy cutoff at the electron mass. Since the result depends only logarithmically on the cutoff, he was able to obtain a result that was quite close to the observed value. Shortly after, fully consistent renormalization of relativistic QED was understood, and with that no ad hoc cutoff is necessary.

Similarly, Renaud finds it necessary to introduce by hand a high energy cutoff; and, like the Lamb shift, the result depends only logarithmically on the cutoff, so he feels justified in drawing some tentative conclusions. But how might one do better? Again, the analogy with the Lamb shift may be instructive. For one thing, some kind of subtraction of the flat space gravitational vacuum interactions should be done, because one is perturbing around that vacuum. For another thing, to master renormalization of QED it was necessary to develop a formalism in which Lorentz and gauge symmetry are manifest. It is plausible that, similarly, to master the physics of the outgoing black hole modes, it will be essential to preserve diffeomorphism symmetry. Not only is that a far more complicated symmetry to handle, but one must also allow for different background metrics, unlike for the Lamb shift in which all physics takes place in Minkowski spacetime. The challenge to improve on Renaud’s calculation is daunting, but the potential reward is great: an account of the quenching of near-horizon correlations could serve as the “Lamb shift” of quantum gravity,⁵ and bring us closer to understanding that theory.

It should be emphasized at the outset that simple perturbation theory in the gravitational constant G will never erase a light cone singularity in a correlation function, so clearly something more is needed to make headway. Gravitational interaction becomes arbitrarily strong in the UV, so it is plausible that nonperturbative effects do produce erasure. However, to detect that, one would presumably need to calculate using a UV completion of general relativity. It would be wonderful if AdS/CFT duality could be deployed to this end, but that is challenging because the near horizon physics is difficult to capture using available holographic technology, but perhaps future developments will make this possible. (See the discussion section for further comments on this.)

In any case, Renaud took a different route. He introduced a large number N of identical scalar fields, which enables effects that are nonperturbative in G to be calculated in a controlled way, at

⁴The difference is small since it depends on a high power of the small electromagnetic coupling constant. In the gravitational case, the coupling grows with energy, hence the effect of a background field might be large, indeed large enough to quench otherwise infinite correlations between fields near and far from a horizon.

⁵Albeit unfortunately lacking a precise measurement to compare to!

next to leading order in $1/N$. In effect, the double series in G and N is organized into a sum of series with the product GN held fixed, the m th series being one in powers of $G^m N$. The $m = 1$ series yields semiclassical gravity, in which the classical metric satisfies the semiclassical Einstein equation, while the $m = 2$ series includes, at the next order in $1/N$, effects of fluctuations of the energy-momentum tensor of the matter, characterized by its two-point correlation function. A toy model explaining how this works is discussed in [15, Section 9.4].

I next review some of the content of Renaud's paper [11]. My presentation will sometimes follow a different path than his, and offer what may be different justifications for the approximations involved. (I also correct a few typos and numerical factors.) Because his reasoning is not everywhere completely clear to me, I may have missed or misunderstood important points, and I may have introduced errors not present in [11]. For these reasons, and since I cover only a portion, I encourage the interested reader to consult Renaud's paper as well.

3.2. Spherical model

The model consists of a massless scalar field coupled to gravity in four spacetime dimensions, restricted to spherical symmetry. (Later, N identical such scalar fields will be introduced in order to exploit a systematic large N approximation when computing a correlation function.) Restricted to spherical symmetry there are no independent gravitational degrees of freedom, and the system is described as effectively two-dimensional. The background metric g_0 is taken to be a black hole formed by collapse of a thin null shell with a macroscopic mass M (although the details of how the black hole forms play no essential role). With spherical fluctuations included, the metric $g_0 + h$ after the black hole has formed is given in one gauge by the line element⁶

$$ds^2 = e^\psi [-f dv^2 + 2 dv dr] + r^2 d\Omega^2 \quad (1)$$

where

$$f = f_0 - \frac{2G\mu}{r} := 1 - \frac{2G(M+\mu)}{r}, \quad (2)$$

with ψ and μ functions of v and r that determine the deviation h from the background g_0 . These spherical metric fluctuations are induced by the scalar field stress tensor. The advanced time coordinate $v = 0$ is chosen to coincide with the collapse of the null shell, so the line element (1) with $M \neq 0$ is applicable for $v > 0$ only.

The spherical mode of the four-dimensional scalar field χ is described by an effective two-dimensional field,

$$\phi := \chi \sqrt{4\pi r^2}, \quad (3)$$

in terms of which the action (up to total derivatives with respect to v or r) is

$$S^{\phi,h} = - \int dv dr \left[\phi_v \phi_r + \frac{1}{2} f \phi_r^2 + \frac{1}{2} (f_r/r) \phi^2 \right], \quad (4)$$

where subscripts denote partial derivative with respect to the subscript variable.⁷ Note that this action depends on the metric perturbation h only via μ , i.e., ψ does not enter. The last term in the integrand of (4) takes the form

$$G \left(\frac{M+\mu}{r^3} - \frac{\mu_r}{r^2} \right) \phi^2. \quad (5)$$

Before adding the fluctuation μ in his presentation, Renaud drops this term on the grounds that it produces a potential barrier that, while it partially scatters ingoing into outgoing modes and

⁶We adopt units with $c = \hbar = 1$.

⁷A comma will be included in the following when necessary to separate a derivative subscript from a previous subscript. Also, here and below, an expression like ϕ_r^2 denotes the square of the derivative (not the derivative of the square).

vice versa, does not affect the near horizon propagation and is therefore not of interest for the consequences of the strong, in-out gravitational interaction. He does not mention the μ terms that occur in (5).

On the static black hole background g_0 , and neglecting the potential barrier term in (4), the scalar field splits into ingoing and outgoing components that are dynamically independent. At the classical level, the action (4) (with potential barrier term neglected) implies the equations of motion

$$(2\phi_\nu + f_0\phi_r)_r = 0. \quad (6)$$

The solutions that satisfy $\phi_r = 0$, i.e., $\phi = \phi(\nu)$, are ingoing modes, which are denoted by $\phi_+(\nu)$. They are constant on the ingoing radial null curves. The other way to solve (6) is to satisfy

$$2\phi_\nu + f_0\phi_r = 0, \quad (7)$$

which implies that ϕ is constant on the outgoing radial null curves (cf. Appendix A). This outgoing mode is denoted ϕ_- , and is a function only of the retarded time u , $\phi_- = \phi_-(u)$.

The gravitational interactions are in the terms of (4) involving f , namely, $\mu\phi_r^2$, $\mu_r\phi^2$, and $\mu\phi^2$. Note that since ϕ_+ depends only on ν in the (ν, r) coordinate system, ϕ_r involves only the outgoing mode ϕ_- . Moreover, whatever frequency $\phi_-(u)$ has with respect to u , its derivative with respect to r diverges at the horizon (cf. (38)):

$$\phi_r|_\nu = -2f_0^{-1}\phi_u|_\nu, \quad (8)$$

so the outgoing modes near the horizon have “high local frequency” with respect to r . Once μ is solved for in terms of ϕ , its ϕ_+ dependence in the $\mu\phi_r^2$ term thus entails strong gravitational interactions between ϕ_- and ϕ_+ . Renaud’s analysis tracks only this interaction. He does not explicitly discuss why. Perhaps he did not notice the other contributions, or perhaps he just chose to simplify the model by retaining only the dominant interaction term. Notice that (after integration by parts on μ_r), the other two terms have the forms $\mu\phi\phi_r$ and $\mu\phi^2$, which are less singular at the horizon since they contain only one or zero factors of ϕ_r . Going forward, I follow Renaud and drop the other terms.⁸

The action for the matter-gravity system on the background g_0 takes the form

$$S = S^\phi + S^g, \quad (9)$$

where S^g is the Einstein–Hilbert action for the metric $g = g_0 + h$. The strongest gravitational interaction between the in and out modes of ϕ is produced by the $f\phi_r^2$ term in (4),

$$S_{\text{int}} = G \int d\nu dr (\mu/r) \phi_r^2. \quad (10)$$

The gravitational constraint equations fix μ and ψ in terms of ϕ (and the background metric). To simplify the computation Renaud includes here only the lowest order contribution in powers of the gravitational constant G . To solve for μ and ψ at this order one could expand S^h to quadratic order in h and integrate out h from the resulting action, which amounts to solving the linearized Einstein equation for μ and ψ . Since ψ does not appear in the scalar field action, it is determined in terms of μ and the background, and is in any case not relevant for the gravitational self-interaction of ϕ , so we may restrict attention to solving for μ .

⁸Renaud explains below equation (17) that, since the interaction is to be computed only to lowest order in G , one may use the on-shell condition $\phi_{+,r} = 0$ in (10), and therefore that interaction is governed by $\mu\phi_{-,r}^2$. (Before equation (16), Renaud also states that equation (16) follows since $\phi_{+,r} = 0$ even in the presence of gravitational interactions. However, to justify the use of the on-shell property it is in any case necessary to appeal to the perturbative argument he gives below equation (17), according to which at lowest order the gravitational interactions are neglected. The relevance of the fact that $\phi_{+,r} = 0$ even in the presence of gravitational interactions is therefore not clear to me.)

Instead of working with a symmetry-reduced Einstein–Hilbert action, we can just use the standard, tensorial Einstein equation. The linearized Einstein tensor component $G_{\nu\nu}^{(1)}$ with the metric (1) is

$$G_{\nu\nu}^{(1)} = \frac{2}{r^2} G\mu_\nu + \frac{f_0}{r^2} [2G\mu_r + \psi + r\psi_\nu + f_0 r\psi_r]. \quad (11)$$

The term in square brackets does not appear in Renaud’s equation (17). Perhaps he neglected it because it is proportional to f_0 , which is very small near the horizon where the interactions of interest are localized. However, in view of the r derivative in μ_r , it is not clear to me that it is subdominant. In particular, using the same partial derivative identity as in (8), we have $f_0\mu_r = -2\mu_u|_\nu$, which is unsuppressed by f_0 . On the other hand, this would receive contributions only from ϕ_- , so would not contribute to the in-out mode interaction. In any case, I will neglect it.

The linearized Einstein equation sets $G_{\nu\nu}^{(1)}$ equal to the metric-linearized part of $8\pi G T_{\nu\nu}$, where the component $T_{\nu\nu}$ of the scalar field energy-momentum tensor is

$$T_{\nu\nu} = \chi_\nu^2 + 2\pi r^2 f (\chi_\nu \chi_r + f \chi_r^2). \quad (12)$$

The second term on the right-hand side also does not appear in Renaud’s equation (17). Again, perhaps he neglects it because it is proportional to f (2), but $f_0\chi_r = -2\chi_u|_\nu$, so it is not actually suppressed. However, as with the above contribution to the Einstein tensor, it would involve only the outgoing field, so would not contribute to the in-out interaction. Neglecting it, this component of the linearized Einstein equation becomes simply

$$\mu_\nu = \phi_\nu^2 \quad (13)$$

after taking the definition (3) into account.

3.3. Gravitational interaction between in and out modes

In general μ depends on both ν and r , so (13) does not fully determine μ . However, as explained above, in the near-horizon region the dominant contribution to the interaction comes from a contribution to μ that depends only on the ingoing mode, which has no r -dependence at fixed ν . Renaud neglects the ϕ_- dependence of μ , so that the solution to (13) becomes

$$\mu_+(v) := - \int_v^\infty dv' \phi_{+,v'}^2 m \quad (14)$$

where the subscript $+$ is Renaud’s notation to indicate that this is the metric perturbation driven by ϕ_+ fluctuations. Since it depends only on ν , μ_+ is nothing but a ν -dependent fluctuation in the mass of the line element (1). I have chosen the integration constant (operator) so that $\mu_+(\infty) = 0$, that is, the metric perturbation vanishes at future null infinity.⁹

Since μ_+ is independent of ϕ_- , equation (7) becomes a linear equation for ϕ_- . Since $2\partial_\nu + f\partial_r$ is an outgoing null vector of the line element (1), the solutions are constant on the modified outgoing null curves, $\phi_-(\nu, r) = \phi_-(\tilde{u})$, where $\tilde{u}(\nu, r)$ is the retarded time in the presence of the gravitational fluctuations caused by ϕ_+ . This fluctuating retarded time satisfies the same equation (7) as does the outgoing mode,

$$2\tilde{u}_\nu + \left(f_0(r) - \frac{2G\mu(\nu, u)}{r} \right) \tilde{u}_r = 0. \quad (15)$$

Renaud imposes the teleological boundary condition that in the asymptotic future $\tilde{u}$ becomes the retarded time coordinate in flat spacetime, i.e., $\tilde{u}(\nu, r) \rightarrow \nu - 2r$ as $\nu, r \rightarrow \infty$ (36). The reason for this choice is presumably that the quantum state far from the horizon is well described in the

⁹Renaud chose instead in his equation (17) to integrate from 0 to ν , in effect setting the perturbation to zero at the time of the shell collapse that forms the black hole. This does not seem to me well justified. Moreover, it leads to a divergent contribution to $\langle \mu_+(\nu) \mu_+(\nu') \rangle$ that he neglects in his equation (38).

semiclassical approximation by perturbative excitation of the flat space vacuum. (A spacetime sketch of the curves at play is shown in Figure 1.) The solution to (15) is characterized, to first order in G , by the variation

$$\delta u := \tilde{u} - u, \quad (16)$$

which satisfies

$$(\delta u)_v|_u = G \frac{\mu(v, u)}{r} u_r|_v = -G \frac{2\mu(v, u)}{r(v, u) - 2M} \quad (17)$$

(where (36) and (37) have been used). The u fluctuation is thus given to first order in the gravitational interaction by

$$\delta u(v, u) = G \int_v^\infty dv' \frac{2\mu(v', u)}{r(v', u) - 2M}. \quad (18)$$

Note that the effect produced by the fluctuations of μ is amplified near the horizon, due to the small denominator in (18). When including only the ϕ_+ dependence of μ , $\mu(v', u)$ in the integrand of (18) is replaced by $\mu_+(v')$ (14).

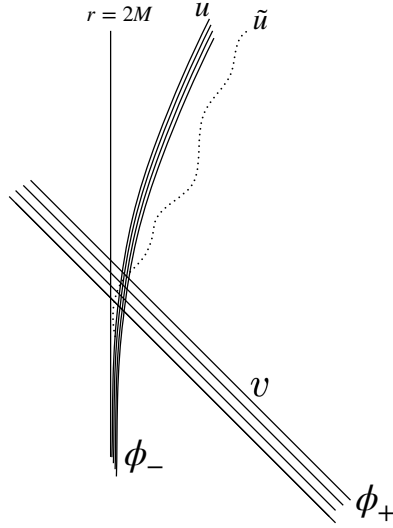


Figure 1. Spacetime sketch of the near-horizon region of a black hole, with some constant phase lines for a pair of ingoing and outgoing modes, $\phi_+(v)$ and $\phi_-(u)$. The curve labeled $\tilde{u}$ is a sample null curve of the metric with dispersion driven by the ϕ_+ vacuum fluctuations.

In the quantum theory we promote (6) to the Heisenberg equation for the field operator (modulo operator ordering that is important in the exact treatment but will be irrelevant in the perturbative treatment we are pursuing). When retaining only the ϕ_+ dependence of μ , the outgoing field operator $\phi_-(\tilde{u}) = \phi_-(u + \delta u)$ becomes an operator-valued function of an operator-valued argument, with δu given by (18), which depends on the field operator ϕ via (14).¹⁰ In the approximations and simplifications adopted by Renaud, the fluctuation δu is entirely induced by the fluctuations of the ingoing mode $\phi_+(v)$. This generates entanglement between the in and out modes which, as we'll see, is responsible for decohering the $\phi_- \phi_-$ correlator.

¹⁰This function of an operator can be made more concrete by expressing it in terms of the Fourier transform $\tilde{\phi}(k)$, as $\phi_-(u + \delta u) = \int \frac{dk}{2\pi} \tilde{\phi}(k) e^{ik(u + \delta u)}$.

The quantum fluctuations of δu are characterized by the variance, $\langle \delta u(v, u) \delta u(v, u) \rangle_C$. To compute this using (18) we first need to compute the connected part of the correlator $\langle \mu_+(v) \mu_+(v') \rangle$ of the metric fluctuations,

$$\langle \mu_+(v) \mu_+(v') \rangle_C := \langle \mu_+(v) \mu_+(v') \rangle - \langle \mu_+(v) \rangle \langle \mu_+(v') \rangle. \quad (19)$$

The metric fluctuation $\mu_+(v)$ is determined by an integral over $\phi_{+,v}^2$ (14). Since δu already has a factor of G , we only require the free field correlator of $\phi_{+,v}^2$, which is determined by the two-point function

$$\langle \phi_+(v_1) \phi_+(v_2) \rangle = -\frac{1}{4\pi} \ln(v_1 - v_2 - i\epsilon) + \text{constant}. \quad (20)$$

It follows that¹¹

$$\langle (\phi_{+,v_1})^2 (\phi_{+,v_2})^2 \rangle_C = \frac{1}{8\pi^2} \frac{1}{(v_1 - v_2 - i\epsilon)^4}, \quad (21)$$

so we have

$$\begin{aligned} \langle \mu_+(v) \mu_+(v') \rangle_C &= \frac{1}{8\pi^2} \int_v^\infty dv_1 \int_{v'}^\infty dv_2 \frac{1}{(v_1 - v_2 - i\epsilon)^4} \\ &= -\frac{1}{48\pi^2} \frac{1}{(v - v' - i\epsilon)^2} \\ &= \frac{1}{48\pi^2} \int_0^\infty d\omega \omega e^{-i\omega(v-v'-i\epsilon)}. \end{aligned} \quad (22)$$

(The second equality is obtained by writing $(v - v' - i\epsilon)^{-4} = -\frac{1}{6} \partial_v \partial_{v'} (v - v' - i\epsilon)^{-2}$.) With this correlator in hand, we are now in a position to compute the variance of δu using (18), which we postpone to the next section.

To conclude this subsection, it is worth noting that, while I discussed the interaction using the Heisenberg picture, Renaud phrased the physics largely in terms of the action. In particular, at first order in G , he argues that the interaction action (10) is

$$S_{\text{int}} = G \int_0^\infty dr \int_0^\infty dv \int_0^v dv' r^{-1} \phi_{+,v'}^2 \phi_{-,r}^2. \quad (23)$$

Having “integrated out” the linearized metric perturbation, the induced gravitational interaction between ingoing and outgoing modes has the form $T_+ \cdot \mathcal{D}^{-1} \cdot T_-$, where $T_\pm$ is the energy-momentum tensor of the ingoing/outgoing modes, and $\mathcal{D}$ is the “kinetic” operator for the metric perturbation. In the present context, which is reduced to spherical symmetry, and restricted to first order in G , this yields (23). It is of key importance that the interaction diverges at the horizon, in the following sense. If $\phi_+ \sim e^{i\omega v}$ and $\phi_- \sim e^{i\lambda u}$, so that ω and λ are the asymptotic black hole rest frame frequencies of both modes, then $\phi_{+,v} \sim \omega \phi_+$, and (using (38))

$$\phi_{-,r}|_v = -2f_0^{-1} \phi_{-,u}|_v \sim f_0^{-1} \lambda \phi_-|_v. \quad (24)$$

The interaction density is thus proportional to $(\omega \lambda / f_0)^2$, which diverges at the (unperturbed) horizon.

4. Quenching of the two-point correlation function

The quantity to be studied for diagnosis of the quenching of trans-Planckian correlations is the two-point correlation function for the outgoing field,

$$\langle \phi_-(x_2) \phi_-(x_1) \rangle. \quad (25)$$

The expectation value is to be taken in the in-vacuum state. The points are specified as follows (see Figure 2): x_1 is located by the spherical radius r and the advanced time v of a radial null

¹¹Renaud has the coefficient $1/16\pi^2$ in his equation (37), but I think the two equivalent Wick contractions result in a factor of 2. In any case, the precise numerical factor is not relevant for his analysis.

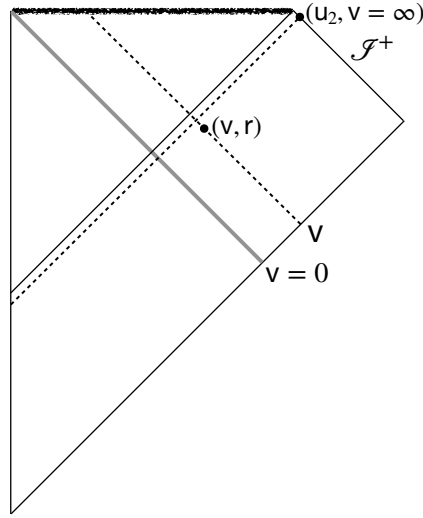


Figure 2. Penrose diagram of black hole formed from collapse of a spherical null shell (thick grey line) at the ingoing null coordinate $v = 0$. The correlation function of interest is located at the points labeled by coordinates (v, r) and $(u_2, v = \infty)$, where r is the spherical radius and u is the outgoing null coordinate. $\mathcal{S}^+$ is future null infinity, which coincides with $v = \infty$, and $u \rightarrow \infty$ at the future endpoint of $\mathcal{S}^+$.

geodesic at $\mathcal{S}^-$, and the point x_2 sits on $\mathcal{S}^+$ at the retarded time u_2 . Since the gauge diffeomorphisms do not act at $\mathcal{S}^\pm$, and r is intrinsically defined (in the spherically truncated context), the correlation function is gauge-invariant. In the absence of the gravitational interaction, i.e., at zeroth order in G , it is proportional to $\ln(u_2 - u(v, r) - i\epsilon)$. As discussed above, the effect of the gravitational interaction is to turn $u(v, r)$ into an operator $\tilde{u}(v, r)$ which acts on the Hilbert space of ingoing modes. The Fourier transform of the u_2 dependence of the two-point function is proportional to $e^{-i\lambda\tilde{u}(v, r)}/\lambda$, so the expectation value

$$\langle \tilde{\phi}_-(v = \infty, \lambda) \phi_-(v, r) \rangle \propto \langle e^{-i\lambda\tilde{u}(v, r)} \rangle \quad (26)$$

characterizes the fate of the two-point function in the presence of the metric fluctuations driven by the quantum fluctuations of the ingoing modes. The challenge now is to compute this quantity including nonperturbative effects in G .

As discussed in Section 3.1, Renaud introduces at this point a large number N of identical scalar fields. An expansion in $1/N$ at fixed GN then allows him to exponentiate the effect of the gravitational interaction without the need to solve the equations of motion beyond linear order in G . It also allows him to import from the Schwinger–Keldysh closed time path formalism [15] a key result: when the environment of a quantum system is “Gaussian”, i.e., governed by a quadratic action, the effect of the environment on the rest of the system is captured by the so-called noise and dissipation kernels, and the effect of the noise kernel is identical to that produced by a Gaussian distribution of noise [16].

The ingoing modes drive fluctuations of the metric variable μ_+ , which produce an effective noisy environment for the outgoing modes. (Renaud focuses on the effect of the noise alone, presumably since it is easily captured with the stochastic method, but a more complete treatment would also include the dissipation.) To implement this scheme one needs to solve (15) for the fluctuating retarded time at first order in G , and then compute the stochastic approximation to the expectation value (26), which will involve all orders of G .

With the retarded time expanded around the background value as $\tilde{u} = u + \delta u$, the expectation value (26) takes the form

$$\langle e^{-i\lambda\tilde{u}(v,r)} \rangle = e^{-i\lambda u(v,r)} \langle e^{-i\lambda\delta u(v,r)} \rangle. \quad (27)$$

In the large N approximation, taking into account just the noise kernel, the quantum expectation value is replaced by the stochastic average with respect to a Gaussian probability distribution,

$$P(\delta u) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\delta u^2/2\sigma^2} \quad (28)$$

where the variance $\sigma^2 = \langle \delta u \delta u \rangle_C$ is the quantum expectation value, and its position dependence is suppressed. This yields the approximation for the Fourier transform of the two-point function

$$\langle \tilde{\phi}_-(v = \infty, \lambda) \phi_-(v, r) \rangle \sim e^{-i\lambda u(v,r)} e^{-\lambda^2 \sigma^2/2}. \quad (29)$$

It just remains to compute σ^2 .

The variance σ^2 receives equal contributions from the N scalars. Using (18) for the δu integral and (22) for the μ_+ correlator we find¹²

$$\begin{aligned} \sigma^2 &= \langle \delta u(v, u) \delta u(v, u) \rangle_C \\ &= 4G^2 N \int_v^\infty dv_1 \int_v^\infty dv_2 \frac{\langle \mu_+(v_1) \mu_+(v_2) \rangle_C}{[r(v_1, u) - 2M][r(v_2, u) - 2M]} \\ &\approx \frac{4G^2 N e^2}{(4M^2)(48\pi^2)} \int_v^\infty dv_1 \int_v^\infty dv_2 e^{\kappa(u-v_1)} e^{\kappa(u-v_2)} \int_0^\infty d\omega \omega e^{-i\omega(v_1-v_2-i\epsilon)} \\ &\approx \frac{G^2 N \kappa^2}{(r/2M-1)^2} \frac{1}{3\pi^2} \int_0^\infty d\omega \frac{\omega}{\omega^2 + \kappa^2} e^{-\omega\epsilon}. \end{aligned} \quad (30)$$

Here $\kappa := 1/4M$ is the surface gravity of the background black hole. The approximate equalities use (36) together with the near-horizon approximation $e^{r/2M} \approx e^1$. The integrand of the ω integral in (30) peaks at $\omega = \kappa$ for $\epsilon = 0$, but in the limit $\epsilon \rightarrow 0$ the integral diverges as $\gamma - \ln(\kappa\epsilon)$, where γ is Euler's constant. If we introduce a UV frequency cutoff $\Lambda = 1/\epsilon \gg \kappa$, the integral becomes approximately $\ln(\Lambda/\kappa)$.¹³ As mentioned in Section 3.1, the UV divergence in this crude model may result from the fact that the flat spacetime gravitational vacuum interactions have not been suitably subtracted. Nevertheless, the logarithmic dependence on the cutoff is very weak, so perhaps a correct physical lesson can be drawn.

The standard deviation σ in (30) depends on v and u only via r , and can be written as

$$\sigma = \frac{\bar{\sigma}}{r/2M-1}, \quad \bar{\sigma} := G\kappa \sqrt{\frac{N \ln(\Lambda/\kappa)}{3\pi^2}}. \quad (31)$$

According to (29), the Fourier component of the two-point correlation function at Killing frequency λ will decay to zero as a Gaussian when $\lambda\sigma \gg 1$, i.e., when

$$r/2M-1 \ll \lambda\bar{\sigma}. \quad (32)$$

The two-point correlation function itself is thus modified from $\ln(u_2 - u(v, r) - i\epsilon)$, which is singular at $u_2 = u(v, r)$, to a function that is regular in the UV. In the near horizon limit we have $r/2M-1 \approx \kappa^2 \ell^2$, where ℓ is the radial proper distance to the horizon (i.e., to the bifurcation surface in the maximal extension of the Schwarzschild metric) on a “static slice” (normal to the Killing vector). The decay condition (32) may thus be expressed as $\ell^2 \ll \lambda\bar{\sigma}/\kappa^2$. Using (31), with the square root taken as $O(\sqrt{N})$, the decay condition becomes

$$\ell \ll \ell_c := \sqrt[4]{N} \sqrt{\lambda/\kappa} \ell_P, \quad (33)$$

¹²Note that σ^2 depends on only the symmetric, real, part of $\langle \mu_+(v_1) \mu_+(v_2) \rangle_C$.

¹³Renaud did not include ϵ in the ω integral, but rather just introduced Λ as the upper limit of integration.

where $\ell_P = \sqrt{\hbar G/c^3}$ is the Planck length. At a frequency equal to that of the Hawking temperature ($T_H = \kappa/2\pi$), the correlator thus dies off when the point x_1 is within $\sim \sqrt[4]{N}$ Planck lengths of the horizon, measured on a static slice.

4.1. *Validity regime questions*

The Gaussian $e^{-\lambda^2 \sigma^2/2}$ that damps the Fourier transform of the two-point function can be expanded as a power series in σ^2 , which according to (30) is proportional to $G^2 N$. Since GN is held fixed, this is a series in powers of $1/N$, which is thus formally convergent, despite containing arbitrarily high orders in G . This series is what Renaud pointed to as the first contribution beyond the semiclassical approximation, and it accounts for the title of his paper. One should ask, nevertheless, what is the relative contribution of the series in powers of $G^m N$, with $m > 2$? Term by term, these series will be smaller than the $G^2 N$ series provided the dimensionless quantity formed with G is small compared to 1. If the quenching effect kicks in at a length scale above the Planck scale, then presumably that dimensionless quantity is small compared to 1. According to (33), that seems indeed to be the case, provided that the divergent quantity $\ln(\Lambda/\kappa)$ that was treated as $O(1)$ in (33) would in fact not appear in a correctly renormalized treatment.

It is encouraging, for maintaining validity of the approximations, that the critical length ℓ_c (33) is parametrically larger than the Planck length, but we should enquire as to the meaning of that length, and as to whether the Planck length is the right length to compare to. First, while ℓ measures proper distance to the bifurcation surface on a static slice of the Schwarzschild spacetime, there is no bifurcation surface in a spacetime with collapse to a black hole. Nevertheless, shortly after the collapse, ℓ is also very close to the proper distance to the surface of the collapsing matter: equation (40) shows that the “missing length” ℓ_1 divided by the full length ℓ_2 to the would-be bifurcation surface decreases exponentially as $e^{-\kappa \Delta \nu}$. Thus ℓ is a meaningful length scale in the collapse spacetime.

But even if $\ell_c > \ell_P$, would that be long enough for the large N perturbation theory to be justified? Since we have assumed a large number of matter fields N , holding GN fixed, perhaps the condition for perturbation theory not to fail is not $\ell_c > \ell_P$ but rather $\ell_c > \sqrt{N} \ell_P$, which is equivalent to $\lambda > \sqrt{N} \kappa$. This would still permit the conclusion that the singularity of the two-point function is quenched, but it may be quenched at a larger value, since the calculation would only imply that Fourier components with $\lambda > \sqrt{N} \kappa$ would be damped.

5. Discussion

The trans-Planckian problem has several facets: the origin of the outgoing black hole modes and the Hawking radiation they carry, the divergent contribution of quantum field entanglement to black hole entropy, and the origin of degrees of freedom in an expanding universe. These are not problems at the level of effective field theory, but they seem problematic when one digs for a deeper level of understanding, in part because of our incomplete understanding of quantum gravity in general. A major motive for attempting to tackle them is that they may provide guidance in the quest for understanding quantum gravity.

This article focused on Renaud Parentani’s analysis of a black hole in quantum gravity, which aimed to extract consequences of the gravitational interaction of ingoing and outgoing modes *in their ground states*. The model is similar to earlier work [3,12,13,17,18] which also studied the gravitational interaction of these modes, but with a key difference in aim. In those works, the focus was on the black hole S-matrix, in particular, the sensitivity of that S-matrix to the addition of an infalling particle. That work was motivated by the black hole information paradox, the question being whether these interactions could play a role in preservation of unitarity. This

question has persisted to recent times, as seen for example in [19,20]. In contrast, Renaud's study was focused on the vacuum, and designed to explore the consequences of quantum geometry fluctuations for the trans-Planckian problem. Nevertheless, the S-matrix (at least in standard interacting quantum field theory) is closely related to vacuum correlation functions, so the two aspects of near horizon physics may be similarly related. Moreover, the discontinuous shift of retarded time in response to the gravitational effect of an incoming null shock, studied in [12,19], is an abrupt limit of Renaud's fluctuation δu (18) of retarded time in response to the quantum gravitational effect of the incoming vacuum modes.

Renaud aimed to make some progress on this very difficult problem by admitting drastic simplifications: the spherical truncation, the neglect of subleading and other effects not central to the in-out-interaction question, and the use of a large N approximation. Within that framework, he uncovered what looks like it might point to a quantum gravity mechanism that quenches trans-Planckian near-horizon correlations. How valid it is remains to be seen, but in my view this approach deserves to be developed and further explored.

It might be fruitful to study the problem in a two-dimensional theory like JT gravity where, rather than relying on a severe truncation of a higher-dimensional theory, one could come closer to a complete analysis. On the other hand, the trans-Planckian problem might have a different status in two dimensions. Perhaps a model similar to Renaud's could be implemented in four-dimensional spacetime without the spherical truncation, using the well-developed methods of the influence functional as applied to gravity [15]. In fact, those methods have been applied in flat spacetime [21], to fields outside black holes [22], and in de Sitter spacetime [23], but not to the sort of near-horizon correlator, including nonperturbative effects in G , that Renaud targets. Furthermore, it would be important to determine whether higher derivative interactions in the effective theory might dominate lower derivative ones, since each derivative for outgoing modes brings in another factor of $1/f_0$ (8).

Another possible avenue to explore would be the case of a black hole in asymptotically anti-de Sitter spacetime, where AdS/CFT duality could be exploited. As a bulk point approaches the horizon in that setting, its boundary dual is a smearing of local operators over a region that extends to infinite time. Its correlation with operators localized at its future light cone cut on the boundary therefore decays exponentially on a thermal time scale T^{-1} , or perhaps on the scrambling time scale $T^{-1} \log S$, (until the very late time $\sim S/T$ determined by the finiteness of the CFT entropy S at the temperature T) [24]. Might that thermal temporal decay in the boundary dual be related to the near-horizon decay that Renaud found? Studies of the effect of shockwave perturbations of the vacuum [12,19] found scrambling of S-matrix elements or cross-horizon correlations, which does seem "morally" related to Renaud's finding for vacuum correlators. In the AdS/CFT context it is also worth mentioning the salient fact that the finiteness of the black hole entropy, and thus a solution to the trans-Planckian problem, is at hand, since finiteness of the dual CFT entropy is perfectly understood, even if the detailed implications for near horizon physics remain currently out of reach.

What if refined calculations were to establish that indeed trans-Planckian near horizon correlations are quenched by gravitational interaction governed by general relativity? What would be the broader implications for quantum gravity? It is already remarkable that the dynamics of general relativity fits tongue in groove with the quantum field phenomenon of Hawking radiation, allowing for consistent black hole thermodynamics and the generalized second law. This consistency was discovered initially at the semiclassical level on the gravity side. But such a deep unity must presumably extend to the level of the interacting quantum gravitational vacuum.

If trans-Planckian physics were not masked by the gravitational quenching of correlations, then black hole horizons would have been windows into the trans-Planckian regime, which would likely have led to a breakdown of the effective field theory description of gravity. Moreover,

it would seemingly have led to an infinite black hole entropy, which would not only disagree with the Bekenstein–Hawking entropy, but would also invalidate the thermodynamic derivation of the Einstein equation as a vacuum equation of state [25,26]. The masking is thus to be expected, and Renaud’s calculation suggests that a mechanism can already be discerned at the level of perturbative quantum gravity, provided that some nonperturbative consequences can be captured by a suitable, justified resummation like that of the large N approximation.¹⁴ In fact, Renaud proposed in [11] that, in the setting with the nonspherical degrees of freedom included, the length scale ℓ_c at which the near-horizon correlations are quenched should also be the length scale at which the entanglement accounting for black hole entropy is cut off. If this were the case, the scaling of the entropy with the number of fields N might be canceled by the N dependence of ℓ_c .¹⁵

Where would this leave us regarding the other facets of the trans-Planckian problem? Would it reveal how the outgoing modes are generated near a black hole horizon? And would it address the question of how new cis-Planckian modes emerge in an expanding universe? Of course I don’t pretend to know the answers, but allow me to speculate.

It seems plausible that the answer to the first of these questions is yes. In the presence of the strong gravitational interactions, ingoing and outgoing modes are thoroughly mixed, and it is not implausible that the presence of a black hole horizon could engender a net “spectral flow” of outgoing modes emerging from the near horizon region. In fact, these words are quite reminiscent of the idea that a chiral diffeomorphism anomaly underlies the phenomenon of Hawking radiation [27–29]. But I have not yet managed to fully understand the line of thought in those cited papers, and in particular the precise role that “mode creation” versus “mode excitation” plays.

As to the second question, it seems that gravitational interaction in the fixed Hilbert space of an effective theory, no matter how strong, is powerless to create new degrees of freedom. But even that is not to say that gravity does not have a role to play in that mysterious process. It may just be that nonperturbative quantum gravity in an expanding universe does not admit a fixed Hilbert space description. There are certainly reasons to suspect that is the case. In a recent talk [30] I sketched a way that such “vacuogenesis” might work, involving the “wave function of the universe”.

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Declaration of interests

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¹⁴A locally Lorentz-invariant mechanism to cloak the UV entanglement by gravitational back-reaction of the entangled pairs was proposed qualitatively in [26]. That seems to be quite different from Renaud’s mechanism, because it does not rely on the spacetime background being a black hole, and is not (at least in any obvious way) related to interactions between ingoing and outgoing modes.

¹⁵Note that the $N^{1/4}$ dependence of ℓ in the spherical reduction result (33) would *not* cancel, since the entanglement entropy would scale as $N/\ell^2 \propto N^{1/2}$ (in four spacetime dimensions).

Appendix A. Schwarzschild, Eddington–Finkelstein and double-null coordinates

The angular part of ds^2 is $r^2 d\Omega^2$ in both coordinate systems, so I omit it. For the background black hole metric we have

$$ds^2 = -f_0 dt^2 + f_0^{-1} dr^2 = -f_0 dv du = -f_0 dv^2 + 2 dv dr \quad (34)$$

where $f_0 = 1 - 2M/r$, and

$$v = t + r_*, \quad u = t - r_*, \quad dr/dr_* = f_0, \quad r_* = r + 2M \ln(r/2M - 1), \quad (35)$$

so in particular

$$u = v - 2r_*, \quad r_* = (v - u)/2, \quad r/2M - 1 = e^{-r/2M} e^{(v-u)/4M}. \quad (36)$$

The relations between partial derivatives with respect to the (v, u) coordinates and the (v, r) coordinates are

$$\partial_v|_u = \partial_v|_r + \frac{1}{2} f_0 \partial_r|_v, \quad \partial_u|_v = -\frac{1}{2} f_0 \partial_r|_v, \quad (37)$$

and

$$\partial_v|_r = \partial_v|_u + \partial_u|_v, \quad \partial_r|_v = -2f_0^{-1} \partial_u|_v. \quad (38)$$

The (v, r) coordinates of two points on a constant Schwarzschild time slice ($t = \text{const}$) satisfy $v_1 - r_{*1} = v_2 - r_{*2}$, which implies that

$$\frac{r_1 - 2M}{r_2 - 2M} = e^{(-\Delta v + \Delta r)/2M}, \quad (39)$$

with $\Delta v = v_2 - v_1$ and $\Delta r = r_2 - r_1$. If both points are near the horizon we have $\Delta r \ll 2M$, and $r - 2M \approx \kappa^2 \ell^2$, where $\kappa = 1/4M$ is the surface gravity and ℓ is the proper distance to the horizon on a constant t slice. In the near-horizon case (39) thus implies that

$$\ell_1 / \ell_2 \approx e^{-\kappa \Delta v}. \quad (40)$$

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