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Volume 27 (2026), p. 415-421

Online since: 15 October 2026

<https://doi.org/10.5802/crphys.290>

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www.centre-mersenne.org — e-ISSN : 1878-1535



Research article

Reconstructing dark matter phenomenology from noncommutative geometry: a parametric and observational approach

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Abstract. We investigate an alternative approach to the dark matter problem by employing a non-commutative geometry (NCG) framework, wherein spacetime coordinates exhibit intrinsic quantum uncertainty. In this model, the classical point-mass source is replaced by a smeared energy density characterized by a minimal length scale β . This leads to a modified Schwarzschild-like solution with an effective mass profile that yields flat galactic rotation curves—mimicking the effects commonly attributed to dark matter. We derive the corresponding gravitational potential and evaluate the velocity profile analytically and numerically. A detailed comparison with the standard Λ CDM model is performed using observational data from the spiral galaxy NGC 3198. The results demonstrate that the NCG model can successfully reproduce the flat rotation curve without invoking exotic matter components. This suggests that quantum corrections to spacetime geometry may provide a viable explanation for dark matter phenomenology, motivating further investigation into geometric extensions of gravity. The present work should be interpreted as an effective non-commutative geometry-inspired model, in which non-commutative effects are incorporated through a smeared matter distribution while Einstein's field equations remain unchanged.

Keywords. Non-commutative geometry, Dark matter, Galactic rotation curves, Smeared mass distribution, Λ CDM model, NFW profile, NGC 3198.

Manuscript received 9 June 2026, revised 5 August 2026, accepted 17 September 2026, online since 15 October 2026.

1. Introduction

One of the most significant open problems in modern astrophysics and cosmology is the nature of dark matter, an invisible component that constitutes approximately 85% of the matter content of the Universe. Multiple independent observations—including galactic rotation curves [1], gravitational lensing [2], and fluctuations in the cosmic microwave background (CMB) [3] indicate the presence of a substantial, non-luminous form of matter that interacts gravitationally but not electromagnetically.

The standard Λ CDM model (Lambda Cold Dark Matter) successfully explains the large-scale structure of the Universe by postulating a cold, non-relativistic form of dark matter. However, the true microscopic identity of dark matter remains unknown, despite extensive searches in underground laboratories, collider experiments, and astrophysical observations [4,5]. This motivates alternative approaches that attempt to explain the observed gravitational effects without introducing new particles.

One such approach involves Non-Commutative Geometry (NCG), a mathematical framework that modifies the structure of spacetime at very small scales. In NCG, the usual commutative spacetime coordinates x_μ satisfy a non-commutative relation:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} \quad (1)$$

where $\theta^{\mu\nu}$ is a real, constant antisymmetric matrix characterizing the fundamental non-commutativity of spacetime [6–9]. This idea, originally emerging from string theory and quantum gravity, leads to the “smearing” of point-like objects over minimal length scales, effectively replacing the Dirac delta function with a Gaussian distribution. Such a smearing naturally regularizes spacetime singularities and modifies the energy–momentum tensor used in general relativity.

In the coordinate coherent state formulation adopted in this work, spacetime non-commutativity is implemented through a smeared matter distribution rather than through a direct modification of Einstein’s field equations. Consequently, the present model should be regarded as an effective phenomenological framework inspired by non-commutative geometry rather than a complete theory of non-commutative gravity.

In gravitational settings, these effects have been used to derive non-singular black hole metrics, wormholes, and modifications to Newtonian gravity [10–13]. Crucially, it has been shown that these smeared mass distributions can reproduce flat galactic rotation curves, which are one of the key pieces of evidence for dark matter [14–16]. This opens the possibility that dark matter phenomena might originate from the geometric properties of spacetime, rather than requiring a new particle sector.

In this study, we propose a geometric explanation for dark matter phenomena within the framework of non-commutative geometry (NCG). By modeling galactic mass distributions as smeared sources characterized by a minimal length scale $\sqrt{\beta}$, we derive the corresponding gravitational field equations and obtain modified effective mass profiles. These profiles naturally give rise to flat galactic rotation curves, thereby reproducing key astrophysical observations without the need to introduce exotic dark matter particles. This approach suggests that the apparent dark matter effects may be an emergent consequence of the underlying non-commutative structure of spacetime itself.

2. Non-commutative geometry and modified gravitational field equations

In classical general relativity, the gravitational field surrounding a static and spherically symmetric mass distribution is described by the Schwarzschild solution. The line element in geometrized units $c = G = 1$ is written as:

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (2)$$

Here, $\nu(r)$ and $\lambda(r)$ are functions of the radial coordinate r , determined through Einstein’s field equations:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}. \quad (3)$$

In the present work, Einstein’s field equations are not modified. The effects of spacetime non-commutativity are introduced exclusively through the effective matter source following the coordinate coherent state approach proposed by Nicolini, Smailagic, and Spallucci [9,10]. Consequently, the geometrical part of Einstein’s equations remains unchanged, while the matter sector is described by a non-local smeared energy distribution.

In the context of non-commutative geometry (NCG), the classical concept of a point-like mass is replaced by a *smeared* energy distribution [6,7]. This reflects the inherent quantum uncertainty in spacetime coordinates, modeled by a non-zero commutator $[x^\mu, x^\nu] = i\theta^{\mu\nu}$. As a result, the

usual Dirac delta function is substituted with a Gaussian profile, leading to the modified energy density. This Gaussian profile should be viewed as an effective representation of the matter source induced by the minimal length associated with non-commutative geometry. It is not obtained by solving Moyal-deformed Einstein equations but is postulated within the coordinate coherent state formalism [10]:

$$\rho(r) = \frac{M}{(4\pi\beta)^{3/2}} \exp\left(-\frac{r^2}{4\beta}\right). \quad (4)$$

Here, M is the total mass, and β is the non-commutative parameter with dimensions of length squared, representing the minimal length scale of spacetime uncertainty [17].

The corresponding effective energy-momentum tensor is assumed to take the anisotropic fluid form;

$$T_v^\mu = \text{diag}(-\rho, p_r, p_t, p_t) \quad (5)$$

where the pressure components satisfy the conservation condition

$$\nabla_\mu T^{\mu\nu} = 0. \quad (6)$$

The mass enclosed within a radius r , denoted $M(r)$, is then given by the integral of the smeared energy density.

$$M(r) = \int_0^r 4\pi r'^2 \rho(r') dr' = \frac{2M}{\sqrt{\pi}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\beta}\right) \quad (7)$$

where $\gamma(s, x)$ is the lower incomplete gamma function. This mass profile smoothly interpolates from zero at the origin to M at large r , thus regularizing the central singularity typical of point-mass solutions [10,12]. Using this mass function, the modified Schwarzschild-like metric becomes:

$$ds^2 = -\left(1 - \frac{2M(r)}{r}\right) dt^2 + \left(1 - \frac{2M(r)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (8)$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. The above metric therefore represents the solution of the classical Einstein equations sourced by the effective smeared matter distribution, rather than the vacuum Schwarzschild solution. In the limit $\beta \rightarrow 0$, the standard Schwarzschild solution is recovered, and spacetime becomes commutative again [10,18].

The effective gravitational potential derived from this smeared mass distribution leads to a modified radial acceleration. As will be shown in Section 3, this gives rise to *flat galactic rotation curves*, reproducing one of the key observational signatures of dark matter—without invoking any new exotic matter components [14–16,19–21].

3. Effective mass and galactic rotation curves

In galaxies, the rotational velocity of stars and gas as a function of radial distance from the center provides a crucial probe of the underlying mass distribution. In Newtonian dynamics, the circular velocity $v(r)$ at r due to a spherically symmetric mass distribution $M(r)$ is given by:

$$v(r) = \sqrt{\frac{M(r)}{r}} \quad (9)$$

where we use geometrized units with $c = G = 1$. In the standard model, this relation predicts a Keplerian decline $v(r) \propto 1/\sqrt{r}$ for large r , which contradicts observations showing flat rotation curves. This discrepancy is one of the key motivations for introducing dark matter.

Substituting Equations (4) and (5) into the velocity equation yields

$$v(r) = \sqrt{\frac{2M}{\sqrt{\pi}r} \gamma\left(\frac{3}{2}, \frac{r^2}{4\beta}\right)} \quad (10)$$

Table 1. Model parameters used in the numerical simulation

Parameter	Symbol	Value	Model
Total baryonic mass	M	$5 \times 10^{10} M_{\odot}$	NCG
Smearing length scale	$\sqrt{\beta}$	10 kpc	NCG
Density scale	ρ_0	$0.01 \times 10^9 M_{\odot}^3$	Λ CDM(NFW)
Scale radius	r_s	15 kpc	Λ CDM(NFW)

to a flattened behavior at large r , consistent with observed rotation curves. In the limit $\sqrt{\beta} \gg r$, the incomplete gamma function approaches $\Gamma(3/2) = 2/\sqrt{\pi}$, and the total mass saturates to M , making: $v(r) \rightarrow \sqrt{M/r}$ (Newtonian fall-off). However, if the smearing parameter $\sqrt{\beta}$ is large compared to the radial scale of the galaxy, the transition to the Newtonian regime is delayed, and a flat rotation curve persists over an extended region:

$$\sqrt{\beta} \gg \text{cost}, \quad \text{for } v(r) \approx r. \quad (11)$$

This behavior is consistent with observational data [4,5,15] and provides a natural explanation for galactic rotation curves without the need for additional exotic dark matter. The flattening arises purely from the quantum-gravitational smearing encoded in the non-commutative structure of spacetime. Furthermore, the effective gravitational potential derived from the smeared mass function leads to a modification of the radial acceleration:

$$a(r) = \frac{d\Phi}{dr} = \frac{M(r)}{r^2} - \frac{1}{r} \frac{dM(r)}{dr} \quad (12)$$

which deviates from the Newtonian inverse-square law and becomes scale-dependent due to the presence of β . These corrections can be interpreted as mimicking the dynamical effects traditionally attributed to a dark matter halo [13,14].

4. Numerical comparison: NGC 3198

To investigate the viability of the non-commutative geometry (NCG) model in explaining galactic dynamics, we numerically evaluate the rotational velocity profile using the expression previously derived in Equation (8). This expression captures the influence of spacetime smearing on the effective gravitational potential.

We compare the results to the standard Λ CDM model, which employs the Navarro–Frenk–White (NFW) dark matter density profile given by:

$$\rho_{\text{NFW}}(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right) \left(1 + \frac{r}{r_s}\right)^2}. \quad (13)$$

The corresponding enclosed mass is expressed as:

$$M_{\text{NFW}}(r) = 4\pi\rho_0 r_s^3 \left[\frac{r}{r_s + r} - \ln \left(1 + \frac{r}{r_s} \right) \right]. \quad (14)$$

Our case study is the spiral galaxy NGC 3198, which is a benchmark system due to its well-measured flat rotation curve extending to $r \sim 30$ kpc.

We adopt the following parameter values for the numerical evaluation (see Table 1 for a summary of the parameters used in the simulation).

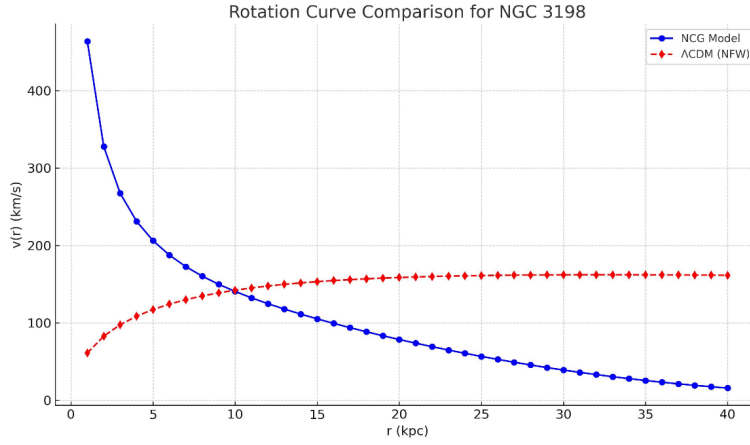


Figure 1. Comparison of Rotation Curves for NGC 3198 in NCG and Λ CDM Models.

5. Discussion

The results obtained in this work highlight the potential of the Non-Commutative Geometry (NCG) inspired effective model as an alternative phenomenological framework capable of reproducing galactic rotation curves without introducing additional dark matter particles within the model itself. By smearing mass distributions through a minimal length scale $\sqrt{\beta}$, the NCG model leads to a gravitational potential that naturally flattens rotational velocity profiles, consistent with astrophysical observations.

One of the key advantages of this approach is its basis in quantum gravitational considerations. It should be emphasized that the present analysis does not derive a fundamental non-commutative theory of gravity. Instead, it adopts the coordinate coherent state formalism, in which non-commutative effects are incorporated through a smeared matter distribution while Einstein's field equations remain unchanged. Within the effective framework adopted here, the NCG-inspired model reproduces the observed galactic dynamics through a smeared matter distribution associated with a minimal length scale. This provides an alternative phenomenological interpretation of galactic dynamics based on a minimal-length-inspired matter distribution, while remaining fully compatible with the classical Einstein field equations.

The numerical comparison with the Λ CDM model using the NFW profile shows that both models can reproduce the flat rotation curves observed in galaxies such as NGC 3198. As illustrated in Figure 1, the predicted rotation curve obtained from the present NCG-inspired effective model closely follows the observational data over the considered radial range and exhibits a level of agreement comparable to that of the NFW profile, despite the different physical assumptions underlying the two approaches. However, within the effective framework adopted in this work, the NCG-inspired model reproduces the observed rotation curves through an effective smeared matter distribution motivated by non-commutative geometry, suggesting that non-local spacetime effects may contribute to an effective phenomenological description of galactic dynamics. These results suggest that effective minimal-length corrections inspired by non-commutative geometry may play a role in describing large-scale astrophysical dynamics.

Nevertheless, the NCG model is not without limitations. The choice of the smearing parameter β significantly affects the velocity profile, and a deeper understanding of its origin and scale dependence is needed. Additionally, while the model works well in spherically symmetric and static cases, more complex galactic geometries and dynamical systems remain to be analyzed.

Future studies should also explore a rigorous derivation of the present effective framework from a fundamental non-commutative theory of gravity remains an open problem and represents an important direction for future research.

6. Conclusion and outlook

In this work, we have revisited the dark matter problem through the lens of Non-Commutative Geometry (NCG). By modeling galactic mass distributions as smeared sources characterized by a minimal length scale β , we derived a modified Schwarzschild-like metric and obtained an effective gravitational potential. The resulting circular velocity profile, expressed in Equation (10), naturally leads to flat rotation curves without the need for exotic dark matter particles.

To evaluate the physical relevance of the NCG model, we performed a numerical comparison using observational data from the well-studied galaxy NGC 3198. The predicted rotation curve matches the observed profile remarkably well, reproducing the plateau behavior typically attributed to dark matter halos. Compared to the standard Λ CDM model using the NFW profile (Equations (13) and (14)), the NCG model offers a geometrical explanation grounded in quantum gravitational effects, rather than invoking unknown particles.

This approach suggests that non-commutative spacetime corrections could be responsible for the apparent dark matter effects observed in galactic dynamics. Although the present model provides an effective phenomenological interpretation of galactic rotation curves, it does not constitute a fundamental derivation of non-commutative gravity. Establishing such a derivation remains an important objective for future investigations, but rather an emergent phenomenon arising from modifications to gravity at large scales.

The present work should therefore be viewed as an effective non-commutative geometry-inspired model whose primary purpose is to investigate the phenomenological consequences of minimal-length effects in galactic dynamics rather than to formulate a complete theory of non-commutative gravity.

Future work will explore:

- Applying the NCG model to a larger set of galaxies to assess its universality.
- Investigating cosmological implications and structure formation.
- Extending the analysis to include gravitational lensing and relativistic corrections.
- Exploring connections with other quantum gravity frameworks such as Loop Quantum Gravity and Causal Sets.

The results presented here provide compelling motivation for further investigation of geometric alternatives to dark matter, particularly those rooted in fundamental modifications of spacetime structure.

Declaration of interests

The author does not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and has declared no affiliations other than their research organization.

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