INSTITUT DE FRANCE
Académie des sciences

Comptes Rendus
Physique

Anne Tanguy

Elasto-plastic behavior of amorphous materials: a brief review

Volume 22, issue S3 (2021), p. 117-133

<https://doi.org/10.5802/crphys.49>

Part of the Special Issue: Plasticity and Solid State Physics
Guest editors: Samuel Forest (Mines ParisTech, Université PSL, CNRS, France) and David Rodney (Université Claude Bernard Lyon 1, France)

Comptes Rendus
Physique
2021, 22, n^{o} S3, p. 117-133
`https://doi.org/10.5802/crphys.49`

Plasticity and Solid State Physics / *Plasticité et Physique des Solides*

Elasto-plastic behavior of amorphous materials: a brief review

Anne Tanguy [a, b]

[a] LaMCos, INSA-Lyon, CNRS UMR5259, Université de Lyon, F-69621 Villeurbanne
Cedex, France

[b] ONERA, University Paris-Saclay, Chemin de la Huniére, BP 80100, 92123 Palaiseau,
France

E-mail: Anne.Tanguy@onera.fr

Abstract. Disordered materials, like metallic glasses or silicate glasses, have an atomistic amorphous structure preventing the formation of extended defects such as *dislocations*. Irreversible deformation in these materials is thus localized, but can organize along shear bands. In this brief review, based on recent publications, we will see if local plasticity can be measured and predicted in disordered atomic assemblies, and in what conditions it can be related to preexisting structural defects. We will then draw a general picture of the plastic mechanical behaviour within the theoretical framework of mechanical instabilities. Finally, we will focus our attention on different scenarii for shear banding in glasses.

Keywords. Plasticity, Constitutive laws, Amorphous materials, Glasses, Eshelby inclusions, Shear bands.

1. Introduction

Amorphous materials are disordered at all scales, meaning that they do not exhibit any characteristic structural lengthscale despite the average interatomic distance. The absence of regularity in the atomistic stacking forbids the direct visualisation of atoms, except in the very specific case of two-dimensional systems, like recently in layered polymorphs of silicon dioxyde based on SiO_4 tetrahedra, covalently bonded to a metallic substrate [1, 2]. The lack of direct *in situ* visualisation makes it difficult to identify properly possible defects responsible for plasticity (irreversible static deformation). Few attempts have been proposed based on image correlation of speckle images [3] or on the evolution of static structure factors upon external deformation [4, 5]. However, while the first still does not allow a quantitative identification of plasticity related defects, the second results only from data spatially averaged over the size of the X-ray beam, that is on micrometric sizes too large to locate isolated nanometric defects. Moreover, the signature of plasticity is not always visible neither in the structure factor, due to the fact that, in amorphous materials, the related structural changes if any are far from being homogeneously distributed, and also because plasticity in amorphous samples does not always induce easily recognizable structural changes [6]. In this context, numerical simulations at the atomic scale [7–14] combined with theories based on the existence of localized plastic rearrangements identified by their residual strains [15, 16] have allowed building in the last thirty years a theoretical picture of plasticity in amorphous materials without referencing to structurally visible defects such as dislocations. The precise signature of plasticity is however strongly dependent on the composition of the glass, that is on the nature of the bonding (degree of covalency for example [6]), on the atomic composition [17, 18], as well as on its thermo-mechanical history [19–21].

Despite the general belief that amorphous materials like glasses are brittle, they indeed all develop plasticity, either at small or large scale, depending on the temperature, and on the applied load (pressure, strain rate). It was measured already in the sixties that silica glasses exhibit plasticity without volume conservation [22]. A homogeneous plastic deformation of 70% (see Figure 1) was recently observed at the micrometer scale, in micropillar experiments in a silica glass [23–25]. In normal glasses, like sodo-silicate glasses, shear bands were observed under the same conditions [17, 20]. At finite temperatures [26] or under high frequency acoustic waves [27], some metallic glasses can even exhibit superplasticity, with up to 160% plastic flow strain without breaking, at centimeter scales. Plasticity may also result from the accumulation of shear bands [28] and may be increased through thermal treatments [29]. Plasticity is thus far from being negligible in the mechanical response of amorphous materials. It is however always related to very high loads and high hardness. Metallic glasses for example are harder than high-resistance steels [30], and more generally amorphous materials are harder than their crystalline counterpart [31]. But this does not mean the absence of plasticity: this only means that amorphous materials support very large amounts of stresses (10 GPa) and strains (10%), before a macroscopic plastic flow. We will see later, that localized plasticity is indeed already present in the linear, apparently reversible, part of the stress–strain curve. After some progressive yielding transition, a plastic plateau related to a macroscopic plastic flow is visible. When the plastic flow is localized along shear bands, the macroscopic stress–strain curve exhibits a well marked overshoot that will be discussed later [13]. The beginning of a macroscopically measurable plastic flow associated with some amount of global residual strain is usually identified by the determination of elasto-plastic frontiers called yield surfaces. Depending on the composition, these yield surfaces show various shapes, indexed by the global irreversible densification [25, 32].

The present article will give a brief overview of the numerical and experimental measurements of plasticity at different scales in amorphous materials, and related models. In this field, many models have been proposed [33–35], but only few of them allow a coherent and pragmatic picture of the right physical processes at play. In the first part, we will discuss the representations of the local atomistic signature of plasticity in amorphous materials. In the second part, we will show that plasticity is the signature of a mechanical instability, in the presence of disorder. In the last part, we will compare different theoretical explanations for the yielding transition and shear banding in glasses. Finally we will conclude by discussing the possibilities to measure the amount of plastic deformation in an amorphous material knowing only its initial and final configurations, as well as to predict the shape and position of plastic rearrangements before they occur.

2. Local atomistic signature of plasticity

The main difficulty related to the identification of an atomistic signature for plasticity in amorphous materials, is related to the lack of simple reference configuration, contrary to crystals. In crystals, the reference configuration is ordered over sufficiently large scales to imprint homogeneously the electronic beams in Transmission Electron Microscopes for example. As such, dislocation lines can be revealed by inclining the incident beam in the proper direction over the sample allowing direct *in situ* imaging [39], and more recently by 3D tomography including the informations on crystal orientation in scanning electron microscopy (SEM) [40]. In amorphous mate-

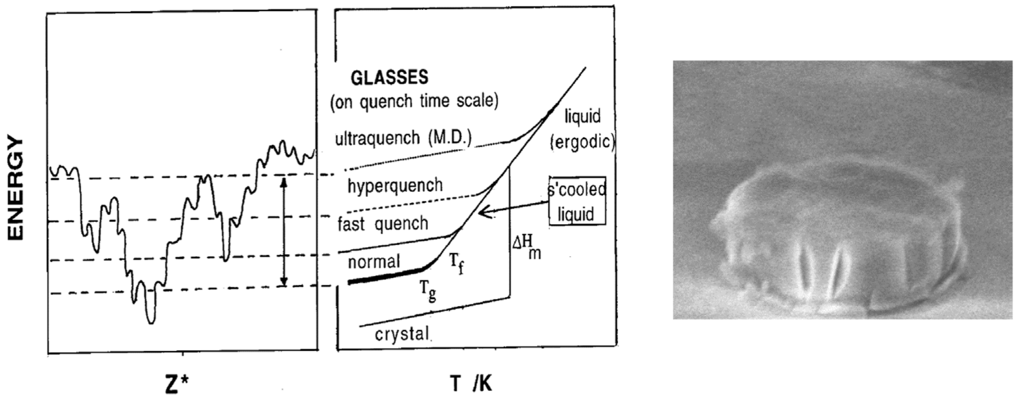

Figure 1. Left: schematic view of the potential energy landscape in the phase space [36,37]. Reproduced with permission from Angell [36]. Copyright (2021) by AIP Publishing. Right: large plastic deformation in a pure silica micro-pillar compressed with a micro-indentor, as detailed in [24]. Private image published with the permission of the author Kermouche.

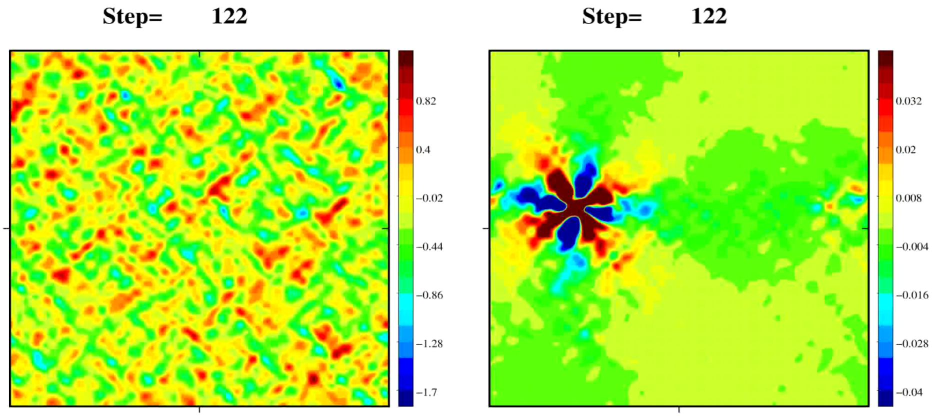

Figure 2. Left: quenched shear stresses σ_{xy} in a model 2D Lennard-Jones glass. Right: incremental shear stress $\delta\sigma_{xy}$ when the same sample is submitted to simple shear with applied $\delta\epsilon_{xy}$ strain–stresses are indicated in Lennard-Jones units as detailed in [38]. Private data published with the permission of the authors Tsamados and Tanguy.

rials, the reference configuration is metastable [36]. It is irregular down to the atomic scale, and this irregularity does not result from mechanical processes, but from cooling conditions since it often reflects the frozen structure of the liquid state [41]. Moreover, this metastable configuration is not unique but there are many equivalent configurations separated by only small energy barriers (Figure 1). It is interesting to note that amorphous samples at mechanical equilibrium in the reference configurational state, are mechanically very heterogeneous (Figure 2). It is characterized first, by a large distribution of quenched (residual) stresses with spatial correlations along some kind of inclined grid (Figure 2) resulting from the fast cooling process, and more precisely from temporal correlations induced by local shear motion and short range elasticity in the liquid state just before the rapid cooling [42]. This distribution of quenched stresses depends on the preparation protocol, such as the quenching process. The level of quenched stresses is very high. It may be more than ten times larger than the incremental stress resulting from ex-

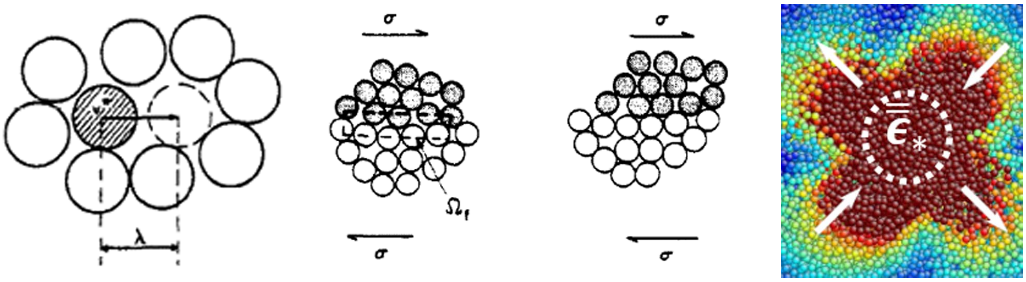

Figure 3. Left: schematic view of the free volume theory from [15]. Reproduced with permission from Spaepen [15]. Copyright (2021) by Elsevier. Middle: schematic view of shear transformation zones theory from [16]. Reproduced with permission from Argon [16]. Copyright (2021) by Elsevier. Right: local displacement field around a plastic rearrangement during the quasi-static athermal simple shear of an amorphous silicon sample from [51]. Private image published with the permission of the authors Albaret and Tanguy.

ternal deformation (*incremental stress* refers to the almost linear local elastic stress increase resulting from the small applied strain step, also referred to as stress rate in some articles [43]), but contrary to the latter, it has neither a significant role on the anomalies of vibrational density of states [38], nor on the local criteria for plastic rearrangements [44]. The incremental stresses have a narrower distribution, and are in general undistinguishable in the landscape of quenched stresses. They look commonly decorrelated from the local values of the residual stresses (Figure 2). However, they play a more important role on the relaxation processes and irreversible motion in glasses [14, 44–47]. Incremental stresses are indeed related to local elastic moduli in Hooke's description of elasticity, and thus to the Hessian matrix (second order derivative) of the Hamiltonian, or indirectly to the dynamical matrix of the system used to determine the instability thresholds at a microscopic level [48]. Spatial heterogeneity in the elastic moduli is responsible for high-frequency (THz) waves scattering in the continuum description of inertial dynamics in heterogeneous solids. These effects suggest a method to predict and measure local irreversible plasticization [49, 50], independent of a static measurement of the atomic scale structure.

By analogy with dislocations, Spaepen [15] and then Argon [16] have proposed two kinds of simple local rearrangements that could carry plastic deformation in amorphous solids. The first one is related to the free volume theory. It consists in voids able to diffuse and flow in the systems and whose activation theory allows recovering the transition from homogeneous to inhomogeneous flow for different strain rates. The second one assumes that residual strains are induced inside shear transformation zones, directly inspired by the theory of disk shape dislocation loops at low temperature and by the 3D spherical Eshelby inclusions theory [53] at higher temperatures (Figure 3). The recent progresses in molecular dynamics numerical simulations, have allowed to store a sufficiently large amount of atomistic data at different strain rates (including athermal quasi-static simulations) to analyze significantly the irreversible rearrangements taking place in an amorphous solid submitted to an external mechanical load, up to few tens of percents of external strain [6, 8, 11, 51, 54–61]. From these simulations, it was definitely proved that plastic deformation was supported uniquely in the core of nanometric size Eshelby like inclusions (Figure 3), with a global residual plastic strain given by the cumulated sum of inclusion eigenstrains [62] (Figure 4). Depending on the glass composition, or cooling procedure, the plastic strain can have a volumic (hydrostatic) and a deviatoric (shear) component. Plasticity in amorphous materials does not take place necessarily at constant volume, as already mentioned. Especially, in silica glasses, it has been related to a local phase transition such as a β to α-cristobalite

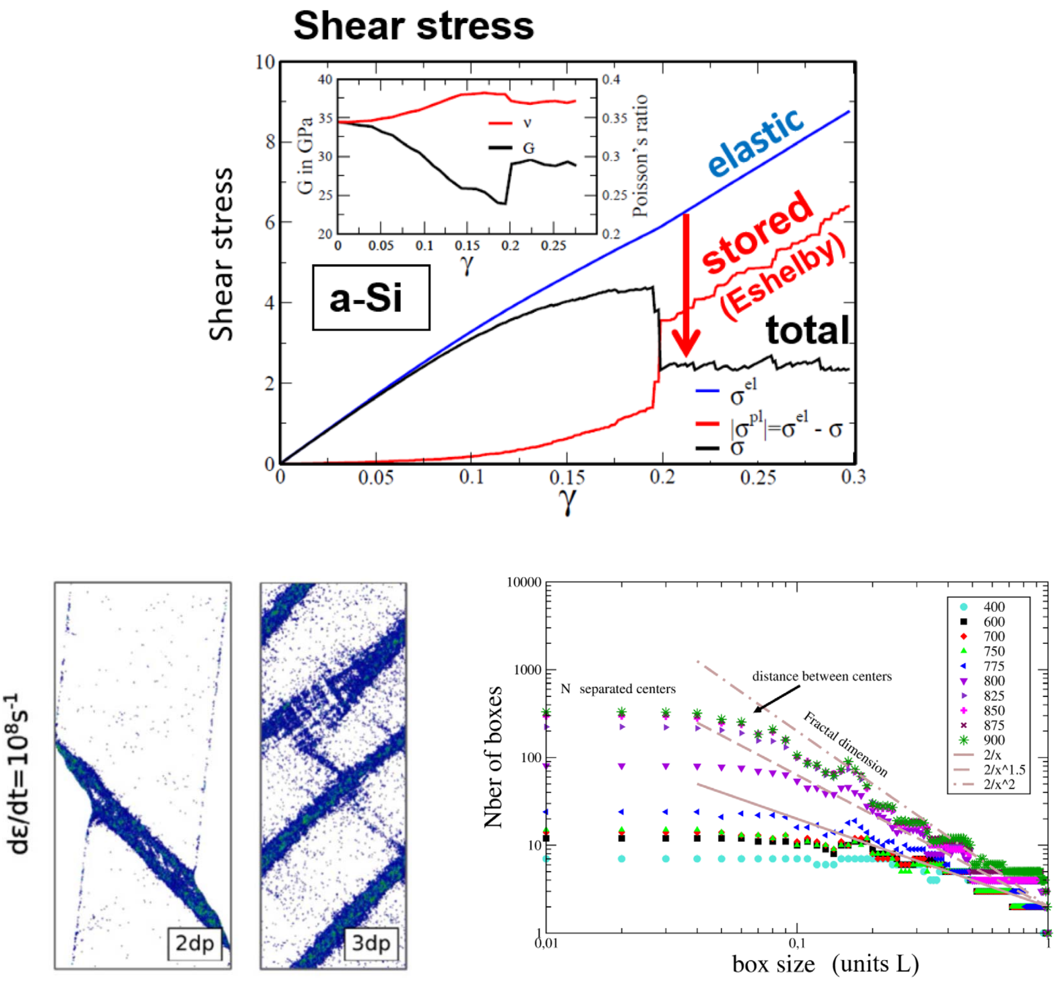

Figure 4. Top: constitutive law and cumulative stress stored inside Eshelby inclusions in a model amorphous Si sample, from [51] (inset: elastic moduli). Private image published with the permission of the authors Albaret and Tanguy. Bottom left: permanent shear band in atomistic models of amorphous bulk metallic glasses (2dp: with free surface, 3dp: with periodic boundary conditions) from [52]. Reproduced with permission from Albe *et al.* [52]. Copyright (2021) by Elsevier. Bottom right: unfolding of shear band from a single vortex, in a model amorphous Si sample: analysis of the Minkowski fractal dimension of the positions of Eshelby cores during the unfolding of the shear band instability, from $D = 1$ to $D = 2$ in a 3D sample. Data from [51]. Private image published with the permission of the authors Albaret and Tanguy.

structure [63], or more generally to a structural buckling process [64–67]. In an original way, the occurrence of the first irreversible rearrangements can occur at very low applied strains [68]. This means that the initial linear behaviour in the stress–strain curve (Figure 4) is not the signature of a non-dissipative process, but rather results from a constant nucleation rate of local irreversible rearrangements, that is not sufficient to induce a measurable plastic strain in the loading direction, even if it may change the value of the linear slope due to the related stress drops [69, 70]. Such a microplastic behaviour [71] was indeed already measured in Brillouin scattering experiments [72, 73], as a source for acoustic absorption, and anomalous pressure sensitivity of the apparent bulk modulus in silica glasses. As will be seen later, the energy barrier crossed to get such isolated rearrangements is very small ≈0.01 eV. Sometimes however, large dissipative events made of a 2D map of local rearrangements (elementary shear bands) may occur. A persistent shear band will lead to a permanent localized deformation which eventually progressively enlarges diffusively upon external load [58, 74, 75]. The spatial localization of plastic events is still a matter of debate. In sodo-silicate glasses for example [32, 70], plastic rearrangements are driven by the motion of sodium ions that are less attached to the atomic network and more mobile, thus creating large local strains easy to convert into dissipative events. In the next section, we will discuss the criteria for the nucleation of plastic rearrangements, whatever their size.

3. Plasticity as a mechanical instability

According to the strict mechanical classification, plasticity is a time-independent process giving rise to energy dissipation [62]. The origin of plastic deformation can be found in instability processes with hysteretic behaviour, as it happens for some types of bifurcations or at the spinodal limits [76–79].

This is the idea behind the old Hill's criterion for materials stability [80] written within the framework of continuum media. Hill's criterion is a condition of stability for the Helmholtz free energy

$$F(Y) = F(X) + \Omega(X)\left\{\tau_{ij}(X)\eta_{ij} + \tfrac{1}{2}C_{ijkl}(X)\eta_{ij}\eta_{kl}\right\}$$

where τ_{ij} is the Lagrangian stress tensor, η_{ij} are the strain components and C_{ijkl} are the elastic moduli. When the deformation is mainly supported by dislocation-like defects, $\eta_{ij} \propto b_i n_j$ where b_i are the Burgers vector components, and n_j are the components of the vector normal to the slip plane. The stability criterion may then be written as

$$\min_{\{b_i, n_j\}} (C_{ijkl} b_i b_k n_j n_l) \geq 0. \tag{1}$$

The limiting case were the inequality in (1) is replaced by an equality marks the onset of instability. Although it was originally developed for macroscopic behavior [43], this criterion was successfully applied to atomistic numerical simulations of crystalline samples under indentation load, to identify the preferred directions for dislocation nucleation, over a limited number of possibilities and with a simplified description of the Helmholtz free energy involving a semi-local acoustic tensor in place of the stiffness tensor [81]. This criterion was also successfully applied to locate plastic rearrangements in model amorphous materials [48], where it was shown a rapid decay to zero of one eigenvalue of the $3N \times 3N$ atomistic dynamical matrix (N is the number of atoms)

$$M_{ij}^{\alpha\beta} = \frac{1}{\sqrt{m_i m_j}} \frac{\partial^2 H}{\partial r_i^\alpha \partial r_j^\beta}$$

responsible for mechanical stability at the atomic level, with r_i^α the position of the particle i in the direction α and m_i its mass. The decay of the eigenvalue appeared at a very small strain 10^{-5} before a plastic rearrangement occurs. As attempted, the plastic rearrangement is initiated in the direction given by the corresponding eigenvector of the dynamical matrix (each line being multiplied by the square root of the corresponding atomic mass). In other terms, it is driven by the vibration mode associated to the smallest vibrational frequency, that becomes unstable at the plastic threshold. In general, these low frequency modes, or "soft modes", are visualized by high strains located inside a nanometric size sphere (the difference with the Eshelby inclusions being due to the fact that before the instability, there is no residual strain inside the core, and thus no interface and no force discontinuity). This theoretical agreement was however difficult

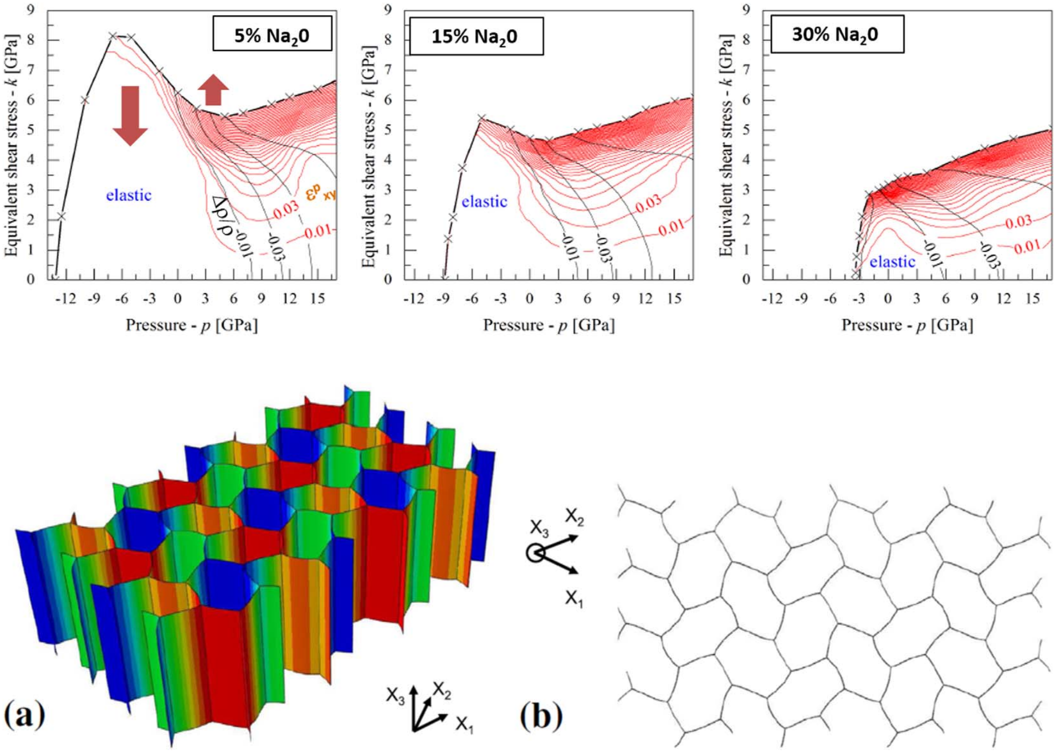

Figure 5. Top: yield surfaces in sodo-silicate glasses for different amounts of residuals plastic strain and different compositions, from [32, 70]. Private figure published with the permission of the authors Molnar and Tanguy. Bottom: buckling of a cellular material: finite element calculation showing the critical mode for isotropic-section hexagonal honeycomb under axial compression with constrained lateral expansion (a) isometric and (b) top view [89]. Reproduced with permission from Lopez Gimenez and Tryantafyllidis [89]. Copyright (2021) by Elsevier.

to detect experimentally due to the very small strain values allowed for its anticipation, and more hope was placed in the local measurement of the elastic moduli, plastic rearrangements taking place preferentially at low values of the Elastic moduli, already identified a few tens of percents strain before the plastic rearrangement occurs [45]. Other authors prefered to refer to "soft spots" [82–84] or to "local non-affinity" [85–87], but, except in very specific cases, the places susceptible to support these large strains (or low elastic moduli), are not simply related to easily identified structural characteristics. This is quite understandable, since they result from a collective vibrational process. Recently, using the nudged elastic band method to measure numerically the local energy barrier separating two metastable equilibrium positions in a large sample of amorphous silicon, Albaret and Rodney [61] have checked that the energy barrier disappears progressively when approaching the critical strain at which a plastic rearrangement occurs. This is in perfect agreement with the description of plasticity as an instability process. Interestingly, they also have observed that the largest energy barrier that is reached between successive plastic rearrangements is about 10 meV only, in agreement with the estimations of the (very small) energy barriers involved in the anharmonic interpretation for Brillouin absorption in glasses [88] and thus to our previous description of microplasticity.

As such, plasticity is thus strictly equivalent to a buckling process, that would take place at the atomic scale [90]. Buckling is indeed given by the cancellation of one eigenvalue in the actual stiffness matrix. Such collapse is known as been a collective process, due to the long-range elasticity taking place in solids. It is also well known by mechanical engineers, that in large scale heterogeneous materials, like cellular or architectured materials for example, buckling process can give rise to a large set of complex collective modes, depending on the material properties, on the orientation of the external load, or on the shape of preexisting defects. In honeycomb lattices for example, the buckling mode shows different degrees of localization, depending on the load applied [89, 91] (see Figure 5). Moreover, it was shown at the buckling threshold, that the normal strength decreases with the applied shear strength, similarly to the yield surfaces observed in silica glasses [25, 32] (see Figure 5). It is clear with such a description, that there is no reason for plasticity to be an isochoric process.

Silica glasses have already been compared to atomistic foams for their gas absorption properties [92]. Their elasto-plastic mechanical properties reinforce this analogy. Already pointed out by Brown [64] and Lambropoulos *et al.* [65], this has been checked recently using a precise measurement of the yield surface with molecular dynamics simulations performed at different pressures [67]. Interestingly, the shape of the yield surface is pressure and composition dependent as can be seen in Figure 5 and in Ref. [70]. By looking in details at the microscopic processes taking place inside the system when entering in the plastic flow zone, it has been shown that the number and the size of plastic cores increase with the pressure, both effects favoring global plastic flow when pressure is increased [67, 70]. But, in the presence of sodium ions immersed in the silicon-dioxyde skeleton, the large majority of plastic events (at least 90%) is supported by these mobile ions, plastic cores containing more ions that the average, especially in the early stages of plasticity where plastic events looked centered on isolated pockets especially rich in sodium [70]. When macroscopic plastic flow occurs, the plastic events are located inside ionic channels, with a sodium density inside the plastic cores only slightly higher than the average one. Note that in the plastic flow regime, the global structural changes in pure silica glasses are sufficient to affect the vibration modes homogeneously at the scale of the sample, giving rise to a well defined signature in the Raman spectrum of the silica glasses, thus confirming the collective character of the related buckling process. However, in some other cases the plastic residual deformation localizes along 2D shear bands. In the next chapter, we will review different explanations for the occurrence of shear bands.

4. Shear banding, or not shear banding

We have seen before, that any dissipative plastic rearrangement is initiated by a plastic instability promoted by a low frequency vibration mode. During the unfolding of the instability [76, 77], residual plastic strain will take place, leading to a new equilibrium position. In some cases, a large event spanning the whole system size may occur before reaching the new equilibrium position. These 2D events in 3D systems, are called “elementary” shear bands [11]. In the athermal quasi-static case, when an elementary shear band occurs, it can formally be decomposed into a succession of Eshelby inclusions aligned along a plane, but these Eshelby inclusions appearing dynamically cannot be separated in a succession of equilibrium positions at constant external load. They are also referred to “avalanches” [93] or “embrionic shear bands” in mesoscopic models of plasticity [52]. It was shown in our molecular dynamics simulations of model amorphous silicon, that the Eshelby inclusions first align, forming a single (1D) vortex, and then activate an alignment of vortices along a plane crossing the whole system (Figure 4). When looking at the vibration modes and at the elastic moduli before such a large event occurs, it is shown that the Eshelby inclusions included in the band, were all included in a set of low frequency vibration modes, whose large strains were often supported by an alignment of low elastic (shear) moduli, percolating along the sample [45]. The occurrence of such a large event induces a large global stress drop $\propto L^2/L^3$ and thus affects the stress–strain relation. The succession of many such events induces a progressive saturation of the stresses, characteristic of the entry into a flow regime, also called the "yielding transition" (Figure 4).

The yielding transition in the plasticity of amorphous materials, can thus be seen at a microscopic level as resulting from the succession of instabilities which manifest themselves as localized rearrangements with various sizes. Each of these rearrangements induces a residual plastic strain, and the yielding transition does not characterize the occurrence of the first dissipative process, but rather a sudden acceleration of plasticity indexed on the residual plastic strain. It was observed with a very good precision [94], that when approaching the yielding transition, the distribution of energy barriers shows a progressive change marked by an increase in the number of the very low energy barriers, thus making the mechanical stability more and more marginal, a bit like in a liquid. Interestingly, the crossing of energy barriers is not facilitated, as in a liquid, from an almost isotropic temperature effect, but from an increase in the number of low energy mechanical paths. By counting the number of cycles with a given strain amplitude to reach a limit cycle under oscillatory shear, Regev *et al.* [95] have shown that the critical strain for plastic flow corresponds to the strain amplitude marking the transition to chaos (infinite number of oscillatory cycles). The yielding transition can also be seen as the percolation of a sufficient number of growing clusters of non-affine deformations, as already mentioned above and also observed experimentally in colloidal glasses [87]. The evolution of the fraction of "mobile" (strongly non-affine) cells spanning the entire amorphous solid as a function of the applied strain, belongs to the universality class of directed percolation and shows evidence of a critical strain acting as a well marked cross-over between two kinds of behaviours: global arrest and collective flow [96]. The yielding transition thus appears as a critical phenomenon, as already suggested thirty years ago by Roux and Hansen [97] on the basis of cellular automaton models of plasticity. Cellular automaton simulations, have now been replaced by more detailed mesoscopic models inspired by dislocation dynamics calculations, describing the accumulation of plasticity as a succession of localized (Eshelby like) events [98] triggered by a realistic description of the long range elasticity in solids [99–103], sometimes combined with different kinds of relaxation processes. The competition between long-range elasticity and disorder (in the local yield stress for example, or in the local residual strain) will converge progressively into a collective organization with either sparse or localized plasticity. Under an imposed stress, the yielding transition corresponds to the divergence of avalanche sizes following an apparent universal distribution [103].

Another transition occurs sometimes at larger strain: under specific conditions, the plastic deformation localizes in the flow regime along a permanent or "mature" shear band. The transition from sparse plasticity to localized plasticity along a permanent shear band refers to a ductile to brittle transition. The unfolding of a permanent shear band is usually related to the occurrence of a stress overshoot in the stress–strain relation, that can give rise to brittle crack under an imposed stress (Neumann boundary conditions) [20, 104]. Permanent shear banding is a very different concept from the elementary shear banding discussed above. Indeed, it is an object that is built gradually, often from an accumulation of elementary sheared bands, but not necessarily. It can appear for example from isolated events, in a succession of oscillatory forcings under specific temperature and structural conditions [105, 106] and its final shape can appear very gradually [107]. Its progressive accumulation share some analogies with damage. Permanent shear bands if any, are thus established after the yielding transition. The different regimes of plastic deformations are summarized in the schematic diagram shown in Figure 6. The occurrence of permanent (mature) shear band depends on numerous factors. Intrinsic factors may be the directionality of bounds [6], the ease of creating free volume or coordination defects inside the glass [108–110], the aging or stability of the glass (for example the initial annealing) [93, 111–114],

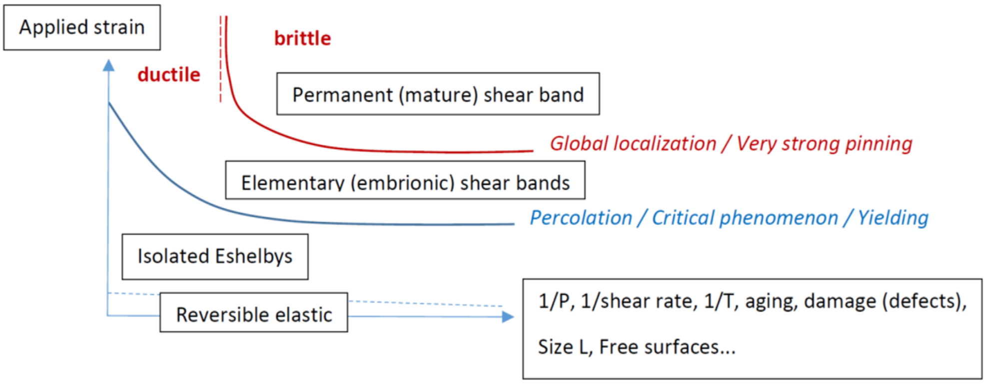

Figure 6. Schematic description of the different mechanical responses of amorphous materials to a prescribed strain.

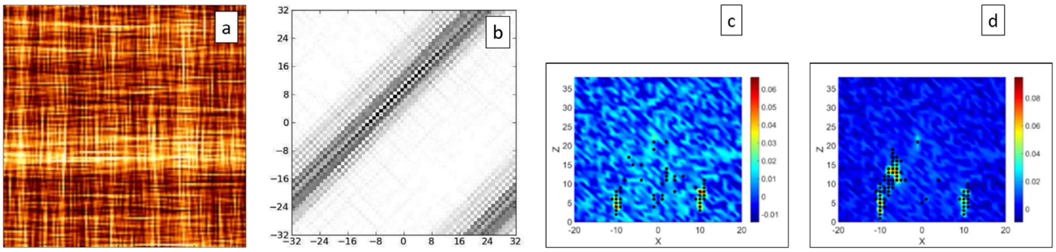

Figure 7. Mesoscopic modeling of plasticity in amorphous materials. (a) Shear banding at the yielding transition from [103]. Reproduced with permission from Budrikis *et al.* [103]. Open Access article. (b) Permanent shear banding from [111]. Reproduced with permission from Vandembroucq *et al.* [111]. Copyright (2021) by APS Publisher. (c–d) Sparse plasticity and shear banding under a nanoindentor for random strain thresholds in $[\epsilon_c, \epsilon_c + Q]$ with (c) $Q = 0.015$ and (d) $Q = 0.01$ from [121]. The indentor is applied on the bottom side of the figure. A set of initial defects representing the adhesion are imposed at the boundary of the contact. In both figures (c–d) the inverse distance to threshold in the local deviatoric strain is shown, after 20 successive plastic events. The location of the past plastic events is identified with black circles. The next plastic event will take place at the smallest distance to threshold (brightest color). Private figure published with the permission of the authors Chen and Tanguy.

the ease to generate avalanches (also refered to positive or negative value of a "weakening parameter") [115]. Extrinsic factors at play are the temperature and the strain rate (both encouraging a more uniform plasticity) [52, 75, 116–118], as well as the size of the system (free surfaces acting as low energy heterogeneous initiators for plastic rearrangements) [52, 119]. The fact that temperature and strain rate have analogous effect on the homogeneity of the deformation can be explained by two factors: first the fact that high frequency mechanical waves in amorphous materials are known to promote local heating [120], and thus act as thermal activators for new sites of plasticity, and second the fact that the activated instabilities have not enough time to relaxe before the activation of new instabilities [59], resulting in a permanent high energy excitation.

While continuous modelizations of this ductile to brittle transition involves a bifurcation [109, 122], it is difficult to quantitatively identify the microscopic mechanisms at the origin of this transition. While some authors claim the existence of universal mechanisms for plastic flow in amorphous materials [103], it seems that these universal features, apart from the yielding transition, result in fact from specific conditions between energy barrier fluctuations and elastic energy. For example, the case of shear banding in metallic glasses under a nanoindenter was extensively discussed [123–125]. However, the universal features observed in [103] are restricted to a small part of the sample. Moreover, the existence of curved elementary shear bands is not obvious at all in this geometry. It depends on the width of the distribution of local yielding energies, compared to the spatial variations of elastic energy [121]: the strong disorder case being insensitive to the specificities of the loading and giving rise to sparse homogeneous plasticity, independent of the presence of the indentor. The extrinsic contact to the indentor (friction, adhesion) is also crucial in this case, since friction or adhesion may act as heterogeneous initiators for plasticity at the surface (Figure 7). Finally, curved permanent shear bands may occur only for a specific amplitude of stress fluctuations compared to continuous stress gradients induced by the indentor inside the sample. Mesoscopic modeling of plasticity is actually described as a depinning phenomenon of an elastic manifold [126], the applied strain acting as a control parameter similar to time, and local plasticity refering to local non-linear depinning giving rise to a local increase of the residual plastic strain (order parameter) in competition with recovering elasticity [126]. As kown for a long time, two different regimes may occur: the weak pinning, and the strong pinning regimes. The weak pinning regime gives rise to collective flow [127], and the strong pinning regime corresponds to sparse localized plasticity. Permanent shear banding would correspond to the extreme case of a very strong pinning with a localisation of plasticity always along the same self-sustaining soft site. For usual disorder, such a situation can only occur with the help of a self-sustaining weakening parameter, or local damage function helping the focus of activity on a single site.

5. Perspectives

To conclude this brief review of recent progresses made on the description of plasticity in amorphous materials, let's first remind the main results. The lack of long-range structural order in amorphous materials forbids the formation of linear defects like dislocations. Elementary processes have been identified in amorphous materials that are equivalent to dislocations in crystals [128]: the Eshelby inclusions, with residual plastic strain confined in a 3D nanometric volumic core. Pennyshape Eshelby inclusions are dislocation loops [53]. In amorphous materials however, the structural signature of the Eshelby inclusions is not always visible. Moreover, they occur for a very small amount of imposed strain, due to the very small amplitude of energy barriers separating the neighboring metastable states in the phase space. Any plastic rearrangements result from an instability, or buckling process. It corresponds also to the loss of positive definiteness of the material tangential stiffness tensor, that is known to induce a loss of ellipticity of the governing differential equations under quasi-static loading, when discontinuous solutions emerge [43]. This continuous description is however unable to identify secondary equilibrium positions and the energy barriers between them and is thus caught in the trap of endless mesh refinement. This is why it is important to base these models on a properly informed microscopic description. Here, we show that before the yielding transition characterizing the beginning of macroscopic plastic flow, amorphous materials undergo the occurrence of several local plastic rearrangements that organize progressively into percolating clusters at the yielding threshold. Two different kinds of transitions occur: first the yielding threshold, and then, depending on the extrinsic or intrinsic capacity of self-sustainability of elementary shear bands, a possible transition to a brittle regime with permanent shear band. The first transition is shown as a (simple) universal critical phenomenon, but the second one yields specific materials properties or loading conditions, difficult to

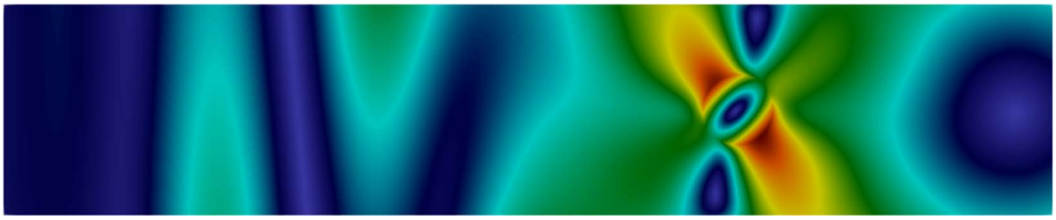

Figure 8. Finite Element calculation of the scattering of an acoustic wave by an Eshelby inclusion from Luo *et al.* (2020) [137]. Private figure published with the permission of the authors Luo, Giordano, Gravouil and Tanguy.

infer without acquiring a large set of experimental data. Among common observations, temperature or strain rate favor homogeneous plastic flow, while aging or mechanical stability, system size and free boundaries will favor the generation of permanent shear band.

These results open new perspectives for materials design. New design of surfaces have been used for example to control the plastic response of brittle bulk metallic glasses [129, 130]. The misalignment of the principal directions of the inclusions eigenstrains as compared to those of the applied strain is a way to explain and to quantify the Baushinger effect (asymmetry of the stress response between positive and negative applied strains) [131, 132]. In general, the qualitative dependence of the plastic flow on the parameters identified in Figure 6 from experimental and atomistic simulations, should not be forgotten in the search for new models, such as accelerated dynamics models [133, 134], and definitely guide the efforts to build them.

Finally, the presence of defects in the permanent shear bands makes them visible with spectroscopic measurements, like Raman spectroscopy [21, 135]. In the absence of permanent shear banding, the description of plasticity in terms of Eshelby inclusions, as well as the connection between heterogeneous elasticity and soft zones, should make it visible (see Figure 8) with high frequency acoustic scattering measurements [49] or using non-linear acoustics [136], thus making possible in the future the non-destructive identification and prediction of plasticity in amorphous materials.

Acknowledgement

This work was supported by the French National Research Agency (ANR) and Ministère de l'Education Nationale et de la Recherche.

References

[1] P. Y. Huang *et al.*, "Imaging atomic rearrangements in two-dimensional silica glass: Watching silica's dance", *Science* **342** (2013), p. 224-227.

[2] C. Buchner *et al.*, "Ultrathin silica films: The atomic structure of two-dimensional crystals and glasses", *Chem. Eur. J.* **20** (2014), no. 30, p. 9176-9183.

[3] J. Li, Z. L. Wang, T. C. Hufnagel, "Characterization of nanometer-scale defects in metallic glasses by quantitative high-resolution transmission electron microscopy", *Phys. Rev. B* **65** (2002), article no. 144201.

[4] H. F. Poulsen, J. A. Wert, F. Neuefeind, V. Honkimaki, M. Daymond, "Measuring strain distributions in amorphous materials", *Nat. Mater.* **4** (2005), p. 33-36.

[5] T. C. Hufnagel, R. T. Ott, J. Almer, "Structural aspects of elastic deformation of a metallic glass", *Phys. Rev. B* **73** (2006), article no. 064204.

[6] C. Fusco, T. Albaret, A. Tanguy, "Role of local order in the small-scale plasticity of model amorphous materials", *Phys. Rev. E* **82** (2010), article no. 066116.

[7] C. A. Schuh, A. C. Lund, "Atomistic basis for the plastic yield criterion of metallic glasses", *Nat. Mater.* **2** (2003), p. 449-452.

[8] C. Maloney, A. Lemaitre, “Subextensive scaling in the athermal, quasistatic limit of amorphous matter in plastic shear flow”, *Phys. Rev. Lett.* **93** (2004), article no. 016001.

[9] J. Rottler, M. O. Robbins, “Unified description of aging and rate effects in yield of glassy solids”, *Phys. Rev. Lett.* **95** (2005), article no. 225504.

[10] F. Albano, M. L. Falk, “Shear softening and structure in a simulated three-dimensional binary glass”, *J. Chem. Phys.* **122** (2005), article no. 154508.

[11] A. Tanguy, F. Leonforte, J. L. Barrat, “Plastic response of a 2D Lennard–Jones amorphous solid: Detailed analyses of the local rearrangements at very slow strain rate”, *Eur. Phys. J. E* **20** (2006), p. 355-364.

[12] I. Procaccia, “Physics of amorphous solids: Their creation and their mechanical properties”, *Eur. Phys. J.: Spec. Top.* **178** (2009), p. 81-122.

[13] D. Rodney, A. Tanguy, D. Vandembroucq, “Modeling the mechanics of amorphous solids at different length scale and time scale”, *Model. Simul. Mat. Sci. Eng.* **19** (2011), article no. 083001.

[14] D. Richard *et al.*, “Predicting plasticity in disordered solids from structural indicators”, *Phys. Rev. Mater.* **4** (2020), article no. 113609.

[15] F. Spaepen, “A microscopic mechanism for steady state inhomogeneous flow in metallic glasses”, *Acta Metall.* **25** (1977), p. 407-415.

[16] A. S. Argon, “Plastic deformation in metallic glasses”, *Acta Metall.* **27** (1979), p. 47-58.

[17] T. M. Gross, M. Tomozawa, A. Koike, “A glass with high crack initiation load: Role of fictive temperature-independent mechanical properties”, *J. Non-Cryst. Solids* **355** (2009), p. 563-568.

[18] G. Molnár, P. Ganster, J. Török, A. Tanguy, “Sodium effect on static mechanical behavior of MD-modeled sodium silicate glasses”, *J. Non-Cryst. Solids* **440** (2016), p. 12-25.

[19] W. Li, Y. Gao, H. Bei, “On the correlation between microscopic structural heterogeneity and embrittlement behavior in metallic glasses”, *Sci. Rep.* **5** (2015), article no. 14786.

[20] T. M. Gross, M. Tomozawa, “Fictive temperature-independent density and minimum indentation size effect in calcium aluminosilicate glass”, *J. Appl. Phys.* **104** (2008), article no. 063529.

[21] C. Martinet, M. Heili, V. Martinez, G. Kermouche, G. Molnar, N. Shcheblanov, E. Barthel, A. Tanguy, “Highlighting the impact of shear strain on the SiO_2 glass structure: From experiments to atomistic simulations”, *J. Non-Cryst. Solids* **533** (2020), article no. 119898.

[22] J. Mackenzie, “High-pressure effects on oxide glasses: I, densification in rigid state”, *J. Amer. Ceram. Soc.* **6** (1963), p. 461-470.

[23] R. Lacroix, G. Kermouche, J. Teisseire, E. Barthel, “Plastic deformation and residual stresses in amorphous silica pillars under uniaxial loading”, *Acta Mater.* **60** (2012), p. 5555-5566.

[24] G. Kermouche, G. Guillonneau, J. Michler, J. Teisseire, E. Barthel, “Perfectly plastic flow in silica glass”, *Acta Mater.* **114** (2016), p. 146-153.

[25] G. Molnár, G. Kermouche, E. Barthel, “Plastic response of amorphous silicates, from atomistic simulations to experiments — a general constitutive relation”, *Mech. Solids* **114** (2017), p. 1-8.

[26] Y. H. Liu, G. Wang, R. J. Wang, D. Q. Zhao, M. X. Pan, W. H. Wang, “Super plastic bulk metallic glasses at room temperature”, *Science* **315** (2007), p. 1385-1388.

[27] X. Li *et al.*, “Ultrasonic plasticity of metallic glass near room temperature”, *Appl. Mater. Today* **21** (2020), article no. 100866.

[28] J. Schroers, W. L. Johnson, “Ductile bulk metallic glass”, *Phys. Rev. Lett.* **93** (2004), article no. 255506.

[29] W. Song, X. Meng, Y. Wu, D. Cao, H. Wang, X. Liu, X. Wang, Z. Lu, “Improving plasticity of the $Zr_{46}Cu_{46}Al_8$ bulk metallic glass via thermal rejuvenation”, *Sci. Bull.* **63** (2018), p. 840-844.

[30] A. I. Salimon, M. F. Ashby, Y. Bréchet, A. L. Greer, “Bulk metallic glasses: What are they good for?”, *Mater. Sci. Eng. A* **375–377** (2004), p. 385-388.

[31] W. A. Phillips, *Amorphous Solids – Low Temperature Properties*, Springer, 1981.

[32] G. Molnár, P. Ganster, A. Tanguy, E. Barthel, G. Kermouche, “Densification dependent yield criteria for sodium silicate glasses: An atomistic simulation approach”, *Acta Mater.* **111** (2016), p. 129-137.

[33] J.-L. Barrat, A. Lemaitre, *Heterogeneities in Amorphous Systems Under Shear*, Oxford University Press, 2010.

[34] P. Sollich, “Rheological constitutive equation for a model of soft glassy materials”, *Phys. Rev. E* **58** (1998), p. 738-759.

[35] D. Pan *et al.*, “Experimental characterization of shear transformation zones for plastic flow of bulk metallic glasses”, *Proc. Natl Acad. Sci. USA* **105** (2008), p. 14769-14772.

[36] C. A. Angell *et al.*, “Vibrational dynamics and thermodynamics, ideal glass transitions and folding transitions, in liquids and biopolymers”, *AIP Conf. Proc.* **708** (2004), p. 473-482.

[37] C. A. Angell, “Energy landscapes for cooperative processes: nearly ideal glass transitions, liquid–liquid transitions and folding transitions”, *Philos. Trans. Royal Soc. A* **363** (2005), p. 415-432.

[38] M. Tsamados, A. Tanguy, F. Léonforte, J. L. Barrat, “On the study of local-stress rearrangements during quasi-static plastic shear of a model glass: Do local-stress components contain enough information?”, *Eur. Phys. J. E* **26** (2008), p. 283-293.

[39] I. M. Robertson, P. J. Ferreira, G. Dehm, R. Hull, E. A. Stach, “Visualizing the behavior of dislocations—seeing is believing”, *MRS Bull.* **33** (2008), p. 122-131.

[40] S. Yamasaki, M. Mitsuhara, K. Ikeda, S. Hata, H. Nakashima, “3D visualization of dislocation arrangement using scanning electron microscope serial sectioning method”, *Scr. Mater.* **101** (2015), p. 80-83.

[41] M. F. Ashby, A. L. Greer, “Metallic glasses as structural materials”, *Scr. Mater.* **54** (2006), p. 321-326.

[42] A. Lemaitre, “Structural relaxation in a scale-free process”, *Phys. Rev. Lett.* **113** (2014), article no. 245702.

[43] R. De Borst, *Encyclopedia of Computational Mechanics*, 2nd ed., John Wiley and Sons, New York, 2017, ch 10.

[44] M. Tsamados, A. Tanguy, C. Goldenberg, J. L. Barrat, “Local elasticity map and plasticity in a model Lennard-Jones glass”, *Phys. Rev. E* **80** (2009), article no. 026112.

[45] A. Tanguy, B. Mantisi, M. Tsamados, “Vibrational modes as a predictor for plasticity in a model glass”, *Europhys. Lett.* **90** (2010), article no. 16004.

[46] E. Lerner, S. Karmakar, I. Procaccia, “Athermal nonlinear elastic constants of amorphous solids”, *Phys. Rev. E* **82** (2010), article no. 026105.

[47] G. T. Barkema, N. Mousseau, “Event-based relaxation of continuous disordered systems”, *Phys. Rev. Lett.* **77** (1996), p. 4358-4362.

[48] C. Maloney, A. Lemaitre, “Universal breakdown of elasticity at the onset of material failure”, *Phys. Rev. Lett.* **93** (2004), article no. 195501.

[49] F. Barra, R. Espinoza-Gonzalez, H. Fernandez, F. Lund, A. Maurel, V. Pagneux, “The use of ultrasound to measure dislocation density”, *JOM* **67** (2015), p. 1856-1863.

[50] H. Luo, A. Gravouil, V. Giordano, A. Tanguy, “Thermal transport in a 2D nanophononic solid: Role of bi-phasic materials properties on acoustic attenuation and thermal diffusivity”, *Nanomaterials* **9** (2019), article no. 1471.

[51] T. Albaret, A. Tanguy, F. Boioli, D. Rodney, “Mapping between atomistic simulations and Eshelby inclusions in the shear deformation of an amorphous silicon model”, *Phys. Rev. E* **93** (2016), article no. 053002.

[52] K. Albe, Y. Ritter, D. Sopu, “Enhancing the plasticity of metallic glasses: Shear band formation, nanocomposites and nanoglasses investigated by molecular dynamics simulations”, *Mech. Mater.* **67** (2013), p. 94-103.

[53] J. D. Eshelby, “The determination of the elastic field of an ellipsoidal inclusion, and related problems”, *Proc. R. Soc. Lond. A* **241** (1957), no. 1226, p. 376-396.

[54] Y. Shi, M. L. Falk, “Strain localization and percolation of stable structure in amorphous solids”, *Phys. Rev. Lett.* **95** (2005), article no. 095502.

[55] C. Maloney, A. Lemaitre, “Amorphous systems in athermal, quasistatic shear”, *Phys. Rev. E* **74** (2006), article no. 016118.

[56] Y. Shi, M. B. Katz, H. Li, M. L. Falk, “Evaluation of the disorder temperature and free-volume formalisms via simulations of shear banding in amorphous solids”, *Phys. Rev. Lett.* **98** (2007), article no. 185505.

[57] K.-W. Park, C.-M. Lee, M.-R. Lee, E. Fleur, M. Falk, J.-C. Lee, “Paradoxal phenomena between the homogeneous and inhomogeneous deformations of metallic glasses”, *Appl. Phys. Lett.* **94** (2009), article no. 021907.

[58] M. L. Falk, C. E. Maloney, “Simulating the mechanical response of amorphous solids using atomistic methods”, *Eur. Phys. J. B* **75** (2010), p. 405-413.

[59] C. Fusco, T. Albaret, A. Tanguy, “Rheological properties versus local dynamics in model disordered materials at low temperature”, *Eur. Phys. J. E* **37** (2014), article no. 43.

[60] S. Patinet, D. Vandembroucq, M. L. Falk, “Connecting local yield stresses with plastic activity in amorphous solids”, *Phys. Rev. Lett.* **117** (2016), article no. 045501.

[61] F. Boioli, T. Albaret, D. Rodney, “Shear transformation distribution and activation in glasses at the atomic scale”, *Phys. Rev. E* **95** (2017), article no. 033005.

[62] A. Zaoui, D. Pineau, D. François, *Comportement Mécanique des Matériaux*, Hermes, Paris, 1995.

[63] L. Huang, J. Kieffer, “Molecular dynamics study of cristobalite silica using a charge transfer three-body potential: phase transformation and structural disorder”, *J. Chem. Phys.* **118** (2003), p. 1487-1498.

[64] C. J. Brown, “The elastic stability of square perforated plates under combinations of bending, shear and direct load”, *Thin-Walled Struct.* **4** (1986), no. 3, p. 239-246.

[65] J. C. Lambropoulos, S. Xu, T. Fang, “Constitutive law for the densification of fused silica, with applications on polishing and microgrinding”, *J. Am. Ceram. Soc.* **79** (1996), p. 1441-1452.

[66] A. Shorey, K. Xin, K. H. Chen, J. C. Lambropoulos, “Deformation of fused silica: nanoindentation and densification”, *Proc. SPIE* **3424** (1998), p. 72-81.

[67] B. Mantisi, G. Kermouche, E. Barthel, A. Tanguy, “Impact of pressure on plastic yield in amorphous solids with open structure”, *Phys. Rev. E* **93** (2016), article no. 033001.

[68] F. Leonforte, R. Boissiere, A. Tanguy, J. P. Wittmer, J.-L. Barrat, “Continuum limit of amorphous elastic bodies iii: three-dimensional systems”, *Phys. Rev. B* **72** (2005), article no. 224206.

[69] B. Mantisi, A. Tanguy, G. Kermouche, E. Barthel, “Atomistic response of a model silica glass under shear and pressure”, *Eur. Phys. J. E* **85** (2012), article no. 304.

[70] G. Molnár, P. Ganster, A. Tanguy, "Effect of composition and pressure on the shear strength of sodium silicate glasses: An atomic scale simulation study", *Phys. Rev. E* **95** (2017), article no. 043001.

[71] C. J. McMahon, *Microplasticity*, Interscience Publishers, New-York, 1968.

[72] S. Hau *et al.*, "Brillouin scattering of vitreous silica under high pressure", *Ann. Phys.* **4** (1995), p. 91-98.

[73] B. Rufflé, G. Guimbretiere, E. Courtens, R. Vacher, "Glass specific behavior in the damping of acoustic-like vibrations", *Phys. Rev. Lett.* **96** (2006), article no. 045502.

[74] E. R. Homer, C. A. Schuh, "Mesoscale modeling of amorphous metals by shear transformation zone dynamics", *Acta Mater.* **57** (2009), p. 2823-2833.

[75] E. R. Homer, C. A. Schuh, "Three-dimensional shear transformation zone dynamics model for amorphous metals", *Model. Simul. Mat. Sci. Eng.* **18** (2010), article no. 065009.

[76] T. Poston, I. Stewart, *Catastrophe Theory and its Application*, Pitman, London, 1978.

[77] Y.-C. Lu, *Singularity Theory and an Introduction to Catastrophe Theory*, Springer-Verlag, 1976.

[78] J. Guckenheimer, P. Homes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcation of Vector Fields*, 3rd ed., Springer-Verlag, New-York, 1997.

[79] H. J. Jensen, Y. Bréchet, B. Douçot, "Instabilities and correlations of an elastic lattice in a random potential", *Europhys. Lett.* **23** (1993), no. 9, p. 623-628.

[80] R. Hill, *The Mathematical Theory of Plasticity*, Clarendon Press, Oxford, 1950.

[81] R. E. Miller, D. Rodney, "On the nonlocal nature of dislocation nucleation during nanoindentation", *J. Mech. Phys. Solids* **56** (2008), p. 1203-1223.

[82] M. L. Manning, A. Liu, "Vibrational modes identify soft spots in a sheared disordered packing", *Phys. Rev. Lett.* **107** (2011), article no. 108302.

[83] A. Ghosh, V. Chikkadi, P. Schall, D. Bonn, "Connecting structural relaxation with the low frequency modes in a hard-sphere colloidal glass", *Phys. Rev. Lett.* **107** (2011), article no. 188303.

[84] J. Ding *et al.*, "Soft spots and their structural signature in a metallic glass", *Proc. Natl Acad. Sci. USA* **111** (2014), p. 14052-14056.

[85] M. L. Falk, J. S. Langer, "Dynamics of viscoplastic deformation in amorphous solids", *Phys. Rev. E* **57** (1998), p. 7192-7205.

[86] M. L. Falk, J. S. Langer, "Deformation and failure of amorphous solidlike materials", *Condens. Matter Phys.* **2** (2011), p. 353-373.

[87] A. Ghosh, Z. Budrikis, V. Chikkadi, A. L. Sellerio, S. Zapperi, P. Schall, "Direct observation of percolation in the yielding transition of colloidal glasses", *Phys. Rev. Lett.* **118** (2017), article no. 148001.

[88] R. Vacher, E. Courtens, M. Foret, "Anharmonic versus relaxational sound damping in glasses. ii. Vitreous silica", *Phys. Rev. B* **72** (2005), article no. 214205.

[89] F. López Jiménez, N. Triantafyllidis, "Buckling of rectangular and hexagonal honeycomb under combined axial compression and transverse shear", *Int. J. Solids Struct.* **50** (2013), p. 3934-3946.

[90] Q.-S. Nguyen, *Stabilité et mécanique non-linéaire*, Hermes Science Publications, 2000.

[91] R. Tounsi *et al.*, "Numerical investigation, experimental validation and macroscopic yield criterion of al5056 honeycombs under mixed shear-compression loading", *J. Impact Eng.* **108** (2017), p. 348-360.

[92] B. Coasne *et al.*, "Poroelastic theory applied to the adsorption-induced deformation of vitreous silica", *J. Phys. Chem. B* **118** (2014), p. 14519-14525.

[93] Z. Budrikis, S. Zapperi, "Avalanche localization and crossover scaling in amorphous plasticity", *Phys. Rev. E* **88** (2013), article no. 062403.

[94] E. Lerner, S. Karmakar, I. Procaccia, "Statistical physics of the yielding transition in amorphous solids", *Phys. Rev. E* **82** (2010), article no. 055103(R).

[95] I. Regev, J. Weber, C. Reichhardt, K. A. Dahmen, T. Lookman, "Reversibility and criticality in amorphous solids", *Nat. Commun.* **6** (2015), article no. 8805.

[96] G. P. Shrivastav, P. Chaudhuri, J. Horbach, "Yielding of glass under shear: A directed percolation transition precedes shear-band formation", *Phys. Rev. E* **94** (2016), article no. 042605.

[97] S. Roux, A. Hansen, "Perfect plasticity in a random medium", *J. Phys. II France* **2** (1992), p. 1007-1021.

[98] J. D. Eshelby, "The elastic field outside an ellipsoidal inclusion", *Proc. R. Soc. Lond. A* **252** (1959), no. 1271, p. 561-569.

[99] B. Tyukodi, D. Vandembroucq, C. E. Maloney, "Avalanches, thresholds, and diffusion in mesoscale amorphous plasticity", *Phys. Rev. E* **100** (2019), article no. 043003.

[100] A. Nicolas, E. E. Ferrero, K. Martens, J.-L. Barrat, "Deformation and flow of amorphous solids: a review of mesoscale elastoplastic models", *Rev. Mod. Phys.* **90** (2018), article no. 045006.

[101] J.-C. Baret, D. Vandembroucq, S. Roux, "An extremal model of amorphous plasticity", *Phys. Rev. Lett.* **89** (2002), article no. 195506.

[102] K. Martens, L. Bocquet, J.-L. Barrat, "Spontaneous formation of permanent shear bands in a mesoscopic model of flowing disordered matter", *Soft Matter* **8** (2012), p. 4197-4205.

[103] Z. Budrikis, D. Fernandez Castellanos, S. Sandfeld, M. Zaiser, S. Zapperi, "Universal features of amorphous plasticity", *Nat. Commun.* **8** (2017), article no. 15928.

[104] M. Seleznev, A. Vinogradov, "Shear bands topology in the deformed bulk metallic glasses", *Metals* **10** (2020), article no. 374.

[105] A. D. S. Parmar, S. Kumar, S. Sastry, "Strain localization above the yielding point in cyclically deformed glasses", *Phys. Rev. X* **9** (2019), article no. 021018.

[106] W.-T. Yeh *et al.*, "Glass stability changes the nature of yielding under oscillatory shear", *Phys. Rev. Lett.* **124** (2020), article no. 225502.

[107] A. Le Bouil, A. Amon, S. McNamara, J. Crassous, "Emergence of cooperativity in plasticity of soft glassy materials", *Phys. Rev. Lett.* **112** (2014), article no. 246001.

[108] C. A. Schuh, T. C. Hufnagel, U. Ramamurty, "Mechanical behavior of amorphous alloys", *Acta Mater.* **55** (2007), p. 4067-4109.

[109] P. Thamburaja, "Length scale effects on the shear localization process in metallic glasses: A theoretical and computational study", *J. Mech. Phys. Solids* **59** (2011), no. 8, p. 1552-1575.

[110] L. Li, E. R. Homer, C. A. Schuh, "Shear transformation zone dynamics model for metallic glasses incorporating free volume as a state variable", *Acta Mater.* **61** (2013), p. 3347-3359.

[111] D. Vandembroucq, S. Roux, "Mechanical noise dependent aging and shear banding behavior of a mesoscopic model of amorphous plasticity", *Phys. Rev. B* **84** (2011), article no. 134210.

[112] M. Popović, T. W. J. de Geus, M. Wyart, "Elasto-plastic description of brittel failure in athermal amorphous materials during quasistatic loading", *Phys. Rev. E* **98** (2018), article no. 040901.

[113] M. Ozawa *et al.*, "Random critical point separates brittle and ductile yielding transitions in amorphous materials", *Proc. Natl Acad. Sci. USA* **115** (2018), p. 6656-6661.

[114] A. Barbot, M. Lerbinger, A. Lemaître, D. Vandembroucq, S. Patinet, "Rejuvenation and shear banding in model amorphous solids", *Phys. Rev. E* **101** (2020), article no. 033001.

[115] K. A. Dahmen, Y. Ben-Zion, J. T. Uhl, "Micromechanical model for deformation in solids with universal predictions for stress–strain curves and slip avalanches", *Phys. Rev. Lett.* **102** (2009), article no. 175501.

[116] F. Varnik, L. Bocquet, J.-L. Barrat, "A study of the static yield stress in a binary Lennard–Jones glass", *J. Chem. Phys.* **120** (2004), p. 2788-2801.

[117] L. Berthier, J.-L. Barrat, "Nonequilibrium dynamics and fluctuation–dissipation relation in a sheared fluid", *J. Chem. Phys.* **116** (2002), article no. 6228.

[118] M. Ozawa, M. Singh, L. Berthier, "Brittle yielding of amorphous solids at finite shear rates", *Phys. Rev. Mater.* **4** (2020), article no. 025603.

[119] F. F. Wua, Z. F. Zhang, S. X. Mao, "Size-dependent shear fracture and global tensile plasticity of metallic glasses", *Acta Mater.* **57** (2009), p. 257-266.

[120] Y. M. Beltukov, D. A. Parshin, V. M. Giordano, A. Tanguy, "Propagative and diffusive regimes of acoustic damping in bulk amorphous material", *Phys. Rev. E* **98** (2018), article no. 023005.

[121] A. Tanguy, P. Chen, T. Chaise, D. Nélias, "Shear banding in a contact problem between metallic glasses", *Metals* **11** (2021), article no. 257.

[122] S. M. Fielding, M. E. Cates, P. Sollich, "Shear banding, aging and noise dynamics in soft glassy materials", *Soft Matter* **5** (2009), p. 2378-2382.

[123] J. Luo, P. J. Lezzi, K. Deenamma Vargheese, A. Tandia, J. T. Harris, T. M. Gross, J. C. Mauro, "Competing indentation deformation mechanisms in glass using different strengthening methods", *Front. Mater.* **3** (2016), article no. 52.

[124] C. Su, L. Anand, "Plane strain indentation of a Zr-based metallic glass: Experiments and numerical simulation", *Acta Mater.* **54** (2006), p. 179-180.

[125] Y. Shi, M. L. Falk, "Stress-induced structural transformation and shear banding during simulated nanoindentation of a metallic glass", *Acta Mater.* **55** (2007), p. 4317-4324.

[126] E. Agoritsas, R. García-García, V. Lecomte, L. Truskinovsky, D. Vandembroucq, "Driven interfaces: From flow to creep through model reduction", *J. Stat. Phys.* **164** (2016), p. 1394-1428.

[127] A. Tanguy, T. Vettorel, "From weak to strong pinning: A finite-size study", *Eur. Phys. J. B* **38** (2004), p. 71-82.

[128] F. R. N. Nabarro, *Theory of Crystal Dislocations*, Clarendon Press, Oxford, 1967.

[129] L. Zhao *et al.*, "Simultaneous improvement of plasticity and strength of metallic glasses by tailoring residual stress: Role of stress gradient on shear banding", *Mater. Des.* **197** (2021), article no. 109246.

[130] R. T. Qu *et al.*, "Macroscopic tensile plasticity of bulk metallic glass through designed artificial defects", *Mater. Sci. Eng. A* **534** (2012), p. 365-373.

[131] E. Lerner, S. Karmakar, I. Procaccia, "Plasticity-induced anisotropy in amorphous solids: The bauschinger effect", *Phys. Rev. E* **82** (2010), article no. 026104.

[132] S. Patinet, A. Barbot, M. Lerbinger, A. Vandembroucq, A. Lemaître, "Origin of the Bauschinger effect in amorphous solids", *Phys. Rev. Lett.* **124** (2020), article no. 205503.

[133] E. R. Homer, D. Rodney, C. A. Schuh, "Kinetic Monte Carlo study of activated states and correlated shear-

transformation-zone activity during the deformation of an amorphous metal”, *Phys. Rev. B* **81** (2010), article no. 064204.

[134] P. Cao, M. P. Short, S. Yip, “Potential energy landscape activations governing plastic flows in glass rheology”, *Proc. Natl Acad. Sci. USA* **17** (2019), p. 18790-18797.

[135] N. S. Shcheblanov, B. Mantisi, P. Umari, A. Tanguy, “Detailed analysis of plastic shear in the Raman spectra of SiO_2 glass”, *J. Non-Cryst. Solids* **6** (2015), p. 6-19.

[136] G. Kapteijns, D. Richard, E. Lerner, “Nonlinear quasilocalized excitations in glasses: True representatives of soft spots”, *Phys. Rev. E* **101** (2020), article no. 032130.

[137] H. Luo, “High frequency thermomechanical study of heterogeneous materials with interfaces”, PhD Thesis, Université de Lyon, 2020.